SECTION III

CHAPTER XIII


Art. 119.—Hydraulic Prime Movers, Water-Wheels.

Although the question of the utilization of natural water powers has always been one of great economic importance, the introduction and perfection of electrical manufacturing processes, and the possibility of transmitting electrical energy without great loss or expense to a great distance from the place of its generation, has of recent years made it practicable to take advantage of many water powers far remote from large centres of industry and has raised the whole question to an altogether higher plane of importance, while the consequent demand for hydraulic prime movers capable of developing large powers in single units, and of satisfying the exhaustive demands of such installations in the way of speed regulation and efficiency, has led to a great transformation in the design of such motors.

The first hydraulic prime mover consisted of a wooden paddle water-wheel dipping into the current of a stream, and as such a motor was only required to do the work previously performed by an animate agency, the power required was small and the efficiency of only secondary importance.

The construction was at first of the most primitive type, but was gradually improved; iron took the place of wood; improvements in design led to increased efficiency; the demand for greater powers led to the necessity for utilizing larger falls and the consequent development of the breast and overshot wheels, until a type of wheel was evolved, which within its limitations was as efficient as the most modern of turbines. Its chief disadvantages lay in its slow speed of rotation, the impossibility of close speed regulation, and in the large size of wheel required for even small powers; and while for the purposes for which the motor was first required these were not serious, the introduction of more modern machinery, more particularly for textile purposes, involved the necessity for a motor which, having a fairly high speed of rotation in order to avoid
HYDRAULIC PRIME MOVERS

excessive loss in gearing, should be capable of close speed regulation and of taking advantage of higher falls and of large quantities of water. For such purposes the water-wheel was almost entirely superseded by one or other type of turbine.

The introduction of electric driving with its large and almost instantaneous changes of load, while giving an additional fillip to the manufacture of high-speed turbines, had its greatest effect in modifying and perfecting the methods of speed regulation, and in increasing the size of the unit, while at the same time rendering it imperative to design a motor which should be highly efficient under a wide range of loads.

The success which has attended the attempt to satisfy these onerous conditions may be inferred when it is remembered that many manufacturers will now guarantee to construct a turbine which shall give an efficiency of over 80 per cent. over a range of loads of 50 per cent., and which shall respond to an increased demand for power of 38 per cent. with less than 3.5 per cent. variation in speed. With smaller load variations the speed variation is almost infinitesimal, and it becomes easy to run a series of alternating current machines in step with such motors.

The design of hydraulic motors has thus proceeded by well-defined stages, the size and efficiency steadily increasing until at the present time a single unit developing 15,000 H.P. and giving an efficiency of 85 per cent. is not at all uncommon, while further development promises to proceed in the direction of still larger units. So far, indeed, as mechanical difficulties affect the question, there appears to be no reason why units developing up to at least 25,000 H.P. should not be constructed directly the demand arises.

Wherever a continuous supply of water at a sufficient elevation, or in motion as in a stream, is available, the potential or kinetic energy which this possesses may be turned into useful work.

Before embarking on any power scheme for utilizing such energy, it is however of the highest importance that the true possibilities of the scheme should be ascertained, for as the usefulness of the supply depends in most cases on its uniformity over long periods of time, the maximum available power is strictly regulated by the least power which is available after the longest probable period of drought.

This minimum supply can only be satisfactorily ascertained by investigation of past records extending over many years. Where such records are not available, every attempt should be made by a close investigation of the rainfall records for the particular districts over a long period of years, and of the character, condition, and area of the gathering ground.
to estimate the minimum supply likely to be available under the worst probable combination of circumstances, and the scheme may then, and not until then, be developed.

The method of utilizing the supply depends largely on its magnitude, form, and locality. Where, though comparatively small, it is continuous, the available horse-power may be largely increased by the formation of a storage reservoir capable of impounding at least a 24 hours' inflow. By this means energy may be utilized for the eight hours or so comprising a working day at a rate greatly in excess of the mean rate of inflow.

Where the natural configuration of the country necessitates the power plant being placed at some distance from the storage reservoir, the supply is usually led through an open canal or ditch having a slight gradient, into a smaller storage reservoir termed the forebay, which is placed as near to the power plant as possible. From the forebay the supply is then taken to the prime mover by means of a closed pipe termed the penstock.

In the case of a water-wheel installation the penstock may consist of an open channel.

The supply of water to the prime mover is regulated by means of sluices or gates, which may either form an integral part of the machine, as in the case of most turbines, or may be fitted in the supply pipe or channel.

After doing work, the water is rejected into a discharge channel termed the tail-race.

The most suitable type of prime mover for any particular case depends on—

1. The quantity of water available.
2. The supply head.
3. The regularity of flow.
4. The possibility of floods.
5. The purpose for which power is required.

Those types in general use consist of—

1. Water-wheels.
2. Turbines.
3. Piston engines.

Each has its own sphere of usefulness, and in determining the type to be adopted each installation demands special consideration, guided by the circumstances peculiar to the case.

In general, the water-wheel is only suitable for small powers and for comparatively low heads and where close speed regulation is not essential. Its efficiency is greatly affected by a variation in the supply and in the
head or tail-race levels. It is of great size and weight in proportion to the power developed, and has a low rotative speed. On the other hand, its construction is simple, its repair inexpensive and easy, and the construction of the supply channel, tail-race, and housing in general inexpensive, while for heads of less than 1 foot it forms the only suitable type of motor.

For all heads above 1 foot, where large power is desired, one or other type of turbine becomes suitable, while in certain cases for comparatively small powers, and where a high head is available and a slow rate of rotation is required, the piston engine is most satisfactory.

In passing through a prime mover, water may do work either by changing potential energy or kinetic energy or pressure energy into work; or by a combination of these processes.

In the overshot water-wheel, for example (Art. 120), rotation is produced almost entirely by the weight of the water; in impulse wheels deriving their motive force from the impact of a high velocity jet of water, work is done solely in virtue of the kinetic energy of the jet; in turbines of the reaction type the pressure energy of the water is partly changed into kinetic energy in the wheel itself, this being absorbed in producing rotation of the wheel; while in a piston engine the water does work in virtue of its pressure, its velocity being so small as to be negligible.

In designing any type of hydraulic prime mover, certain general principles should be borne in mind.

(a) All shock, whether of water on moving or stationary surfaces, or on water moving with a low velocity, should be avoided as being productive of loss of energy in eddy formation.

This may be prevented by arranging that as far as possible any stream of water on meeting a solid surface is moving tangentially to the surface, and that passages conveying the working fluid are not subject to abrupt changes of sectional area or of form.

(b) Abrupt changes in the direction of motion are productive of eddy formation and should be avoided by designing passages and channels with as far as possible an uniform or gradually changing curvature.

(c) Frictional losses should be reduced to a minimum by reducing the area of the wetted surfaces to a minimum compatible with easy curves of flow, and by reducing the relative velocity of flow over such surfaces to a minimum.

(d) As far as possible, the motive fluid should be rejected devoid of energy, and therefore moving with as low an absolute velocity as will suffice to carry it out of the motor.

The possibility of conforming to these general principles varies with the
type of motor. In general, apart from mechanical friction, water-wheels suffer chiefly from the causes outlined in sections (a), (b), and (d); turbines from those in sections (a), (c), and (d); while losses due to shock and eddy production, (a) and (b), are all-important in piston engines.

These types of prime mover will now be considered somewhat in detail.

Art. 120.—The Overshot Water-Wheel (Fig. 185).

In this type of wheel water is supplied near the highest point of the circumference to a series of buckets formed by vanes connected at each end to circular shrouds, the bottoms of the buckets being formed by the inner circumference of the wheel. For convenience of construction the vanes are often made of wood and in two parts, the inner part being radial and the outer inclined to this at an angle depending on the speed of the wheel and the velocity of the supply stream.
A preferable form of bucket is that indicated at P (Fig. 185). Here the vanes, usually of metal, are made in a single piece, and have a continuous curvature throughout.

