CHAPTER X.


ART. 85.—FLOW IN AN OPEN CHANNEL.

The term "open channel" includes all rivers, artificial canals, aqueducts, and conduits, and, in addition, sewers and pipes of whatever section which run partially full, and which consequently do not present a solid boundary to every side of the contained liquid. The force producing flow cannot now be provided by any external head, but is solely due to the slope or gradient of the channel.

If a circular pipe be laid almost horizontally and if the surface level of water flowing through the pipe be allowed to rise, the change from the state in which the flow is governed by the laws appertaining to an open channel, to that in which the ordinary laws of pipe flow hold, is not abrupt, and it is to be inferred that a general formula is deducible which by satisfactory adjustment of constants shall fit either type of flow.

Still, the comparative simplicity of the conditions holding in the case of a circular pipe, the complications which must of necessity be introduced where water flows through a channel of uneven section, and the ease with which accurate observations are made in the one case and the difficulty with which even such a fundamental observation as the difference in level at points widely distant is accurately determined in the other, render it impossible that the laws governing the flow in open channels should be so definite and of such universal application as those already considered.

Assuming the resistance, $R$, to flow, to be proportional to the wetted perimeter $P$ of the channel, this may be expressed as

$$ R = f' S v^n, $$

where $f'$ is a coefficient depending on the condition of the surface and probably, from analogy to pipe flow, also on the velocity $v$, while $n$ is a number probably varying from 1.79 to 2.00, depending on the surface and on the velocity $v$, and $S = P \times l$, where $l$ = length of channel.

E.A.
If \( A \) = sectional area of channel beneath water line, and if we assume the resistance to be equally divided over the area we have, if \( p = \text{resistance per unit area of the stream} \)

\[
p = f' \cdot \frac{P}{A} \cdot l v^m.
\]

Here \( \frac{A}{P} \) or \( m \) is the hydraulic mean depth of the section.

If the channel be of uniform slope \( \frac{h}{l} \), where \( \frac{h}{l} = i = \sin \theta \) (\( \theta \) being the angle of inclination), then the weight of water in this length \( l \), per unit area of the channel, being \( W' l \) lbs., the resolved part of this weight in the direction of motion = \( W i \frac{h}{l} \) = \( W'h \) lbs.

\[
\therefore \text{If the velocity is constant so that this force is entirely expended in overcoming friction and not in producing acceleration, we have}
\]

\[
W h = f' \cdot \frac{P}{A} \cdot l v^m
\]

\[
h = \frac{f''}{m} \cdot \frac{l v^m}{m}
\]

\[
\text{or } \frac{h = f l v^m}{2 g m}
\]

In an open channel \( n \) may be taken as being approximately equal to 2, so that the formula becomes

\[
h = \frac{f l v^a}{2 g m}
\]

This may be written in the form adopted by Chezy, viz.,

\[
v = \sqrt{\frac{2 g}{f}} \cdot \frac{h}{l} \cdot m = C \sqrt{\frac{m}{i}}
\]

where

\[
C^a = \frac{2 g}{f}
\]

Many experiments have been devoted to determining the values of \( C \) or of \( f \) for channels having different physical characteristics, and the results of the more important of these are as follow, the numerical values of the coefficients obtained by the various observers being collected and tabulated on pp. 293—297.

Darcy and Bazin (1855—9), as the results of experiments carried out on the Bourgoyn Canal, gave \( C \) the value \( \frac{1}{\sqrt{a + \frac{b}{m}}} \) where \( a \) and \( b \) (p. 293) vary only with the material and condition of the bed and sides of the channel.
FLOW IN OPEN CHANNELS

These channels were of many different forms and dimensions; were lined with different materials, and had slopes varying from 0.01 to 0.01.

Prony, from experiments by Chezy and Dubuat on earthen channels and on wooden channels of small section put

$$a v + b v^2 = m i$$

or

$$C = \frac{1}{\sqrt{b + \frac{a}{v}}}.$$  

The corresponding value of $f = 2 g \left(b + \frac{a}{v}\right) = A + \frac{B}{v}$

where

$$\begin{align*}
\frac{1}{a} &= 22,472. \quad A = 0.00607, \\
\frac{1}{b} &= 10,607. \quad B = 0.00286.
\end{align*}$$

Eytelwein, from experiments on the Rhine channel, gave the same type of formula, his coefficients being

$$\frac{1}{a} = 41,211. \quad A = 0.00719,$$

$$\frac{1}{b} = 8,975. \quad B = 0.00156.$$  

In both these cases the unit of length is 1 foot. With moderate velocities $ar$ is small compared with $b v^2$ and may be neglected, when the formulae reduce to

Prony

$$v = \frac{1}{\sqrt{b}} \sqrt{m_i} = 103 \sqrt{m_i}. \quad f = 0.00607.$$  

Eytelwein

$$v = \frac{1}{\sqrt{b}} \sqrt{m_i} = 95 \sqrt{m_i}. \quad f = 0.00714.$$  

These coefficients, being independent of the condition of the surface, are obviously only applicable to channels having the same physical characteristics as those experimented upon.

Bazin (1897), as the result of a very large number of experiments on canals and conduits of all sections and dimensions, deduced for $C$ the value

$$\frac{157.6}{N} \text{ in foot units.}$$

$$1 + \frac{N}{\sqrt{m}}$$

$N$ varying with the character of the surface.

Values of $N$ for different types of surface are given on p. 293.

This gives a value of $f = 0.00259 \left\{1 + \frac{2N}{\sqrt{m}} + \frac{N^2}{m}\right\}$.  

$\sigma 2$
This form of equation is in common use in France, and has given good results with velocities not exceeding 4 feet per second.

*Ganguillet* and *Kutter* deduced the coefficient:

\[
C = \frac{41.6 + \frac{0.00281}{i} + \frac{1.8112}{N}}{1 + \left(41.6 + \frac{0.00281}{i}ight) \cdot \frac{N}{\sqrt{m}}}
\]

in foot units.

The equation being identical with that used for pipe flow. *N* depends on the character of the surface, and has values given on p. 293.

The complication of this formula is largely due to an attempt to make it conform to the results of experiments made on the Mississippi. In certain of these, however, the results should have been corrected for the error introduced by the use of double floats, while in others the slope of the water surface was too slight to be measured with any degree of accuracy.

More recent investigations render it extremely doubtful whether the value of *C* does depend on *i* as this formula indicates, and the simpler formula of *Bazin* would appear to give results at least as accurate, except possibly in the case of very large channels. The formula is, however, of very general application in Great Britain, India, Germany and the United States, and the inconveniences due to its complication are removed by the use of hydraulic tables which have been prepared giving the values of *C* for practically all values of *i* and of *N*.