**Theory of Action.** -Let \( H \) be the total head available. A certain proportion of this head must first be utilized in giving the supply water sufficient velocity to carry it into the wheel buckets. If \( h_1 \) is this head, the velocity will then be given by \( v = \sqrt{2g h_1} \).

A small clearance must be allowed between the highest point of the wheel and the bottom of the inlet channel, this clearance \( \delta_1 \) being usually about 1 inch.

A rather larger clearance \( \delta_2 \) must in general be allowed between the lowest point of the wheel and the level in the tail-race, so as to prevent submergence of the wheel buckets in time of flood.

In general \( \delta_2 \) is about 6 inches, but depends largely on the special circumstances of the plant.

Finally, a certain proportion \( h_2 \) of the head must be devoted to producing velocity of flow along the tail-race, so as to give a free discharge from the wheel.

The outer diameter of the wheel buckets is thus limited to \( H - (h_1 + h_2) - (\delta_1 + \delta_2) \).

Let \( R = \) outer radius of wheel.

" \( r = \) inner "

Then \( 2R = H - (h_1 + h_2) - (\delta_1 + \delta_2) \).

The depth of buckets depends on the diameter, breadth and velocity of wheel, and on the quantity of water to be utilized.

Let \( Q = \) quantity of water per second in cubic feet.

" \( \omega = \) angular velocity of wheel in radians per second.

" \( b = \) breadth of wheel,

and since the buckets are never completely filled with water, let \( x = \) fraction of bucket volume occupied by water. Generally \( x \) lies between \( \frac{1}{2} \) and \( \frac{3}{8} \).

Then we have, neglecting the volume occupied by the wheel vanes,

\[ Q = \frac{x (R^2 - r^2)}{2} \pi b \omega \] cubic feet.

\[ \therefore \] \[ Q = \frac{x (R^2 - r^2)}{2} \omega b \] cubic feet per second.

\[ \therefore \] \[ r = \sqrt{R^2 - \frac{2Q}{\omega b x}} \] giving \( r \), and therefore \( R - r \), the depth of the buckets.

To avoid excessive loss under low heads this should be as small as practicable.
Efficiency of Wheel.—Soon after the buckets pass the centre line of the wheel they begin to empty, at \( A \) (Fig. 185), while they are completely emptied by the time they reach the position \( B \), where the outer part of the bucket is horizontal. If then \( h \) is the mean vertical distance through which the water is carried before being discharged, the work done in virtue of its weight = \( WQh \) foot lbs. per second.

It may readily be shown that the efficiency is a maximum when the peripheral velocity is one half that of the inflowing stream. A considerable increase in speed has, however, very little effect on the efficiency. The most important effect of such an increase is due to the tendency to a premature emptying of the buckets by increased centrifugal action.

To prevent loss by splash and shock at entrance to the buckets, the vane angles should be so arranged that the relative motion of water and vane at entrance is parallel to the tip of the vane. Thus if \( ab \) (Fig. 185) represents the velocity \( r \) in direction and magnitude, and if \( bc \) represents the linear velocity, \( \omega L \) of the vane, then \( ac \) represents the velocity of the water relative to the vane, and for the water to enter by sliding along the vane this should be parallel to \( ac \) at entrance.

The general practice is to form the bucket tips so as to make an angle of 25° to 80° with the tangent to the circumference at the tip. This gives a bucket which retains the water for a vertical distance equal to about 8 of the wheel diameter.

If this angle be \( \beta \), the bucket is not completely emptied until the wheel has turned through 180° — \( \beta \).

Having drawn in the profile of the buckets, if a straight line \( df \) (Fig. 185) be drawn through the bucket tip so as to enclose an area \( def \times \) bucket area), the water will begin to escape from the bucket when \( e \) becomes horizontal. When \( fc \) becomes horizontal the bucket is completely emptied. Thus with radial vanes the water is entirely emptied from a bucket by the time it has fallen to the level of the wheel axle.

Water may be retained in the buckets until these are near the lowest
point of the wheel, by enclosing the discharge side of the wheel below A in a closely fitting shrouding, with as small a clearance as possible (Fig. 185a), and by this means the efficiency may be increased. This method has the further advantage that the relative velocity of discharge is now increased by the head of water in the buckets, and as the direction of this motion is opposite to that of the wheel, the absolute velocity of discharge, and therefore the kinetic energy rejected, is reduced.

Effect of Varying the Tail-Race Level.—If the tail-race level rises so that the wheel is partially submerged, considerable resistance to rotation is caused if the vanes move in the opposite direction to that of flow in the tailrace. By reversing the direction of inflow, however (Fig. 186), the direction of the tail-race flow becomes the same as that of the buckets, and these may now become partially submerged without serious loss of efficiency.

Further advantage may be taken of the fall by arranging a masonry breastwork A (Fig. 187) to fit the wheel closely, the tail-race level being now slightly below the level at which each bucket finally empties itself. The compartment B is kept dry either by draining into a sump where practicable, or by means of a small pump driven by the wheel.

To avoid running at inconveniently low speeds either of two expedients may be adopted. The first is to run the wheel at a speed from 50 to 100 per cent. in excess of that giving maximum efficiency, when, since velocities are small, the amount of energy wasted by shock at entrance is not considerable.

The second expedient consists in giving $h_1$ a larger value, and in taking
the supply by means of a closed guide passage of suitable depth out of the bottom of an open forebay (Fig. 188).

The velocity of efflux is then increased, and the velocity of rotation may be increased proportionately. The diameter of the wheel is reduced, and also its width for a given volume of water. A cheaper wheel, and one rotating at a more convenient speed for transmission purposes is thus obtained. The efficiency is, however, slightly reduced, since losses due to shock and friction are increased by the higher velocities, as are the losses due to rejection of kinetic energy in the tail water, while in addition, the increased centrifugal action, tends to empty the buckets sooner. The latter action, however, while reducing the efficiency of the wheel, tends to make it, to a certain extent self-governing under variable loads, since a diminution in load, followed by an increase in speed, tends to empty the buckets.
The following results of experiments by Smeaton indicate to what extent the efficiency is lowered by increasing the proportion of the total head absorbed in giving velocity energy to the supply water.

<table>
<thead>
<tr>
<th>Diameter of Wheel \ Total Head $H$</th>
<th>Proportion of Total Head absorbed in giving Velocity to supply Water</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>.90</td>
<td>.10</td>
<td>.73</td>
</tr>
<tr>
<td>.84</td>
<td>.16</td>
<td>.69</td>
</tr>
<tr>
<td>.80</td>
<td>.20</td>
<td>.66</td>
</tr>
<tr>
<td>.73</td>
<td>.27</td>
<td>.62</td>
</tr>
<tr>
<td>.68</td>
<td>.32</td>
<td>.59</td>
</tr>
</tbody>
</table>

Regulation is usually performed by a sluice governing the discharge from the penstock.

The overshot wheel is well suited for small powers and heads ranging from about 15 to 50 feet, and when working under suitable conditions gives efficiencies up to about 80 per cent. As the head diminishes, the larger proportional loss of head necessitated by the depth of the buckets and by the clearances $\delta_1$ and $\delta_2$ renders the wheel less efficient, and for heads between 15 feet and 6 feet the breast wheel becomes more suitable.

**Art. 121.—The Breast Wheel (Fig. 189).**

Here the wheel itself is almost identical with the overshot. The principles of its construction and of the design of its buckets are the same, but water is now admitted to the buckets at some point in the breast of the wheel.

As before, the supply may be brought to the wheel either in an open supply channel under a comparatively low head not exceeding 1 foot, in which case the supply is led on below the wheel centre, or by means of a closed supply pipe under a greater head, when the supply is led on above the centre. The general arrangement in each case is indicated in Fig. 189 a and b. The water is prevented from escaping from the buckets before reaching the bottom of the wheel by means of a breastwork of masonry, the clearance between the wheel and masonry being reduced to the minimum possible, usually about $\frac{3}{8}$ inch. The necessity for this breast-work renders the wheel more expensive than the overshot. Regulation is performed by throttling the supply by means of sluices arranged as indicated in Figs. 189 a and b.
As in the case of the overshot wheel, the maximum efficiency is obtained when the bucket angles are arranged so as to give entry without shock and when the peripheral speed of the wheel is one half \( v \cos a \). Here \( a \) is the angle which the direction of the inflowing stream makes with the tangent to the wheel at the point of entry.

For maximum efficiency the level in the tail-race should be the same as that in the buckets at discharge. In this type of wheel, on account of the manner of filling the buckets, special provision must be made for letting air out as the water rushes in. To this end air vents are usually formed in the inner circumference of the buckets. The wheel is capable of more accurate speed regulation under varying heads than is the overshot, and its efficiency under favourable circumstances may be as high as 65 per cent.

Art. 122.—The Side Wheel.

Where the fall is between 3 feet and 6 feet, the breast wheel becomes unsuitable because of the smallness of its diameter, and in such a case the
Sagebiem wheel may be adopted. This wheel (Fig. 190) has buckets formed by a series of flat vanes, which are tangential to a circle concentric with the wheel itself. The buckets are open top and bottom, and are of comparatively great depth. The water enters the buckets with a velocity sensibly the same as that in the approach channel, the vane angles being determined as indicated in the figure, so that the vanes enter the water without shock.

A circular casing is provided in which the wheel works with little clearance, and which connects the head and tail-race. Then, neglecting leakage between this casing and the vanes, each bucket retains its supply until it passes its lowest point of the wheel, after which communication is made with the tail-race, and the level in the bucket falls to that of the tail-race water. For maximum efficiency the wheel should be designed so that the level in the bucket on reaching the bottom of the wheel is the same as that in the tail-race.

If \( v \), represents the relative velocity of water and bucket at entrance, the water will rise initially to a height in the bucket above that in the head-race, this height being given by \( \frac{v^2}{2g} \), and the depth of bucket should be such that in time of flood this does not cause flow to take place over the inner end of the buckets.

In this wheel the velocity of rotation is proportional to the flow, and the wheel is thus capable of dealing with large quantities of water. It is however, unfitted for driving a variable load, since an increase in load, by
causing a diminution in speed, reduces the flow through the wheel, and thus reduces the energy supply when it is most needed. Its peripheral velocity is low, and on that account it is well adapted for driving a pumping plant or milling machinery where the load is uniform and where high velocities are not required.

The chief losses occur during the emergence of the vanes from the tailrace. Owing, however, to the slow speed of the wheel, all hydraulic losses are low and an efficiency of up to 80 per cent. may be obtained under favourable circumstances, though the large size of the wheel and its comparatively costly construction very largely counterbalance the advantages of high efficiency.

**Art. 123.—The Undershot Wheel.**

For heads below 3 feet the undershot wheel is preferable. In its simplest form this consists of a wheel carrying a series of flat radial vanes around its circumference (Fig. 191). These, dip into the water flowing either through an open channel or through a penstock of slightly greater width than the wheel, and are arranged so as to clear the bottom of the penstock by about \( \frac{1}{4} \) inch. In some cases, the penstock itself is curved to fit the wheel, and leakage past the buckets, in the interval between successive buckets arriving at the lowest point of their path, is then largely prevented.

In this wheel, work is done solely in virtue of the kinetic energy of the moving stream, the force on the moving vanes being due to the change of momentum produced in the stream by their presence.

For maximum work the peripheral velocity should be one half the velocity of flow of the stream, in which case the theoretical efficiency is .5.

Owing to mechanical friction and hydraulic losses this efficiency is, however, reduced to about 35 per cent., and the maximum efficiency is obtained with a value of \( v_p \), slightly less than \( \frac{1}{4} v \) (Fig. 191).
ART. 124.—THE PONCELET WHEEL.

If the vanes of an undershot wheel, instead of being radial, are inclined backward so as to make an angle $\beta$ at the tips with the tangent to the circumference, and if $\alpha$ be the angle between the direction of the approach stream and this tangent, then by suitable adjustment of $\alpha$, $\beta$, and the speed of rotation, the loss due to shock at entrance may be prevented, and at the same time the discharge water may be given a backward velocity relative to the wheel, thus reducing the absolute velocity of discharge and the loss by rejection of kinetic energy to the tail-race.

With flat vanes the best results are obtained when $\beta$ is about $80^\circ$, and efficiencies of up to 55 per cent. may then be obtained.

In the Poncelet wheel (Fig. 193) these vanes, instead of being flat, form arcs of circles, and with this type of wheel efficiencies of from 60 to 70 per cent. are usual.

To determine the correct vane angles, let

$AB$ (Fig. 192 a) represent the absolute velocity of water at entrance.

$CB$ represent the absolute velocity of vanes.

Then $AC$ represents the velocity of water relative to vanes, and to avoid shock, the vane tips should be parallel to $AC$.

Again, at the discharge point, let

$B'C'$ = velocity of vanes.

$C'A'$ = velocity of water relative to the vanes.

Then $B'A'$ = absolute velocity of discharge.
Let \( v = \) initial velocity of water.

\[ v_f = \text{final} \quad v_r = \text{relative} \quad u = \text{velocity of buckets}. \]

On entering the buckets the relative velocity is \( v_r \), and in virtue of this the water will rise to a height approximately equal to \( \frac{v_r^2}{2g} \) feet above its normal level.

It then falls through this height relative to the wheel under the action of gravity, so that on leaving the wheel its relative velocity will again be approximately equal to \( v_r \). This assumption neglects the effect of friction and of eddy formation in the buckets, both of which tend to make the final less than the initial relative velocity. Assuming, however, for simplicity that \( v_r \) is the same at inlet and at discharge, we have

\[ AC = C'A' = v_r \]
\[ CB = B'C' = u. \]

For \( v_f \) to be as small as possible with a given value of \( v_r \), evidently \( A'B' \) should be perpendicular to \( B'C' \).

The two diagrams may now be combined graphically so as to give the most suitable angles \( \alpha \) and \( \beta \) by making \( A'C' \) coincide with \( AC \).

Thus, draw \( cd \) (Fig. 192) \( = u \) and produce \( bc \) to \( b' \), making \( b'c = cb \). From \( b' \) draw a perpendicular to \( b'b' \), and with \( b \) as centre describe an arc of a circle with \( v \) as radius to cut this at \( a \). Join \( ac \).

Then \( ec = \sqrt{v_r} \); \( \overrightarrow{ad} = a \angle acb' = \beta \).

From the figure it is evident that for minimum loss of kinetic energy at discharge we must have \( v \cos \alpha = 2u \).

\[ \therefore \quad u = \frac{v}{2} \cdot \cos \alpha. \]

In practice \( \alpha \) is usually made about \( 15^\circ \).

\[ \therefore \quad \cos \alpha = 0.965; \quad \therefore \quad u = 0.488 v. \]

The theoretical efficiency is given by

\[ \eta = \frac{WQ}{g \left\{ v^2 - v_f^2 \right\}} \]
\[ \therefore \quad \eta = \frac{WQ}{g \left\{ v^2 \right\}} \]

But from the figure we have

\[ v^2 - v_f^2 = 4u^2 \]

\[ \therefore \quad \eta = \frac{4u^2}{v^2} = \cos 2\alpha \]
PONCELET WHEEL

::: With \( a = 15^\circ \), the maximum theoretical efficiency is \( 93 \).

Again (Fig. 192),

\[
\frac{v}{2} \cos a \sin \beta = v \sin (\beta - a)
\]

\[
\therefore \frac{v}{2} \cos a \sin \beta = v \sin (\beta - a)
\]

\[
\therefore \cot \beta \tan a = \frac{1}{2}
\]

\[
\therefore \tan \beta = 2 \tan a
\]

If

\[
a = 15^\circ, \tan \beta = 5558
\]

\[
\therefore \beta = 28^\circ 2'
\]

Actually, because of hydraulic losses caused by relative motion of the mass of water in the buckets, and which are proportional to \( v_r^2 \), it is found advisable to diminish the relative velocity slightly by giving the vanes a velocity varying from \( .5 \) to \( .6 \) \( v \). Under these circumstances the maximum working efficiency is obtained.