By altering the value of *i* in this formula from 0.001 to 0.01, the value of *C* is altered by less than 1 per cent. For streams of fairly rapid slope the value of *i* may then be taken as sensibly equal to 0.001, in which case the value of *C* simplifies to

\[
C = \frac{44.4 + \frac{1.811}{iN}}{1 + \frac{44.4}{N} \cdot \frac{1}{\sqrt{m}}}
\]

In very large rivers the flow is sensibly independent of the character of the bed, and for such a case *Manning* gives *C* the value

\[
C = 62 \left(1 + \frac{\sqrt{m}}{7} - \frac{0.05}{\sqrt{m}}\right)
\]

in foot units.

\[
C = 34 \left(1 + \frac{\sqrt{m}}{4} - \frac{0.03}{\sqrt{m}}\right)
\]

in metre units.

From the nature of the case it would appear hopeless to obtain any.

---

1 *Inst. Civil Engineers of Ireland, December 4, 1889.*
strictly mathematical solution for flow in open channels and rivers of irregular section, and even to observe and record correctly the physical data required is almost impossible. An examination of the results given by an application of the various formulae shows that for abnormal sections or velocities a difference of 50 per cent. is not uncommon.

Coefficients for Flow in Open Channels.

<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>N.</td>
</tr>
<tr>
<td>Smooth cement or planed timber</td>
<td>-0.00046</td>
<td>-0.000045</td>
<td>-100</td>
</tr>
<tr>
<td>Unplaned timber, flumes, slightly tuberculated iron, ashlars and well-laid brickwork</td>
<td>-0.00058</td>
<td>-0.000133</td>
<td>290</td>
</tr>
<tr>
<td>Rubble masonry and brickwork in an inferior condition. Fine gravel well rammed</td>
<td>-0.00073</td>
<td>-0.00060</td>
<td>883</td>
</tr>
<tr>
<td>Rubble in inferior condition. Canals with earth beds in very good condition</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canals with earth beds in good condition</td>
<td></td>
<td></td>
<td>1.540</td>
</tr>
<tr>
<td>Ditto in moderate condition</td>
<td>-0.00085</td>
<td>0.0035</td>
<td></td>
</tr>
<tr>
<td>Canals and rivers in rather bad order</td>
<td></td>
<td></td>
<td>2.355</td>
</tr>
<tr>
<td>Ditto in very bad order</td>
<td>-0.0012</td>
<td>-0.0070</td>
<td>3.170</td>
</tr>
</tbody>
</table>

Here a channel is said to be in very good order when it is free from boulders, hollows in its bed and banks, sharp bends, snags and weed
HYDRAULICS AND ITS APPLICATIONS

When badly choked with weeds the value of $N$ in Kutter's formula may become much greater than 0.035. The following values of $N$ in the latter formula are taken from Jackson's tables, and are probably slightly more accurate than those given above.

- Planed timber accurately jointed—glazed or enamelled surfaces $\cdot 009$
- Smooth cement or plaster $\cdot 010$
- Unplaned timber well jointed—new brickwork well laid $\cdot 012$
- Unglazed stonework iron—brick and ashlar $\cdot 013$
- Wooden troughs with battens inside, $\frac{1}{2}$ inch apart $\cdot 015$
- Rubble set in cement $\cdot 017$

If any of these are in bad order the next higher value for $N$ is to be taken.

For convenience in applying the results of these formulae the values of $v$ in the formula $h = \frac{f \ell v^2}{2gy}$, and of $C$ in the formula $v = C\sqrt{\frac{m}{i}}$ have been calculated and are tabulated opposite.

The following table gives values of $C$ in the formula $v = C\sqrt{\frac{m}{i}}$, calculated from Bazin's formula in which $G = \frac{157.6}{1 + \frac{N}{\sqrt{m}}}$.

<table>
<thead>
<tr>
<th>Hydraulic mean Depth (m ft.)</th>
<th>Values of $N$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.109</td>
</tr>
<tr>
<td>1.0</td>
<td>0.290</td>
</tr>
<tr>
<td>1.5</td>
<td>0.333</td>
</tr>
<tr>
<td>2.0</td>
<td>1.54</td>
</tr>
<tr>
<td>2.5</td>
<td>2.35</td>
</tr>
<tr>
<td>3.0</td>
<td>3.17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$m$ ft.</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
<th>8.0</th>
<th>10.0</th>
<th>15.0</th>
<th>20.0</th>
<th>25.0</th>
<th>30.0</th>
<th>50.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>187</td>
<td>142</td>
<td>145</td>
<td>146</td>
<td>147</td>
<td>148</td>
<td>149</td>
<td>150</td>
<td>151</td>
<td>152</td>
<td>152</td>
<td>158</td>
<td>154</td>
<td>155</td>
<td>155</td>
<td>156</td>
</tr>
<tr>
<td>of $N$.</td>
<td>112</td>
<td>122</td>
<td>128</td>
<td>131</td>
<td>133</td>
<td>135</td>
<td>138</td>
<td>140</td>
<td>141</td>
<td>143</td>
<td>145</td>
<td>147</td>
<td>148</td>
<td>149</td>
<td>149</td>
<td>150</td>
</tr>
<tr>
<td>Character of Surface</td>
<td>Darcy</td>
<td>Barn</td>
<td>Kutter</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$m = 5$</td>
<td>$m = 1$</td>
<td>$m = 2$</td>
<td>$m = 5$</td>
<td>$m = 1$</td>
<td>$m = 2$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Smooth cement or planed timber</td>
<td>$f = 0.9354$</td>
<td>$0.325$</td>
<td>$0.310$</td>
<td>$0.345$</td>
<td>$0.318$</td>
<td>$0.304$</td>
<td>$0.634$</td>
<td>$0.264$</td>
<td>$0.219$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unplaned timber, flutes, slightly tuberculated iron, asphalt and well laid brickwork</td>
<td>$f = 0.0514$</td>
<td>$0.152$</td>
<td>$0.164$</td>
<td>$0.515$</td>
<td>$0.431$</td>
<td>$0.376$</td>
<td>$0.017$</td>
<td>$0.396$</td>
<td>$0.320$</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rubble masonry and brickwork in an inferior condition—fine gravel well rammed</td>
<td>$f = 0.241$</td>
<td>$0.156$</td>
<td>$0.663$</td>
<td>$0.227$</td>
<td>$0.070$</td>
<td>$0.053$</td>
<td>$0.129$</td>
<td>$0.854$</td>
<td>$0.663$</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rubble in inferior condition. Canals with earth beds in perfect condition</td>
<td>$f = 0.052$</td>
<td>$0.167$</td>
<td>$0.130$</td>
<td>$0.260$</td>
<td>$0.167$</td>
<td>$0.130$</td>
<td>$0.167$</td>
<td>$0.130$</td>
<td>$0.167$</td>
<td></td>
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</tr>
<tr>
<td>Canals with earth beds in good condition</td>
<td>$f = 0.550$</td>
<td>$0.280$</td>
<td>$0.167$</td>
<td>$0.312$</td>
<td>$0.210$</td>
<td>$0.151$</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canals with earth beds in moderate condition</td>
<td>$f = 0.550$</td>
<td>$0.280$</td>
<td>$0.167$</td>
<td>$0.312$</td>
<td>$0.210$</td>
<td>$0.151$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canals and rivers in rather bad order. weeds on bottom</td>
<td>$f = 0.979$</td>
<td>$0.328$</td>
<td>$0.302$</td>
<td>$0.779$</td>
<td>$0.450$</td>
<td>$0.266$</td>
<td>$0.710$</td>
<td>$0.452$</td>
<td>$0.305$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canals and rivers in very bad order—choked with boulders</td>
<td>$f = 25.6$</td>
<td>$35.0$</td>
<td>$45.1$</td>
<td>$28.8$</td>
<td>$37.8$</td>
<td>$49.2$</td>
<td>$3.1$</td>
<td>$37.6$</td>
<td>$46.0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Rational Formula.