The construction of the wheel is substantially as shown (Fig. 193). The buckets are open both at their inner and outer circumferences, and to prevent water at impact from flowing into the inner part of the wheel, the depth of buckets should be not less than \( h + \frac{v_r^2}{2g} \), where \( h \) = thickness of stream. In practice \( h \) should not exceed 9 inches.

But

\[
r_r = \frac{n \sin a}{\sin (\beta - a)}
\]

\[
\therefore \text{taking} \ a = 15^\circ, \beta = 30^\circ \ (\text{approx.}), \text{we have} \ r_r = n = .55 \ v \ (\text{approx.})
\]

and taking \( v^2 = 2g \ H \), where \( H \) is the supply head, we have

Depth of buckets = \( .3 \frac{v_r^2}{2g} + h \)

\[
= .3 \ H + h.
\]

In practice the depth is usually taken as \( \frac{H}{3} + h \).

The spacing varies so as to give about forty-eight buckets in the circumference, the maximum spacing being about 16 inches. The most suitable arc of water contact is about 30°. Since at all points of this arc the direction of the approach stream should make an angle \( \beta \) with the tangent to the circumference, the approach channel is not straight but has a bed curved so as to fulfil this condition.

To draw this curve, let \( K \) and \( M \) (Fig. 198) be the extremities of the arc of contact, and from \( K \) and \( M \) draw \( K \ O \) and \( M \ O \) perpendicular to the required directions of the stream, i.e., making an angle \( a \) with the normals at \( K \) and \( M \), and intersecting in \( O \). Then an approach bed,
having as profile the arc described with centre $O$, and passing through $M$, will give the required inclination at $K$ and $M$, and approximately for all points between.

Regulation is performed by an adjustable sluice $S$, whose position is regulated by some form of centrifugal or float governor. In order to prevent loss by leakage past the vane tips the supply bed is extended to form a circular lip closely fitting the wheel. This should be no longer than is necessary to give contact with two vanes at once. If made too long, the water on flowing down the vanes, instead of freely escaping with relative velocity $v_r$, meets this lip, its relative velocity is destroyed, and on

emerging from the lip it escapes with the forward velocity of the vane as in the ordinary type of undershot wheel.

In common with all impulse wheels, for efficient working the vanes must not be submerged, and the level of the supply bed must be sufficiently above that of the tail-race to obviate any flooding. This wheel is well adapted for heads between 4 and 7 feet. Its speed is fairly high, and the consequent fly-wheel effect renders it easy of regulation under variable loads.

Its part-gate efficiency is high if the head be kept fairly constant, but with very variable heads, owing to the variation in velocity of approach, its efficiency is not so good.
ART. 125.—THE PELTON WHEEL.

The Pelton wheel (Fig. 194) is the only form of water-wheel which is adopted for use with high heads, and where a limited supply of water under such a head is available it often forms the most suitable type of prime mover. In such a case the turbine proper, with the exception of the Girard type (Art. 129), is unsuitable, as will be seen later (Art. 128), while where the supply water is charged with sand or similar matter in suspension, as is not unusual, the Pelton wheel, on account of the simplicity of its construction and of the ease with which its buckets can be renewed, has manifest advantages over the Girard turbine. The pressure water is supplied through a pipe line terminating in one or more nozzles which play on to a series of buckets fixed around the periphery of the wheel.

The latter is a development of the old hurdy-gurdy of the Pacific Slope. This consisted of a wheel having a rim to which a series of flat plates were fixed radially, the jets from one or more nozzles impinging freely on these and causing rotation. Under these conditions, the theoretical efficiency cannot exceed 50 per cent. (p. 378), while in practice loss by splashing, friction, etc., reduces this to about 30 per cent.

1 By courtesy of Messrs. Gilbert, Gilkes & Co., Ltd., Kendal.
The first important improvement in the machine was made about 1870, when the flat plates were replaced by hemispherical cups fixed alternately on each side of the centre line of the wheel with their concave sides to the jet. This was known as the Cascade wheel (Fig. 195). By this means the jet was deflected backwards, and the theoretical possibilities of the wheel were at once doubled (see p. 378), the maximum theoretical efficiency becoming unity.

The next step was to replace these cups by a series of concave buckets mounted on the centre line of the wheel, fitted with knife-edged ridges to split the jet and having surfaces curved so as to give the jet its backward deflection as smoothly and uniformly as possible. Several types of bucket have been designed with this end in view and some of the more successful are illustrated in Fig. 196 (a, b, and c) and in Fig. 197 (a and b).

While the type of bucket fitted with a lip, as shown in Fig. 196 (b) and in Fig. 197 (a), is common in practice, it does not satisfy the conditions necessary for high efficiency so well as the bucket which omits that portion of the lip in the line of the jet as shown in Figs. 196 (a and c) and 197 (b). The lip and ridge of the bucket deflect the jet in two planes, approximately at right angles, and as the paths of the streams thus formed cross, a certain amount of energy is dissipated by their impact. Furthermore, the lip tends to deflect the jet radially inwards towards the rim of the wheel, in which case some fouling of the succeeding bucket is inevitable. Relative tests of buckets (Fig. 197, a and b) fitted
to a wheel developing 375 B.H.P. under 860 feet head, and having a 2-inch jet on a 35-inch pitch circle, showed an efficiency about 6 per cent. greater with bucket $b$ than with bucket $a$.\(^1\)

In practice an efficiency of unity is impossible of attainment for several reasons.

(1) In order that the discharge from one shall clear the back of the following bucket, the jet cannot be deflected through the full 180°, the actual deflection usually being 160°. Thus kinetic energy is rejected in virtue of the motion of the water parallel to the axis of the wheel at discharge. To prevent this loss becoming large, the buckets should not be spaced too closely together.

(2) The relative velocity of water and bucket at discharge is less than at the point of impact because of skin friction, while windage causes the actual velocity of impact to be less than that theoretically equivalent to the head at the nozzle. Both these causes have the effect of reducing the pressure on the bucket, and to obviate this loss as far as possible the wetted surface of the buckets should be a minimum, and therefore the number of buckets should be as small as is consistent with continuous impact, while they should be made no larger than is necessary to give the required change of direction with easy curves and without shock. Also the surface should be as smooth and well finished as possible.

To reduce windage the jet should be circular in section, since this gives the minimum perimeter per unit area of cross section, while it has the further advantage of being the most stable form of jet. Other forms

---

ultimately tend to the circular, and in doing so tend to become unsteady.

(3) Sharp corners and uneven curves in the buckets cause loss of energy by eddy formation.

(4) Splashes on entering buckets, if unsuitably designed (reduced by reducing the number of buckets).

(5) The jet, being placed tangential to the pitch circle of the buckets, meets and remains in contact with each bucket in turn through some appreciable angle of rotation. The angle at which the jet meets the bucket, and also the angle of discharge, will in consequence vary as the wheel rotates, and it follows that unless the buckets are designed so as to give normal impact on the ridge and a discharge which is tangential to the wheel, and unless the speed of the wheel is regulated so that the backward velocity of discharge is equal to the forward velocity of the buckets, an excess of kinetic energy will be rejected to the tail-race.

Theoretically, assuming the angle of deflection to be 180° and neglecting the effect of friction, the most efficient speed of the buckets is one-half that of the jet (p. 378). When allowance is made for the effect of friction, as on p. 371, the most efficient bucket speed is seen to be slightly less than this. In practice the most efficient bucket speed is found to be from 44 to 48 times that of the jet, the higher ratio being possible with the more efficient buckets.

Efficiency of Pelton Wheel.

Let \( u \) = peripheral speed of buckets at pitch circle.

\[
= \frac{2\pi rN}{60}
\]

where \( r \) = perpendicular distance from axis of jet to centre of wheel; \( N \) = revs. per minute.

Let \( v \) = initial velocity of jet.

Let \( v_2 \) = final absolute velocity.

Let \( v_r \) = relative velocity of jet and bucket at entrance.

Let \( v_r' \) = relative velocity at discharge.

Let \( a \) = mean angle between jet and tangent at point of contact.

Let \( \gamma \) = total angle of deflection of jet.

Then initial velocity of jet in direction of tangent at point of impact \( v \cos a \).

Component, parallel to tangent at discharge, of final velocity relative to bucket \( 2v_r \cos \gamma \).

Absolute velocity in this direction at discharge \( = v + 2v_r \cos \gamma \).
PELTON WHEEL

\[ \frac{1}{g} \left( e \cos \alpha - u - 2v_r \cos \gamma \right). \]

\[ = \frac{u}{g} \left( v \cos \alpha - u - 2v_r \cos \gamma \right) \text{ ft. lbs.} \]

\[ \text{Efficiency} = \frac{u}{gh} \left( v \cos \alpha - u - 2v_r \cos \gamma \right) \text{ ft. lbs.} \]

The loss due to friction and eddies in buckets = \[ \frac{r^2_r - r'^2}{2g} \text{ ft. lbs. per lb} \]

where \( v_r = \sqrt{v^2 + u^2 - 2vu \cos \alpha} \) (Fig. 199).

The loss due to rejection of kinetic energy = \[ \frac{v^2}{2g} \text{ ft. lbs. per lb} \]

where \( v^2 = v^2 + 2v^2 + 2vu \cos \gamma \).

Experiments indicate that in the average wheel \( v_r \) may be as low as from 5 to 6 times \( v_r \). It is certain, however, that in a well-designed bucket, having a ratio of bucket width to jet diameter not less than about 3:8, this ratio is greater, and approximates to 7.5.

Taking \( a = 10^\circ; \gamma = 160^\circ; u = 46 \, v; v_r = 75 \, v_r \); this makes \( r^2_r = 55 \, v \); \( 2v_r = 41 \, v \); and the hydraulic efficiency = 84, a value agreeing closely with the results of the best modern practice.

<table>
<thead>
<tr>
<th>Test Number.</th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
<th>4.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolutions per minute .</td>
<td>302</td>
<td>300</td>
<td>301</td>
<td>299</td>
</tr>
<tr>
<td>Output in kilowatts</td>
<td>962</td>
<td>1724</td>
<td>2296</td>
<td>2527</td>
</tr>
<tr>
<td>Diam. of jet, ins.</td>
<td>4.36</td>
<td>5.61</td>
<td>6.56</td>
<td>6.80</td>
</tr>
<tr>
<td>Mean velocity of jet, f.s.</td>
<td>221.4</td>
<td>222.8</td>
<td>224.2</td>
<td>227.4</td>
</tr>
<tr>
<td>Ratio of bucket to jet velocity</td>
<td>4.06</td>
<td>4.10</td>
<td>4.06</td>
<td>3.99</td>
</tr>
<tr>
<td>Discharge, c.f.s.</td>
<td>22.9</td>
<td>38.2</td>
<td>52.9</td>
<td>57.9</td>
</tr>
<tr>
<td>Nozzle efficiency, per cent.</td>
<td>95.8</td>
<td>96.8</td>
<td>98.2</td>
<td>98.6</td>
</tr>
<tr>
<td>Relative velocity of jet entering bucket .</td>
<td>132.1</td>
<td>134.0</td>
<td>135.1</td>
<td>138.9</td>
</tr>
<tr>
<td>&quot; leaving &quot;</td>
<td>77.5</td>
<td>79.2</td>
<td>65.1</td>
<td>68.9</td>
</tr>
</tbody>
</table>

**Distribution of Energy of Jet.**

| Useful work, per cent. | 74.4 | 74.2 | 70.4 | 68.1 |
| Bucket friction and eddies, per cent. | 23.0 | 23.2 | 27.7 | 29.2 |
| Kinetic energy in discharge, per cent. | 1.1 | 1.0 | 1.8 | 1.9 |
| Other hydraulic losses, due to splash at entrance, changing angle of application of jet, etc. | 1.5 | 1.6 | 1.1 | 0.8 |
HYDRAULICS AND ITS APPLICATIONS

The above table shows the distribution of energy as obtained in a series of tests on a wheel of which the following are the details:

- Number of buckets: 15; angle $\alpha = 4^\circ 41' 5''$.
- Width: 19.5 inches; angle $\gamma = 166^\circ$.
- Projected area: 252 square inches; head, 810 feet.
- Radius of pitch circle: 33.5 inches.
- Outside tip of buckets: 40.5 inches.
- Revolutions per minute: 300.
- Nozzle—needle nozzle, with tip 7.5 inches diameter.

The large value of the bucket losses, particularly in trials 3 and 4, is undoubtedly due partly to the comparatively small ratio of width of bucket to diameter of jet, and partly to the design of bucket, which is shown in Fig. 198.

The following table, showing results obtained in recent tests of Pelton wheels, well illustrates the extent to which the high efficiency is maintained at part loads in a well-designed wheel.

| No. | Number of Nozzles | Fall in Metres | Revolutions per Minute | Efficient Horsepower at Full Gate | $a$ | $b$ | $a$ | $b$ | $a$ | $b$ | $a$ | $b$ | $a$ | $b$ | $a$ | $b$ | $a$ | $b$ |
|-----|------------------|----------------|-----------------------|----------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| X    | 2                | 90             | 395                   | 396                              | 1000| 76.0| 82.3| 79.7| 86.5| 81.6| 74.4| 82.6| 30.6| 84.2| 81.2| 77.0|
| XI   | 2                | 189            | 760                   | 360                              | 1000| 85.6| 85.9| 84.9| 84.9| 84.9| 85.2| 81.2| 87.8| 86.0| 42.0| 87.8| 86.0| 32.0| 77.0|
| XII  | 2                | 590            | 686                   | 376                              | 1000| 85.7| 86.9| 86.7| 86.7| 86.7| 85.5| 82.8| 85.1| 82.8| 27.1| 86.0| 86.0| 14.2| 75.9|
| XIII | 1                | 860            | 3,710                 | 680                              | 1000| 86.4| 86.9| 86.4| 86.4| 86.4| 85.1| 81.2| 81.2| 46.7| 81.2| 81.2| 22.2| 75.9|

Form of Buckets.—Where the dividing ridges of the buckets are straight in profile, these are not fixed radially but are inclined backwards from the direction of rotation at such an angle as to give normal incidence on the first impact of the jet. If placed radially the jet would be deflected into the rim of the wheel during the first half of the period of impact and would tend to produce serious inefficiency.

A type of construction which is more theoretically correct, and which is found to give better results in practice, is indicated in Fig. 199.

Let \( ac = v \) = velocity of jet.
Let \( bc = u \) = velocity of bucket.
Then \( ab \) = relative velocity of jet and bucket.
If \( \omega \) = angular velocity of wheel, and if \( r \) = radius at the point of impact, we have \( u = \omega r \) and \( bc \) is perpendicular to \( oc \).
Draw \( dc \) parallel to \( ab \). Then \( dc \) represents the direction of the jet relative to the moving bucket.
For the jet to leave the bucket with zero absolute tangential velocity, its final direction must be parallel to, and the component parallel to the plane of the wheel of its final relative velocity must be equal to \( cb \).
then $\overline{ac}$, the bisector of the angle $\overline{b \ v \ d}$, be a normal to the surface of impact at $r$, the required conditions will be fulfilled, the jet striking the vane in the direction $\overline{ac}$ and leaving (relatively to the wheel) in the direction of $\overline{bc}$. The direction and magnitude of $\overline{ac}$, and the direction of $\overline{bc}$ being fixed, if the final relative velocity be approximately known, the magnitude of $\overline{bc}$ and therefore the most efficient speed of rotation may be determined. In a well-designed bucket the final relative velocity may be taken as $\cdot 75$ of the initial. If $c$ and $k$ be the first and last points at which the jet impinges on the ridge and if a third point $l$ be taken midway between $c$ and $k$, the directions of such normals as $\overline{cg}$ may be determined for these three points, and a smooth curve drawn through these points and having the required normals will give the correct curve for a longitudinal section of the receiving edge of the bucket. In general, a circular arc with centre at $p$, the intersection of the normals through $c$ and $k$ will give a very close approximation to the curve.