From analogy with pipe flow it would appear probable that a formula of the type

\[ h = \frac{f l v^n}{(A)^x} = \frac{f l v^n}{m^x} \]

where, as in the case of a pipe, \( n \) is in general less than 2, would most nearly represent the law of channel flow.

Claxton Fuller\(^1\) has determined the values of \( f, n, \) and \( x \) from many experimental results of Darcy, Bazin, Smith, Stearns, and other observers, and the following table is abridged from values given by him:

<table>
<thead>
<tr>
<th>Form of Section</th>
<th>Material of Surface</th>
<th>( n )</th>
<th>( x )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular</td>
<td>Smooth neat cement</td>
<td>1.75</td>
<td>1.167</td>
<td>0.0000676</td>
</tr>
<tr>
<td>Rectangular</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circular</td>
<td>Cement and sand</td>
<td></td>
<td></td>
<td>0.0000787</td>
</tr>
<tr>
<td>Rectangular</td>
<td>Smooth ashlar</td>
<td></td>
<td></td>
<td>0.0000904</td>
</tr>
<tr>
<td>Circular</td>
<td>Bare metal pipes with rivetted joints</td>
<td>1.77</td>
<td>1.18</td>
<td>0.0000871</td>
</tr>
<tr>
<td>Rectangular</td>
<td>Rough brickwork</td>
<td>1.80</td>
<td>1.20</td>
<td>0.0000977</td>
</tr>
<tr>
<td>Rectangular</td>
<td>Unplanned timber</td>
<td></td>
<td></td>
<td>0.0000944</td>
</tr>
<tr>
<td>Circular</td>
<td>Rough Brickwork or ashlar</td>
<td></td>
<td></td>
<td>0.0001122</td>
</tr>
<tr>
<td>Rectangular</td>
<td>Lined with fine gravel</td>
<td>1.90</td>
<td>1.33</td>
<td>0.0001202</td>
</tr>
<tr>
<td>Rectangular</td>
<td>&quot; &quot; &quot; coarse gravel</td>
<td>1.96</td>
<td>1.40</td>
<td>0.0001521</td>
</tr>
<tr>
<td>&quot; &quot; &quot; Rubble masonry</td>
<td></td>
<td>2.10</td>
<td>1.50</td>
<td>0.0001862</td>
</tr>
<tr>
<td>&quot; &quot; &quot;</td>
<td></td>
<td></td>
<td></td>
<td>0.0002240</td>
</tr>
</tbody>
</table>

While further investigation may slightly alter these values, it is extremely probable that the final solution of the problem of flow in regular channels will be found in this type of formula.

The pipe flow formula of Thrupp

\[ h = \frac{C^n v^n l}{m^x} \]

where \( m \) is the hydraulic mean depth is also applicable to channel flow, the following being the values of the quantities \( C^n, n, \) and \( x \). When \( m \) is small, \( x + a \sqrt{\frac{v}{m}} - 1 \) should be substituted for \( x \).

\(^1\) Feider, "Calculations in Hydraulic Engineering," Part II. (Longmans & Co.), 1902.
FLOW IN OPEN CHANNELS

<table>
<thead>
<tr>
<th>Surface</th>
<th>n.</th>
<th>( C_n )</th>
<th>( x )</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neat cement, semi-circular section</td>
<td>1.74</td>
<td>( 0.000680 )</td>
<td>1.165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brickwork, well laid</td>
<td>1.95</td>
<td>( 0.000494 )</td>
<td>1.190</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unplanned plank</td>
<td>2.00</td>
<td>( 0.000600 )</td>
<td>1.220</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brickwork, rough</td>
<td>2.00</td>
<td>( 0.000714 )</td>
<td>1.230</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lined with fine gravel</td>
<td>2.00</td>
<td>( 0.000780 )</td>
<td>1.250</td>
<td>( 5670 )</td>
<td>50</td>
</tr>
<tr>
<td>Chiselled masonry</td>
<td>2.00</td>
<td>( 0.001399 )</td>
<td>1.320</td>
<td>( 2215 )</td>
<td>20</td>
</tr>
<tr>
<td>Lined with coarse gravel</td>
<td>2.00</td>
<td>( 0.001246 )</td>
<td>1.320</td>
<td>( 7882 )</td>
<td>60</td>
</tr>
<tr>
<td>Earth in fair condition</td>
<td>2.00</td>
<td>( 0.002085 )</td>
<td>1.410</td>
<td>( 1365 )</td>
<td>100</td>
</tr>
<tr>
<td>Rough earth</td>
<td>2.00</td>
<td>( 0.002360 )</td>
<td>1.440</td>
<td>( 1518 )</td>
<td>100</td>
</tr>
</tbody>
</table>

The results obtained by this formula compare very well with those of Fidler, the latter probably on the whole giving the better results.

Prof. G. S. Williams, adopting this exponential formula, gives \( n \) and \( x \) constant values respectively equal to 1.9 and 1.25. The formula then becomes \( h = \frac{K}{m} \cdot \frac{l}{v^x} \), or \( v = C \cdot m^{\frac{1}{m}} \cdot \frac{n}{l} \), where \( K \) and \( C \) have the following values:

<table>
<thead>
<tr>
<th></th>
<th>( C )</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very smooth channels</td>
<td>180 to 190</td>
<td>( 0.0030 ) to ( 0.0035 )</td>
</tr>
<tr>
<td>Unplanned plank</td>
<td>145 ( \cdot ) 165</td>
<td>( 0.0045 ) ( \cdot ) ( 0.0050 )</td>
</tr>
<tr>
<td>Sewer cock</td>
<td>140 ( \cdot ) 160</td>
<td>( 0.0050 ) ( \cdot ) ( 0.0075 )</td>
</tr>
<tr>
<td>Brick sewers</td>
<td>130 ( \cdot ) 160</td>
<td>( 0.0050 ) ( \cdot ) ( 0.0080 )</td>
</tr>
<tr>
<td>Earth channels</td>
<td>60 ( \cdot ) 80</td>
<td>( 0.010 ) ( \cdot ) ( 0.020 )</td>
</tr>
<tr>
<td>Rough natural channels</td>
<td>40 ( \cdot ) 50</td>
<td>( 0.020 ) ( \cdot ) ( 0.050 )</td>
</tr>
</tbody>
</table>

ART. 86.—CRITICAL VELOCITY IN AN OPEN CHANNEL.

So far it has been assumed that in channel flow the resistance to motion is proportioned to some power of the mean velocity approximating to the second, but while this is undoubtedly true in all natural streams having a fairly rapid slope it is in all probability not the case where velocities are very low.

It might, in fact, be inferred from analogy with pipe flow that below some “critical” velocity the resistance will be proportional to the first power of the velocity. The clear glassy non-distortive reflecting surface observed in any long straight reach of a deep and sluggish stream tends to strengthen this inference, while the behaviour of small particles of suspended matter appears to show almost conclusively that at low speeds motion takes place in st.
HYDRAULICS AND ITS APPLICATIONS

Experiments by Mr. E. C. Thropp\(^1\) on the Thames and the Kennet, the former having a mean depth of 7·6 feet and the latter of 2·4 feet, showed that velocities of 665 feet per second in the former, and 64 feet per second in the latter case, were below the critical, and that for all smaller values the velocity was practically proportional to the surface slope. The experiments did not, however, indicate the point at which the velocity-slope law changed.

Although, owing to the difficulty of measuring such small differences of head as are involved in flow at low velocities, accurate determinations are not practicable, yet the values quoted above show that this critical velocity is immensely high compared with that calculated from Reynold's formula (p. 55) for a cylindrical pipe of the same hydraulic mean depth. The existence of a critical velocity would at once explain the great discrepancy which in some cases exists between the results of experiments on channels having similar physical characteristics, but with very different velocities of flow.

ART. 87.—FORM OF CHANNEL.

Since for a channel of given sectional area \(A\), the hydraulic mean depth \(A / P\) varies with the form of its section, while the resistance to flow increases as \(A / P\) diminishes, it becomes important to determine what form of channel will give the maximum value of \(A / P\) for a given value of \(A\), since this will be the channel of maximum discharge for a given slope. Further, if this sectional area is a minimum, the cost of excavation is a minimum, and since in general the perimeter increases with the sectional area, the cost of pitching the faces of the channel is also a minimum. Theoretically, the best form of channel is one in which the bed is a circular arc, since this gives a minimum ratio of wetted perimeter to sectional area.

An investigation into the properties of different sections will be simplified if the coefficient \(C\) in the formula \(v = C \sqrt{A/P} \cdot i\), be assumed constant for a given surface. On this assumption we have:

\[
Q = A v = C \sqrt{A^2 \cdot P} \cdot i \text{ cub ft per sec.}
\]

For \(v\) to be a maximum, \(A / P\) must be a maximum, so that \(d \left(\frac{A}{P}\right) = 0\)

\[
P d A - A d P = 0.
\]

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Again, for Q to be a maximum, \( \frac{A^3}{P} \) is to be a maximum, so that
\[
d \left( \frac{A^3}{P} \right) = 0.
\]
\[
\therefore 3 P \cdot A^2 \cdot d A = A^3 \cdot d P = 0.
\]
Where \( A \) is fixed, \( d A = 0 \) and both conditions are satisfied if \( d P = 0 \).

\[\text{(2)}\]

**Example.**

(1) **Rectangular Channel** — Breadth \( 2b \) — depth \( d \).

Here
\[
A = 2b \cdot d \quad : \quad P = 2b + 2d = \frac{A}{d} + 2d
\]
\[
\therefore \quad \frac{d P}{d(d)} = -\frac{A}{d^2} + 2.
\]

Putting \( A = 2b \cdot d \) and equating \( \frac{d P}{d(d)} \) to zero, we have \( b = d \).

Putting the full breadth = \( B \), we have \( d = \frac{B}{2} \), i.e., for maximum flow the depth must equal one-half the breadth.

We then have
\[
Q = C \sqrt{\frac{A^3}{P} \cdot i} = C \sqrt{\frac{8b^3 \cdot d^3 \cdot i}{2b + 2d}} = C \sqrt{2i \cdot b^2} = \frac{C}{4} \sqrt{i B^5_2}
\]

\[\text{(1)}\]

(2) **Trapezoidal Channel.** — Fig. 129.

Let \( b = \) half bottom breadth; \( d = \) depth; \( s = \) cotangent of angle of slope of sides.

Then \( A = 2b \cdot d + s \cdot d^2 \).

\[ P = 2(b + d \sqrt{1 + s^2}). \]

For \( Q \) to be a maximum with a given area of channel or for the cost of construction to be a minimum, it is necessary that \( \frac{d P}{d(d)} = 0 \).

But
\[
P = 2 \left\{ \frac{A}{2} \cdot d - \frac{s}{2} \cdot d^2 + d \sqrt{1 + s^2} \right\}
\]
\[
\therefore \quad \frac{d P}{d(d)} = 2 \left\{ - \frac{A}{2} \cdot d^2 - \frac{s}{2} + \sqrt{1 + s^2} \right\}
\]
For maximum value of $Q$

$$\sqrt{1 + s^2} = \frac{2bd + sda}{2d^2} + \frac{s}{2}$$

$$= \frac{b}{d} + s$$

$$(1 + s^2)d^2 = (b + s d)^2$$  \hspace{1cm} (2)

But from the figure it will be seen that if a circle having its centre in the surface of the water can be drawn to touch the sides and bottom, we have

$$mn = b + s d; \quad mp = mq = s d; \quad np = d,$$

and since $(mn)^2 = (mp)^2 + (np)^2$, the above equation is satisfied. These proportions then give the best results.

The value of $s$ depends largely on the material in which the channel is excavated. The following may be taken as the minimum permissible values.

Earthen canal with faced sides $s = 1.0$.

"" "" "" natural "" $s = 1.5$.

"" "" "" in light soil $s = 2.0$.

The latter value is usually adopted for all unfaced earthen sides.

Substituting this value of $s$ in equation (2) we have

$$5d^2 = (b + 2d)^2$$

$$\therefore d = 4.24\ b$$  \hspace{1cm} (3)

\[\therefore\] For maximum discharge $d = 2.12\ B$, where $B$ is the bottom breadth.

We then have

$$Q = C \sqrt{\frac{A^3}{b}}$$

$$= C \sqrt{\frac{(2bd + s d^2)^3}{2(b + d)\sqrt{1 + s^2}}}$$

Substituting for $d$ from (2) we have

$$Q = C b^\frac{5}{2} \frac{(2\sqrt{1 + s^2} - s)}{\sqrt{2(1 + s^2 - s^2)^3}} \cdot \sqrt{A}$$

and giving $s$ the value 2

$$Q = 204 C b^\frac{5}{2} \text{ cub. ft. per sec.}$$

if $C$ and $b$ are taken in foot units

or

$$Q = 36.1\ C B^\frac{5}{2}.$$
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Whatever the slope of the sides, the trapezium of best shape will be that in which the sides are made tangent to the circle of radius \( r = d \) having its centre in the surface, and as may be readily shown, all such channels have the same hydraulic mean depth \( \frac{d}{2} \). It follows then that the velocity of flow when the channel is full will be independent of the slope of the sides, and will depend solely on the gradient of the bed. The discharge of any two trapezoidal channels of the best form and of the same gradient and depths, will, when running full, be proportional to their respective mean widths.