Strictly, since the path of the mid particles of the jet relative to the bucket is given by $\overline{cgs}$, the normal at $q$ should bisect this angle. If, however, the curve through $q$ be made parallel to $\overline{ck}$, the approximation to the correct curve will be sufficiently near.

A close approximation may also be obtained by determining graphically the points of intersection of the bucket with the axis of the jet for different positions of the bucket, and by drawing a smooth curve such that the axis of the jet is normal to the curve in every position of the bucket.

To prevent the jet striking the back of the bucket, this should be everywhere above the line $\overline{dc}$, while to reduce splash on passing through the jet the edge at $c$ should be as sharp as possible.

In modern practice the width of the buckets is between three and five times the diameter of the jet, the ratio diminishing as the size of jet increases, while the wheel diameter should not be less than about ten times the jet diameter. If, on settling the number of revolutions and peripheral speed, and hence the diameter of a wheel, this is less than the required multiple of the diameter of a single jet to give the required power, duplicate jets should be used.

**Number of Buckets.**—For minimum loss these must be as few as is consistent with the jet being wholly intercepted for all bucket positions, so that the entering bucket may entirely intercept the jet before the leaving bucket begins to free itself. From this consideration, a simple geometrical construction shows that if $n$ be the minimum possible number of buckets, $R$ the extreme outer radius over the receiving edges,
if the buckets, \( r \) the pitch circle radius, and \( t \) the thickness or diameter of the jet, \( n \) is given very approximately by the relationship

\[
n = \frac{\pi \cdot \frac{t}{2}}{\sqrt{1 - \left(\frac{r + \frac{t}{2}}{R}\right)^2}}
\]

If \( R = r + s \), so that \( s \) is that portion of the bucket projecting beyond the pitch circle, we have

\[
n = \frac{\pi \cdot \frac{t}{2}}{\sqrt{1 - \left(\frac{r + \frac{t}{2}}{r + s}\right)^2}}
\]

Giving \( s \) a value ranging from 0.6 to in the case of a wheel of less than 3 feet diameter to 0.65 t where the wheel is upwards of 6 feet in diameter, values of \( n \) in close accord with modern practice are obtained. Generally, values of \( n \) given by the formula \( n = k \sqrt{\frac{r}{t}} \), where \( k \) ranges from 7.0 to 8.0 as the wheel diameter decreases from 6 feet to 3 feet, will be found to give results which are sufficiently near for all practical purposes. The theoretical value of \( s \) thus being obtained, a little additional overlap is usually given to allow for any slight variation in the axial position of the jet.

**Speed Regulation.**—Since the efficiency of a Pelton wheel, or other impulse wheel, depends on the maintenance of the correct ratio of peripheral velocity of bucket and velocity of jet, if high efficiency is to be expected at all loads the method of governing must be such as to keep the latter velocity as nearly as possible constant. Where this is the case, there is no reason, except for the greater proportional effect of windage and mechanical friction at part loads, why the efficiency should not be independent of the load. Where, however, the jet velocity is variable, the efficiency falls off considerably as this departs from its theoretically correct value, and for this reason the impulse wheel, while giving excellent part-load efficiencies under a constant supply head, is unsuited for situations in which the percentage variation of head is likely to be great. Since this is more likely to be the case under a low supply head, it affords one reason why the impulse wheel is not in general advisable under such conditions.

The **Speed Regulation** of a Pelton wheel is usually performed in one of four ways.
(1) The stream may be deflected so as to partially miss the buckets at part load, either by swivelling the nozzle, which is then carried on a ball- and-socket joint, or by a stream deflector placed between the nozzle and the wheel. In the former case, owing to the friction at the swivelling joint a considerable force is required to deflect the nozzle, and in consequence the governor must be fitted with a hydraulic relay cylinder, as at C (Fig. 207). The piston rod of this cylinder carries the nozzle, and the governor by regulating the supply of pressure water to one side or other of this piston, also regulates the position of the nozzle.

This method of regulation has the disadvantage of being wasteful of energy at part load, while the nuisance caused by the discharge of the jet directly into the wheel pit may be very great.

On the Pacific slopes, however, many of the water companies require that a constant flow through the pipes be maintained, in order that a constant supply may be delivered over a weir to a ditch of lower level, and in this case the deflecting nozzle affords the most suitable means of speed regulation. The method possesses a further advantage in that it avoids all action of the nature of water ram in the pipes.

(2) The velocity of the jet may be reduced by means of a throttle valve placed behind the nozzle. This is not to be recommended, since the contraction and subsequent enlargement of the stream which occurs at the valve is wasteful of energy, while the variation in the velocity of the jet tends to inefficiency in working.

Further, since the sudden closing of the valve causes a corresponding increase of pressure throughout the pipe due to water hammer (p. 222), this method of governing should never be adopted without the addition of
some suitable protecting device, such as a stand pipe, relief valve, or pressure regulator, to the pipe line, this being placed as near to the valve as possible.

(3) A portion of the jet may be cut off at the nozzle by means of a sharp-edged sluice sliding across the orifice, or the section of the jet may be reduced by means of a needle regulator (Figs. 200—202). This consists of a cylindrical needle of tapering section fitted inside the nozzle axially with the jet. The water flows through the annulus between the needle and the nozzle, forming a solid cylindrical jet on leaving the needle. By axial regulation, the latter may be adjusted so as to fill the orifice either partially or wholly.

While giving a slightly greater loss by friction than the sluice regulation, a more stable jet is obtained, and on the whole needle regulation is to be preferred.

It is highly important, however, that the position of the needle in the nozzle should be perfectly central, or the form and efficiency of the jet may be seriously affected. Also the needle must be supported so as to prevent all vibration and consequent distortion of the jet. A further point to be noted is that the minimum section of the discharge channel should occur exactly at the tip of the nozzle for all positions of the needle. This may be illustrated by reference to Figs. 200 a and b. In a this condition is satisfied. In b this minimum cross section occurs at H A.
some point "\( p \)." After passing this point the area increases, with consequent tendency to unsteadiness of the jet and to loss of energy in eddy formation. Further details as to the properties of such jets are given on p. 459.
By either method of regulation the velocity of efflux is maintained approximately constant, and the efficiency is therefore only slightly affected at low loads, the quantity of water used being approximately proportional to the load.

On the other hand, the inertia of the supply column tends to prevent close governing unless a relief valve, or some such device, is fitted near to the nozzle, while care should always be taken, as explained on p. 285, that the closing of the nozzle actually does diminish the supply of energy to the wheel.

One device which prevents a rise in pressure following any sudden closing of the regulating nozzle is shown in Fig. 201. Here the relay cylinder $C$ is supplied with oil or water under pressure, this supply being regulated by a valve operated by the governor link. Any increase in speed is then accompanied by the admission of pressure water to the right-hand side of the piston. This forces the needle over to the left, reducing the supply of water to the wheel, and at the same time moves the cylinder itself to the right against the resistance of the springs at $S$, and so opens the by-pass valve $V$. In this it is aided by the pressure on the valve itself, so that the pressure is quickly relieved. The motion of the cylinder relatively to its valve moreover tends to cut off the supply of pressure water to the right-hand side of the piston, while the motion of the needle is utilized to bring the governor link back into its central position. This equalizes the pressure on the two sides of the piston, and the cylinder itself, under the action of the side springs $S$, returns to its central position, at the same time closing the by-pass valve. The whole apparatus is now ready to respond to a further change of speed in either direction. Some relay returning device of this nature is indispensable if hunting is to be prevented (Art. 139).

An extremely neat device for the same purpose is illustrated in Fig. 202,\(^1\) and is shown in Fig. 203\(^1\) as fitted to a twin Pelton wheel.

Here the horizontal governor lever $AB$ is not connected to any fixed fulcrum, but is pivoted at $A$ on the end of a plunger working in the dashpot $C$. At $B$ it is connected to the spindle of the regulating valve, $F$ being a fixed fulcrum. A subsidiary lever connects the end of the plunger working in the dashpot $P$ with the anchor link $L$ and with the governor collar, this being solely for the purpose of steadying the motion of the governor.