The following table indicates how the top and bottom widths for a section of this type, vary with the slope of the sides:

<table>
<thead>
<tr>
<th>Slope.</th>
<th>Angle of inclination of sides.</th>
<th>Width.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Top.</td>
</tr>
<tr>
<td>0 to 1</td>
<td>90°</td>
<td>2·000 ( d )</td>
</tr>
<tr>
<td>.25 to 1</td>
<td>75° . 58'</td>
<td>2·062 ( d )</td>
</tr>
<tr>
<td>.5 to 1</td>
<td>63° . 26'</td>
<td>2·236 ( d )</td>
</tr>
<tr>
<td>.75 to 1</td>
<td>53° . 8'</td>
<td>2·500 ( d )</td>
</tr>
<tr>
<td>1·0 to 1</td>
<td>45° . 0'</td>
<td>2·828 ( d )</td>
</tr>
<tr>
<td>1·5 to 1</td>
<td>38° . 41'</td>
<td>3·606 ( d )</td>
</tr>
<tr>
<td>2·0 to 1</td>
<td>26° . 31'</td>
<td>4·472 ( d )</td>
</tr>
<tr>
<td>2·5 to 1</td>
<td>21° . 48'</td>
<td>5·385 ( d )</td>
</tr>
<tr>
<td>3·0 to 1</td>
<td>18° . 26'</td>
<td>6·325 ( d )</td>
</tr>
</tbody>
</table>

Circular Section.—Fig. 130.

Let \( d \) = diameter of circle.

,, \( \theta \) = angle at centre subtended by wetted perimeter.

Then

\[ A = \frac{d^2}{8} (\theta - \sin \theta) ; \quad P = \frac{d^2}{2} \theta \]

\[ \therefore \frac{A}{P} = \frac{d}{4} \left( 1 - \frac{\sin \theta}{\theta} \right) \]

Suppose \( d \) fixed.

For maximum velocity

\[ \frac{d}{d \theta} \left( \frac{A}{P} \right) = 0 \]

\[ \therefore \theta = \tan \theta \]

\[ \therefore \theta = 257\frac{1}{2}°. \]
For maximum discharge
\[ \frac{d}{d \theta} \left( \frac{A^3}{P^2} \right) = 0 \]

\[ \therefore 3P \frac{dA}{d \theta} - A \frac{dP}{d \theta} = 0 \]

\[ \therefore 3 \theta (1 - \cos \theta) = \theta - \sin \theta \]

\[ 2 \theta - 3 \theta \cos \theta + \sin \theta = 0. \]

The value of \( \theta \) which satisfies this equation is 308°, so that a circular conduit will give its maximum discharge when the depth of water is about 95 of the diameter, the discharge then being about 5 per cent. greater than when completely full.

The discharge corresponding to any depth of water is given by
\[ Q = C \sqrt[3]{\frac{A}{P}} \cdot i \]

\[ = C \sqrt[3]{\frac{d^3}{256} \left( \frac{\theta - \sin \theta}{\theta} \right)} \cdot i \]

\[ = \frac{C}{16} \cdot d^{\frac{5}{2}} \cdot i^{\frac{1}{2}} \sqrt{\frac{(\theta - \sin \theta)^3}{\theta}} \]

when \( \theta = 180° = \pi \)

\[ Q = \frac{C}{5.1} \cdot d^{\frac{5}{2}} \cdot i^{\frac{1}{2}} \text{ cub. ft. per sec.} \]

The semi-circular section when running full has a hydraulic mean depth of \( \frac{d}{4} \), and since this is greater than that of any other form of channel of the same area, this section is well fitted for an open channel.

Where a polygonal channel is used, the hydraulic mean depth is greatest when the sides and bottom of the channel are designed so as to be tangent to a circle having its centre in the water line. The trapezoidal section and rectangular section of greatest flow, are particular cases of this. Where vertical sides are to be used the most suitable form of bottom consists of a circular arc, concave upwards.

**Channel of Constant Mean Velocity.**—Where the depth of water in a channel may vary within wide limits, it is in general desirable to design this so that the velocity of flow may be as nearly as possible independent of the depth. Otherwise, in an open canal, the velocity may become so great as to damage the sides and bottom by scouring (Art. 97), while in a sewer, with low heads, the velocity may become insufficient to produce
the necessary flushing. On the assumption that \( v = C \sqrt{m \cdot i} \), where \( C \) = constant, the only essential condition to be satisfied for \( v \) to be independent of the depth is that the hydraulic mean depth shall also be independent of the depth of water.

Thus the required channel must have sides formed by a continuous curve such that the area bounded by the sides, and any two horizontals varies as the length of the arcs intercepted between these horizontals. No curve can be found to satisfy these conditions, though close approximations may be obtained.

Obviously, a rectangular section of great depth compared with its width would satisfy the conditions approximately, and especially if its

![Diagram](image)

**Fig. 131.**

bottom were constructed so as to offer less resistance per unit area than its sides.

A construction which gives accurate results between certain limits may, however, be obtained as follows.

In Fig. 131 let \( x \) be the half breadth of the section at a height \( y \) above \( M'M \) where the half breadth is \( b \), and let \( s \) be the length of the arc \( MN \)

The position of the axis \( M' \ M \) and the breadth \( b \) are usually fixed from a consideration of the minimum discharge to be expected through the channel, a trapezoidal channel having an upper breadth \( M'M (= 2b) \) being designed to take this minimum discharge when running full. Let \( m \) be the hydraulic mean depth of this small channel, let \( p \) be its half perimeter, and \( a \) its half area. It is required to continue the sides of this channel so as to give a section for which the hydraulic mean depth \( A / P \) shall be equal to \( m \) for all depths of water.
Considering the section of the complete channel to one side of the axis $O'O$, we have

$$A = a + \int x \, dy \quad \text{constant} = m$$

$$\therefore \quad a + \int x \, dy = m (p + s).$$

Differentiating, this gives $x = m \frac{ds}{dy}$.

But

$$\frac{ds}{dy} = \sqrt{1 + \left( \frac{dx}{dy} \right)^2}$$

$$\therefore \quad x^2 = m^2 \left[ 1 + \left( \frac{dx}{dy} \right)^2 \right]$$

$$\therefore \quad \frac{dy}{m} = \frac{dx}{\sqrt{x^2 - m^2}}$$

Integrating this we have

$$y = m \cosh^{-1} \frac{x}{m} + D$$

$$= m \log_e \left\{ x + \sqrt{x^2 - m^2} \right\} + D'.$$

But $x = b$ when $y = 0$

$$\therefore \quad D = -m \cosh^{-1} \frac{b}{m}$$

$$D' = -m \log_e \left\{ b + \sqrt{b^2 - m^2} \right\}$$

$$\therefore \quad y = m \left( \cosh^{-1} \frac{x}{m} - \cosh^{-1} \frac{b}{m} \right)$$

$$= m \log_e \left\{ \frac{x + \sqrt{x^2 - m^2}}{b + \sqrt{b^2 - m^2}} \right\}. \quad (1)$$

From equation (2) the curve of the side may be plotted by calculating values of $y$, corresponding to a series of values of $x$.