On a sudden increase in speed, following a reduction in load, the governor collar lifts and the valve spindle is depressed, admitting water.

\(^1\) By courtesy of Messrs. Gilbert Gilkes & Co., Ltd., Kendal.
behind the relay piston and forcing the spear rod into the nozzle. This spear rod is connected with the dashpot C, which itself works in the outer fixed casing K, by a series of links and a bell-crank lever not shown in the sketch, and as the spear rod moves to the right the dashpot is lifted, raising at the same time the fulcrum A. During this portion of the motion, there is a slight downward motion of the dashpot plunger and fulcrum relative to the cylinder. As the motion of the latter ceases, however, the plunger is gradually lifted by the weighted lever W, bringing down the pin at B, and returning the valve to its central position with the governor lever also in its central position.

Any further motion of the spear rod is thus stopped until the wheel has had time to readjust itself to the changed conditions, when the whole arrangement is again ready to adjust itself to any fresh change of speed.

Water ram on closing the nozzle may, if necessary, be prevented by a special automatic device of the makers. In this the spear rod is directly connected through a link with a dashpot plunger, the cylinder of which is vertical, is capable of axial movement, and which is itself connected to a small needle valve which is opened by any upward motion of the cylinder. The main relief valve is slightly overbalanced hydraulically so
as to remain closed whatever the pressure in the main. If, however, the spear rod closes the nozzle rapidly, the sudden motion of the dashpot plunger sucks up the dashpot cylinder and with it the small needle valve. This allows water to escape from above the main relief valve, which is then lifted by the excess pressure on its under side and permits of free discharge from the body of the nozzle.

The dashpot cylinder now begins to fall by its own weight, closing the needle valve and thus the relief valve, the time of closing being adjusted by regulation of the dashpot orifices to suit the length of the supply pipe line.

This system has the great advantage that its working is quite independent of any rise in pressure in the main, but rather anticipates any such possible rise.

Fig. 204 shows details of a device working on exactly the same principle, and applied in this case to the twin Pelton wheel shown in Fig. 205. Here the cross lever $L$ is connected to the piston rods of the two relay cylinders, and carries the dashpot rod $A$. Its connected plunger works in the weighted dashpot $C$, which itself carries the needle valve $V_1$. Pressure
water supplied through the small pipe $P$ keeps the main escape valve $V$ closed so long as the valve $V_1$ is closed. If this valve is opened, however, by a sudden upward motion of $I$, the pressure above the main valve is relieved, and the valve opens, relieving the pressure at the nozzles $NN$.

The valve $V_2$ permits of a sudden depression of $I$, without unduly stressing the dashpot rod $A$.

A modification of the needle method of regulation is indicated in

![Diagram](image)

Fig. 205.—Double Tangential Impulse Wheel, 500 H.P. at 375 Revolutions under 262 feet head; wheel diameter 3.28 feet. The Kubel Electric Power Plant, St. Gall.

Fig. 205, which shows a section of a double tangential wheel of 3.28 feet diameter developing 500 H.P. under a head of 262 feet. The nozzle is rectangular in section, while its upper side is formed by a pivoted flap whose position is regulated by that of the piston of a relay cylinder actuated by the governor. The area of this piston is so large that when its upper side is relieved of pressure, the upward pressure on its lower

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1 By courtesy of Messrs. Escher, Wyss & Cie., Zurich.
face, which is in communication with the nozzle, is sufficient to close the flap. Pressure water from the nozzle is supplied to the regulating valve V (Fig. 206) which regulates the pressure on the top of piston P. If the speed of the wheel increases the governor sleeve rises and the lever $AC$ turns about the fulcrum $B$, depressing valve $V$ and putting the upper side of piston $P$ into communication with the exhaust. This piston rises, closing the nozzle, and also lifting the point of attachment $P'$ of the link $FB$. This raises $B$ and also $C$ and brings the valve $V$ into a new position of equilibrium, in which it is prepared to take control of any fresh change of speed. A striking feature of this installation is that it is fitted with a draught tube (Art. 134), and works under a suction head of 22 feet.

(1) A modern and very common method of regulation is illustrated in Fig. 207. Here a combination needle and deflecting nozzle is used, the needle being set by hand, so as to take the maximum load likely to occur during any hour, while the deflection takes care of any variation of load up to this peak. With a very variable load, such as occurs in electric lighting plants, considerable economy may thus be effected, while the possibility of water hammer is eliminated.

A self-regulating wheel which has been tried with good results as regards speed regulation consists of two discs mounted side by side on the same shaft and capable of relative sidelong motion. These are kept in position by springs, and each carries a series of half buckets which fit together when the discs are close together and then form ordinary Pelton buckets. An increase in speed, by the consequent increased centrifugal force on masses mounted on bell-crank levers connected to the wheel, produces a
relative sidelong motion of the discs, which part in the middle and allow a portion of the jet to pass through to waste.

The complication introduced by this device, together with the waste of energy common to any such method of governing, form the chief drawbacks to the scheme.

The table on the opposite page gives some details of typical Pelton wheel installations of comparatively recent date.

If desired, two or three jets may be arranged so as to play on a single wheel, and the power obtained is then practically proportional to the number of jets. In such a case the sliding hood provides the most convenient method of speed regulation.

For heads above 400 feet, and for powers in single units up to about 2,000 B.H.P., the Pelton wheel is by far the most suitable type of prime mover, while for units up to 15,000 H.P. and with heads ranging from 100 to 400 feet, it is for many purposes to be preferred to its only serious rival, the inward radial flow or Francis turbine. In view of its combined simplicity, efficiency, and ease of regulation, it is probably the most perfect of all hydraulic prime movers, and this may be the more readily granted when the difficult conditions under which it works are remembered. Taking a jet of water to all intents and appearances as rigid as a rod of glass, and, in virtue of its enormous velocity, possessing almost infinite destructive possibilities; dropping it almost without splash into the tail-race divested of practically the whole of its kinetic
<table>
<thead>
<tr>
<th>Locality</th>
<th>Available steam at nozzles in feet</th>
<th>Number and Diameter of Wheels—Details of Governing Device, etc.</th>
<th>Number and no. of nozzles on each wheel</th>
<th>Revolutions per minute</th>
<th>Speed of Buckets</th>
<th>Velocity of Jets f.s.</th>
<th>Horse Power of each wheel</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cauvery Falls</td>
<td>882.5</td>
<td>2 wheels, 4' 6&quot; diam., 24 buckets to each wheel.</td>
<td>2 nozzles, each 5' × 4&quot;</td>
<td>300</td>
<td>70.7 f.s.</td>
<td>154.7 f.s.</td>
<td>1,015</td>
<td>75% guaranteed, 70% at half gate.</td>
</tr>
<tr>
<td>Pike's Peak Power Company, Colorado</td>
<td>1,160</td>
<td>1 wheel, 60' diam.</td>
<td>1 nozzle, 1' diam.</td>
<td>450</td>
<td>132 f.s.</td>
<td>270 f.s.</td>
<td>226</td>
<td>83%</td>
</tr>
<tr>
<td>Los Angeles, 1907</td>
<td>210</td>
<td>4 wheels, 10' diam.; 18 buckets, each 25' wide; combined needle and deflecting nozzles</td>
<td>1 nozzle, 7' diam.</td>
<td>250</td>
<td>131 f.s.</td>
<td>225 f.s.</td>
<td>5.00 f.s (approx.)</td>
<td>?</td>
</tr>
<tr>
<td>University of California, 1904</td>
<td>50.6</td>
<td>1 wheel, 15' diam.</td>
<td>1- 4' nozzle</td>
<td>276</td>
<td>132 f.s.</td>
<td>211 f.s.</td>
<td>173.9%</td>
<td>82.3%</td>
</tr>
<tr>
<td>McGill University, 1897</td>
<td>235</td>
<td>1 wheel, 19' diam.</td>
<td>1- 53' nozzle</td>
<td>746</td>
<td>21 f.s.</td>
<td>59.5 f.s.</td>
<td>121.4 f.s.</td>
<td>78.9%</td>
</tr>
<tr>
<td>Grant Valley</td>
<td>285</td>
<td>1 wheel, 72' diam.</td>
<td>1- 1-56' nozzle</td>
<td>225.5</td>
<td>79.8 f.s.</td>
<td>153.7 f.s.</td>
<td>87.3%</td>
<td>?</td>
</tr>
<tr>
<td>Owne Avon, 1906</td>
<td>98</td>
<td>1 wheel, 30' 6&quot; diam., governed by throttle valve placed behind nozzles, 24 buckets</td>
<td>2 nozzles, 41' and 43' diam.</td>
<td>25</td>
<td>37.7 f.s.</td>
<td>78.3 f.s.</td>
<td>140</td>
<td>?</td>
</tr>
<tr>
<td>Pike's Peak Hydro-Electric Company, Manitoba</td>
<td>2,176</td>
<td>5 wheels, each 7' 6&quot; diam. on pitch wheel, 7' 3&quot; diam. over all. Pressure at nozzles when operating 40 lbs. sq. inch; stat. pressure, 1,047 lbs. sq. inch; regulation by combined needle and deflecting nozzles, 36 buckets</td>
<td>1 nozzle, 21' diam.</td>
<td>450</td>
<td>173 f.s.</td>
<td>368.2 f.s.</td>
<td>2,600 (approx.)</td>
<td>?</td>
</tr>
<tr>
<td>Plant No. 8 San Joaquin Light and Power Company, 1906, Doble wheels</td>
<td>355</td>
<td>2 wheels, each delivering 1,750 H.P., governing by Doble needle regulating nozzles. 55' 4&quot; diam.</td>
<td>1 nozzle, 6' diam.</td>
<td>350</td>
<td>72.0 f.s.</td>
<td>150.0 f.s.</td>
<td>353</td>
<td>77.8%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>494</td>
<td>79.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>715</td>
<td>81.4%</td>
</tr>
<tr>
<td>Cornell University</td>
<td>134</td>
<td>2 main wheels of 250 B.H.P., Doble needle regulating nozzles. 6' 6&quot; diam.</td>
<td>1 nozzle, 7' diam.</td>
<td>124</td>
<td>42.2 f.s.</td>
<td>91.5 f.s.</td>
<td>120</td>
<td>78.7%</td>
</tr>
<tr>
<td>Cornell University</td>
<td>134</td>
<td>2 Exciter wheels of 50 B.H.P., Doble needle regulating nozzles. 8' 1' diam.</td>
<td>1 nozzle, 8' diam.</td>
<td>200</td>
<td>42.0 f.s.</td>
<td>91.5 f.s.</td>
<td>120</td>
<td>78.7%</td>
</tr>
</tbody>
</table>