Since $v = C \sqrt{m} i$ this velocity with any given gradient may be adjusted to any given value by designing the small channel so as to give the required value of $m$. The only restriction is that $m$ cannot exceed $b \div 2$, this being its value when the lower channel is semicircular, or rectangular with a breadth equal to twice its depth.

**Example.**

To design a channel to give an uniform velocity of flow of 4 feet per second, the half breadth $b$ being 2·5 feet, and $C$ having a value 90.

Here, assuming a rectangular section for the lower channel, of depth 2·5 feet, we have $m = 1·25$ feet.

$$\therefore \quad 4 = 90 \sqrt{1·25} i$$

$$\therefore \quad i = 0.0158.$$
Again when \( x = 5 \) we have

\[
y = 1.25 \log_e \left( \frac{5 + \sqrt{25 - 1.5625}}{2.5 + \sqrt{6.25 - 1.5625}} \right)
\]

\[
= 1.25 \log_e \left( \frac{5 + \sqrt{38.4375}}{2.5 + \sqrt{4.6875}} \right)
\]

\[
= 1.25 \log_e 2.108
\]

\[
= 1.25 \times 2.302 \log_{10} (2.108)
\]

\[
= 933 \text{ feet.}
\]

Obtaining a series of such values of \( y \), corresponding to definite values of \( x \), the section may be constructed. For this particular example the following table shows how the half breadth of the section increases with the depth:—

<table>
<thead>
<tr>
<th>( x )</th>
<th>2.5</th>
<th>5.0</th>
<th>10.0</th>
<th>15.0</th>
<th>20.0</th>
<th>30.0</th>
<th>50.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0.0</td>
<td>0.933</td>
<td>1.817</td>
<td>2.320</td>
<td>2.520</td>
<td>2.882</td>
<td>3.300</td>
</tr>
</tbody>
</table>

The curve is a portion of a catenary, and, writing its equation in the form

\[
\frac{y - D}{m} = \cosh^{-1} \frac{x}{m},
\]

or

\[
m \cosh \left( \frac{y - D}{m} \right) = x,
\]

it will be seen that this catenary has its axis parallel to and at a distance \((-D) = m \cosh^{-1} \frac{b}{m}\) below the axis \(M' M\), while its vertex \(P\) is at a horizontal distance \(m\) from the centre line \(O O'\) (Fig. 131).

In a closed channel, or sewer, it is impossible to make the mean depth, and therefore the velocity, constant for all depths of water. To approximate to this as far as possible the egg shaped sewer (Fig. 182) is often used. In section this consists of two circular arcs centred at \(A\) and \(B\), and connected by a second pair of circular arcs centred

H.A.
at \( C \) and \( C' \). The proportions often adopted in practice are indicated in Fig. 132.

Figure 133 \( a \) and \( b \) shows sections which are sometimes adopted for large sewers to the same end, the hydraulic mean depth being fairly high even with a small discharge.

**Effect of varying \( m \) or \( i \).**

**Assuming**

\[
v = C \sqrt{m \cdot i}
\]

we have

\[
\frac{dv}{dm} = \frac{C}{2} \sqrt{\frac{i}{m}}
\]

or

\[
dv = \frac{C}{2} \sqrt{\frac{i}{m}} \cdot dm = \frac{v}{2m} \cdot dm
\]

\[
\therefore \quad \frac{dv}{v} = \frac{dm}{2m}
\]

Thus a small increase in \( m \) produces one-half its percentage increase in \( v \).

Similarly it may be shown that \( \frac{dv}{v} = \frac{di}{2i} \), so that in calculating \( r \), and therefore the discharge, from this formula, any error in the value

**Fig. 133.**

assumed for the slope will lead to one-half the proportional error in the estimated discharge.
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Again, since

\[ Q = C \sqrt{\frac{g^3}{P}} \cdot i \]

\[ \frac{d Q}{d A} = \frac{3}{2} C \sqrt{\frac{i}{P}} \cdot A^2 \]

\[ \therefore \frac{d Q}{Q} = \frac{3}{2} \frac{d A}{A} \]

so that, neglecting the small increase in the wetted perimeter accompanying an increase in depth, any small increase in the cross-sectional area produced by such a change in depth will be accompanied by 1.5 times this proportional increase in the discharge.

Again assuming \( A \) to be known and a small error to be made in the estimated value of \( P \), we have

\[ \frac{d Q}{d P} = C \sqrt{i} \left( -\frac{1}{2} \frac{A^3}{P^2} \right) \]

\[ \therefore \frac{d Q}{Q} = -\frac{1}{2} \frac{d P}{P} \]

so that this error leads to one-half the proportional error in the estimation of \( Q \).

Since these errors should be severally small and may all occur in the same direction, the total possible error will be equal to their sum.

Thus if the probable error in the estimation of \( i = p \) per cent.

\[ \therefore \] \( A = q \) per cent.

\[ \therefore \] \( P = r \) per cent.

The possible area in the estimation of the discharge, assuming \( C \) to have its correct value, will be given by

\[ \left( p^2 + \frac{r}{2} + 1.5 q \right) \text{ per cent.} \]

ART. 88. - GENERAL EQUATION OF FLOW IN AN OPEN CHANNEL.

Consider a steady stream of cross-sectional area \( A \), flowing over a bed having an inclination \( \theta \) to the horizontal, where \( \sin \theta = \text{slope} = i \).

Let \( A B \) (Fig. 134) be any stream tube, the vertical depths of \( A \) and \( B \) below the surface being \( y_A \) and \( y_B \).

Let \( h_B \) be the loss of head in this stream tube from \( A \) to \( B \), due to frictional resistances.

\[ x \ 2 \]
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Then applying Bernoulli’s equation of energy, we have

\[
\frac{p_A}{W} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{W} + \frac{v_B^2}{2g} + z_B + \alpha h_B.
\]

Let the difference in the level of the water surface over the points A and B be \( r \).

Then \( z_A + y_A = z_B + y_B + r \)

\[
\therefore z_A - z_B = r + y_B - y_A
\]

\[
\therefore \frac{p_A}{W} + \frac{v_A^2}{2g} + r + y_B - y_A = \frac{p_B}{W} + \frac{v_B^2}{2g} + \alpha h_B.
\]

If the stream is sensibly parallel over the length \( A \) \( B \), as will be the case if \( \theta \) is not large, we have

\[
\frac{p_A}{W} = y_A \quad \text{and} \quad \frac{p_B}{W} = y_B
\]

\[
\therefore \frac{v_A^2}{2g} + r = \frac{v_B^2}{2g} + \alpha h_B
\]

\[
\therefore r = \frac{v_B^2 - v_A^2}{2g} + \alpha h_B
\]

If now we imagine the area \( A \) divided into \( n \) elementary sections, each equal to \( \frac{A}{n} \) we get, for each stream tube of area \( \frac{A}{n} \):

\[
\begin{align*}
A_n \cdot r &= \frac{A}{n} \cdot \frac{v_B^2 - v_A^2}{2g} + \frac{A}{n} \cdot \alpha \cdot h_B,
\end{align*}
\]

and, summing these over the whole section,

\[
\Sigma_n \left( \frac{A}{n} \cdot r \right) = \Sigma_n \left( \frac{A}{n} \cdot \frac{v_B^2 - v_A^2}{2g} \right) + \Sigma_n \left( \frac{A}{n} \cdot \alpha \cdot h_B \right).
\]

If we assume that the velocity is constant over any cross section, the above equation reduces to

\[
A \cdot r = A \cdot \frac{v_B^2 - v_A^2}{2g} + \alpha h_B
\]

where \( \alpha h_B \) is the total frictional loss between the cross sections at \( A \) and \( B \).

This is still true of the whole mass of water in the stream if the velocity at a cross section is not uniform, provided that the distribution of velocity is such that the total kinetic energy at that section is equal to the mass of water multiplied by the square of the mean velocity at the section. In this case \( v_B \) and \( v_A \) become the mean velocities. Experiments by Messrs. Fteley and Stearns on the flow of water in the Sudbury conduit, 9 feet wide and 3 feet deep, in which the velocity was measured at 97 different points in a cross section, gave results showing that the error in assuming this to be true was less than 1 per cent. The error
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will be greater in a shallow channel having a rough bed, but in general
the results calculated on this assumption may be taken as substantially
correct.

If \( P \) = wetted perimeter of section, and if \((\bar{v})^2 \) = mean square of the
velocity from \( A \) to \( B \)

\[
A H_B = \frac{f (\bar{v})^2}{2 g} \cdot P \cdot A \cdot B,
\]

\[
\therefore \ r = \frac{r_h^2 - r_A^2}{2 g} + \frac{f (\bar{v})^2}{2 g} \cdot \frac{P}{A} \cdot A \cdot B.
\]

If \( A B = \delta l \) we have \( r = \frac{d r}{d l} \cdot \delta l \), while if \( r_A = r, v_B = v + \frac{d r}{d l} \cdot \delta l \)
and \((r)^2 = (r + m \frac{d r}{d l} \cdot \delta l)^2\), where \( m \) is less than unity.

So that, neglecting small quantities of the second order

\[
\frac{d r}{d l} \cdot \delta l = \frac{2}{2 g} \left( \frac{d v}{d l} \right) \delta l + \frac{f (\bar{v})^2}{2 g} \cdot \frac{P}{A} \cdot \delta l
\]

or

\[
\frac{d r}{d l} = \frac{r}{g} \cdot \frac{d v}{d l} + \frac{f (\bar{v})^2}{2 g} \cdot \frac{P}{A} \cdot \delta l.
\]  

(1)

This is the general equation of flow in an open channel, \( v \) being the
mean velocity at a cross section, and though the assumptions made in its
conception are not altogether justified by the result of experiment, yet it
forms a useful guide and is capable of a wide range of application in the
general problems of channel flow.

If \( h \) is the depth of water at \( A \) (measured vertically from the surface),
the depth at \( B \) is given by

\[
h = \frac{d r}{d l} \cdot \delta l + i \cdot \delta l.
\]

Again the depth at \( B \) is given by

\[
h + \frac{d h}{d l} \cdot \delta l
\]

\[
\therefore \ i \cdot \delta l = \frac{d r}{d l} \cdot \delta l + \frac{d h}{d l} \cdot \delta l
\]

\[
\therefore \ \frac{d r}{d l} = \frac{d h}{d l}.
\]

Substituting this value in (1) we get

\[
i = \frac{d h}{d l} = \frac{r \cdot d r}{g \cdot d l} + \frac{f (\bar{v})^2}{2 g} \cdot \frac{P}{A},
\]

giving the general equation in terms of the slope of the bed.

The physical interpretation of this equation is that the total loss of
(potential and pressure) energy per unit length of the channel, due to the
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fall in the level of the bed and in the depth of the water, is equal to the increase of kinetic energy together with the loss in friction per unit length of channel.

For uniform flow such as occurs in a culvert with slope, velocity, and depth of water constant, we have \( \frac{dh}{dl} = 0 \), \( \frac{dv}{dl} = 0 \)

\[
\therefore \ i = \frac{f \cdot v^2 \cdot P}{2 \cdot g \cdot A}
\]

or the total potential energy is absorbed in overcoming frictional resistances. This gives the relation between the slope and the velocity, or the discharge \( Q \), for since \( v = \sqrt{\frac{2 \cdot g \cdot i \cdot A}{f \cdot P}} \)

\[
\therefore \ v \cdot A = Q = \sqrt{\frac{2 \cdot g \cdot i \cdot A^3}{f \cdot P}}
\]

so that for a given slope we can find the section for \( r \) to be a constant, and to give any required discharge.

With a rectangular section, breadth \( b \), we get for either uniform or non-uniform flow, if \( b \) is constant and if \( Q \) is constant

\[
Q = r \cdot b \cdot h = \text{const.}
\]

\[
\therefore \ r \cdot h = \text{const.}
\]

\[
\therefore \ h \cdot \frac{dr}{dl} + r \cdot \frac{dh}{dl} = 0
\]

\[
\therefore \ h \cdot \frac{dr}{dl} = -r \cdot \frac{dh}{dl}
\]

Substituting this value, equation (2) becomes

\[
\frac{dr}{dl} = -r^2 \cdot \frac{dh}{dl} = \frac{\frac{r^2}{g \cdot h}}{d \cdot l} + \frac{f \cdot r^2 \cdot P}{2 \cdot g \cdot A}
\]

\[
\therefore \ h \cdot \frac{dr}{dl} = \frac{2 \cdot g \cdot A}{1 - r^2}
\]

(3)

Here \( \frac{r^2}{g \cdot h} \cdot \frac{dh}{dl} \) still represents the rate of increase of kinetic energy with length, and shows that the K. E. increases when the depth diminishes, i.e., when \( \frac{dh}{dl} \) is negative.

If \( b \) be great in comparison with \( h \), we may write \( \frac{P}{A} = \frac{b}{h} \cdot \frac{2 \cdot h}{b} = \frac{1}{h} \)

(approximately), especially if, as is very usual in open channels, the bottom is rougher than the sides.
FLOW IN OPEN CHANNELS

Substituting this value in equation (9) we have, for such a channel,

\[
\frac{d h}{d l} = \frac{i - \frac{f v^2}{2 gh}}{1 - \frac{v^2}{gh}} = \frac{1 - \frac{f v^2}{2 gh i}}{1 - \frac{v^2}{gh}}
\]  

(4)

This gives the slope of the water surface, and from a solution of this equation, the profile of this surface may be determined.