In each case the velocity at the nozzle has been calculated from the formula \( v = cv \sqrt{\frac{y}{a}} \), where \( cv = 985 \).
energy; the whole affords an unique example of the possibilities of engineering science.

**Design of Pelton Wheel.**

**Example.**

To design a Pelton wheel to work under an effective head of 500 feet and to develop 800 H.P. at 260 revolutions per minute.

Assuming a coefficient of velocity = 985, the velocity of efflux of the jet = $985 \times \sqrt{500 \times 64.4} = 177$ feet per second.

Taking the velocity of the pitch circle of the wheel as 46 times that of the jet, we have

Peripheral velocity of wheel = 81.3 feet per second.

∴ Radius of pitch circle = \( \frac{81.3 \times 300}{2 \pi} = 2.158 \) feet.

∴ Diameter of pitch circle = 4 feet 3\( \frac{3}{4} \) inches.

Next assuming an efficiency of 85 per cent., we have the energy passing the nozzle per second given by

\[ \frac{800 \times 550}{85} \text{ ft. lbs.} = 518,000 \text{ ft. lbs.}, \]

and since each cubic foot of water contains

\[ \frac{62.4 \times (177)^{2}}{2g} \text{ ft. lbs.} = 30,380 \]

ft. lbs. in the form of kinetic energy, this requires

\[ \frac{518,000}{30,380} = 17.06 \text{ cubic feet per second.} \]

The required area of the nozzle is thus

\[ \frac{17.06}{177} = 0.0964 \text{ sq. ft.} \]

\[ = 13.89 \text{ sq. ins.} \]

giving a jet diameter of 4.20 inches.

Taking \( n = 7.5 \sqrt{\frac{r}{t}} \), this gives the number of buckets as equal to 7.5 \( \sqrt{\frac{25.9}{4.22}} = 18.6 \), or say 20 for convenience in balancing.

Next applying the formula \( n = \frac{\pi}{\sqrt{1 - \left(\frac{r + \frac{t}{2}}{r + s}\right)^{2}}} \), we get, on

substituting for \( n, r \) and \( t \), on reduction \( s = 2.5 \) inches, giving the amount by which the buckets must project beyond the pitch circle for continuous impact. For safety it is usual to increase this slightly, say to 2.75 inches, giving an extreme wheel diameter of 4 feet 9\( \frac{1}{2} \) inches.

The buckets would in this case be about 21 inches wide.
Art. 126.—Jets from Needle-Nozzle.

The presence of the central needle in a nozzle provided with needle regulation causes a reduction in the velocity of the central filament, and to this extent tends to reduce the efficiency of the jet. Fig. 208 shows the velocity obtained at different points in the cross section of a jet obtained respectively from a plain conical nozzle, a ring nozzle
and a Pelton wheel nozzle with needle regulator. From these it appears that the central velocity at a point distant ½ inch from the tip of the needle is only 68 of the maximum velocity. At a section 3½ inches from the tip this ratio becomes 90, while when the distance is 9½ inches it becomes 96.

At mid opening (diameter 1·25 inches) the coefficient of velocity diminishes slightly as the head increases, from about 992 with 28 feet head to 978 with 120 feet head. With a given head the velocity was slightly the greatest with the nozzle half open. The efficiencies in these experiments varied from 964 to 998. The maximum jet diameter was 1·50 inches.

Experiments on a larger nozzle, giving a jet up to 7 inches diameter under heads up to 850 feet, showed the following results:

<table>
<thead>
<tr>
<th>Distance from centre of jet (inches)</th>
<th>0·0</th>
<th>0·5</th>
<th>1·0</th>
<th>2·0</th>
<th>3·0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity (feet per second)</td>
<td>212·7</td>
<td>228·7</td>
<td>229·3</td>
<td>229·9</td>
<td>227·8</td>
</tr>
</tbody>
</table>

EXAMPLES

The coefficient of velocity increased from 0.971 to 0.989 as the nozzle area was increased, the coefficient of discharge diminishing at the same time from 0.965 to 0.838, and the efficiency increasing from 0.958 to 0.986. Fig. 209 shows the shape of needle and tip used in these experiments.

(1) A Pelton wheel working under an effective head of 2,100 feet is 86" diameter and is supplied through a single 3/4" nozzle. Determine the necessary number of revolutions of the wheel for approximately maximum efficiency and the probable horse-power, assuming an efficiency of 83 per cent.

Answer.  
1,060 revolutions per minute.  
97.5 H.P.

(2) A Pelton wheel develops 140 B.H.P. under a head of 98 feet. The wheel is 20' 0" in diameter, and is supplied by two nozzles. Determine the number of revolutions per minute and the necessary nozzle diameter, if the efficiency is 80 per cent.

Answer.  
35 revolutions per minute.  
Diam. = 4.3 ins.

(3) Show that the efficiency of a Pelton wheel is theoretically equal to

\[
\frac{2\pi r N}{60 g h} \left( C_v \sqrt{2 \frac{g}{h} \frac{N}{60}} - \frac{2\pi r N}{60} \right) \left( 1 - k \cos a \right)
\]

Where \( r \) = mean radius of bucket circle.

\( N \) = revolutions per minute.

\( h \) = effective head at nozzle.

\( C_v \) = coefficient of velocity at nozzle.

\( k \) = ratio of relative velocity at exit from and entrance to buckets.

\( a \) = total angle through which jet is deflected.