Thus for steady uniform flow \( \frac{d h}{d l} = 0 \), i.e., the depth is constant throughout.

\[ \therefore \frac{1 - \frac{f v^2}{2 gh i}}{1 - \frac{v^2}{gh}} = 0 \]

or

\[ h = \frac{f v^2}{2 g i} \]  

(5)

ART. 89.—NON-UNIFORM FLOW.

If, as is usual, the motion while being constant at a point, varies from point to point in the length of the stream, so that \( \frac{d v}{d l} \) and therefore \( \frac{d h}{d l} \) is not zero, let \( H \) be the equivalent depth of an uniform stream of breadth \( b \) which would give the same discharge as the variable stream in unit time.

Then

\[ H = \frac{f V^2}{2 g i} \]

from (5)

where

\[ Q = V b H \text{ or } V = \frac{Q}{b H} \]

\[ \therefore \frac{Q^2}{b^2 H^2} = \frac{2 g i H}{f} \]

or

\[ \frac{Q^2}{b^2 H^2} = \frac{v^2}{h^2 f} = \frac{2 g i H^3}{h^2 f} \]

Substituting \( \frac{2 g i H^3}{h^2 f} \) for \( v^2 \) in (4), this becomes

\[ \therefore \frac{d h}{d l} = i \left\{ \frac{1 - \frac{H^3}{h^3}}{1 - \frac{H^3}{h^3} - \frac{2 i}{f}} \right\} \]  

(6)

This is the differential equation to the curve forming the longitudinal profile of the surface.

Since the value of \( \frac{d h}{d l} \) depends both on the ratio \( \frac{h}{H} \) which may be artificially adjusted in any stream, and on the ratio \( \frac{2 i}{f} \) which is fixed.
once the slope and physical condition of the bed is fixed, an investigation of any particular case of flow must take into account both these factors.

The surface curves corresponding to a few particular cases of flow will now be investigated.

First suppose \( \frac{2i}{f} \) less than unity, the state of affairs existing in a channel of slope less than \( \frac{f}{2} \).

The following may be taken as approximate values of \( f \) at such velocities as are common in practice (Bazin), the foot being the unit of length.

<table>
<thead>
<tr>
<th>Surface Type</th>
<th>Value of ( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth cemented surface</td>
<td>0.0080</td>
</tr>
<tr>
<td>Ashlar or brickwork</td>
<td>0.0087</td>
</tr>
<tr>
<td>Rubble masonry</td>
<td>0.0065</td>
</tr>
<tr>
<td>Earth</td>
<td>0.0120</td>
</tr>
</tbody>
</table>

so that for \( \frac{2i}{f} \) to be less than unity, with a rubble masonry channel the slope would not exceed 0.0025 feet per foot, or 1 in 310.

Case 1 (a). Let \( \frac{2i}{f} < 1 \) and also \( h^8 < \frac{2i}{f} H^8 \).

Both numerator and denominator of the right-hand side of equation (6) are now negative, so that \( \frac{d}{dl} \) is positive, i.e., the depth of water increases down stream.

Also as \( h \) increases it finally reaches the critical value \( \sqrt[8]{\frac{2i}{f}} H \). Here the denominator becomes equal to zero, and in consequence the value of \( \frac{d}{dl} \) becomes \( \infty \), or the surface curve at this point becomes vertical (Fig. 185), and the phenomenon known as the standing wave is produced.

In the figure, suppose the dotted line \( SS' \) to be drawn parallel to the bed, to represent a depth \( \sqrt[8]{\frac{2i}{f}} H \). If by means of a sluice we can
get the water surface below this level, then on passing the sluice the depth of water increases as shown. Finally, when \( h = \sqrt{\frac{2g}{f} \cdot II} \), the curve should be vertical where it intersects this line. Before this limit is reached, however, the hypothesis that the stream lines are sensibly parallel ceases to be even approximately true, and the curve becomes modified as shown in the dotted lines. This is exemplified in the case of a sluice fitted in a channel having a very small slope.

If \( h_1 \) and \( h_2 \) be the depths of the stream before and after its sudden change of level, the value of \( h_2 \) may be calculated.

Let \( v_1 \) and \( v_2 \) be the velocities at sections \( h_1 \) and \( h_2 \).

It is not now legitimate to assume that the loss due to shock at the sudden change of section is \( (v_1 - v_2)^2 \) as in the case of pipe flow, since the pressure over the area \( F' \cdot E' \) (Fig. 136) is no longer uniform and equal to that from \( F' \) to \( I' \) but varies with the depth, and hence one of the fundamental assumptions made in deducing this formula is unjustified.

On applying the equation of momentum, however, to this particular case, we have

Difference of forces acting in the direction of motion, \( p_1 A_1 - p_2 A_2 \),

on the faces \( C \cdot D \) and \( G \cdot H \)

where \( p_1 \) and \( p_2 \) are the mean pressures over the areas \( A_1 \) and \( A_2 \).

Then

\[
p_1 = \frac{W h_1}{2}, \quad p_2 = \frac{W h_2}{2}.
\]

The change of momentum per second, in passing the sections \( C \cdot D \) and \( G \cdot H \)

\[
= \frac{W}{g} \left\{ A_2 v_2^2 - A_1 v_1^2 \right\}.
\]

Also \( A_1 v_1 = A_2 v_2 \), and if the section is rectangular, \( h_1 r_1 = h_2 r_2 \), so that on equating the momentum per second to the force producing it, we have

\[
\frac{v_1^2}{2} - \frac{v_2^2}{2} = \frac{1}{2} \frac{h_1^2}{h_2} \left( h_2 - h_1 \right) \frac{h_2}{2} = \frac{1}{2} h_1 \frac{h_2}{h_1} h_2.
\]

\[
\frac{v_1^2}{2} = \frac{h_2^2 - h_1^2}{2} h_2 = \frac{h_2^2 + h_1^2}{2} h_2.
\]
If \( h_2 - h_1 = x \) this reduces to
\[
\frac{v^2}{g} = \frac{(x + 2 h_1)(x + h_1)}{2 h_1},
\]
\[\therefore x = \sqrt{\frac{h_1^2}{4} + \frac{2 h_1 v^2}{g}} - \frac{3}{2} h_1.
\]

This gives the height of the standing wave.

An explanation of the production of the standing wave may be found as follows. An examination of equation (3) shows that \( \frac{d h}{d l} \) can only be infinite and a standing wave formed when \( v^2 = g h \), or when
\[
\frac{v^2}{g} \frac{d h}{d l} = \frac{d h}{d l}.
\]

But
\[
\frac{v^2}{g} \frac{d h}{d l} = v \frac{d v}{g} \frac{d h}{d l} = -v \frac{d v}{g} \frac{d l}{d l} = -\frac{d}{d l} \left( \frac{v^2}{2 g} \right)
\]
so that the standing wave is produced when
\[
-\frac{d}{d l} \left( \frac{v^2}{2 g} \right) = \frac{d h}{d l}
\]
i.e., when the rate of decrease of kinetic energy is equal to the rate of increase of potential and pressure energy due to an increase in the depth, or vice versa.

Until this point is reached the rate of decrease of kinetic energy is greater than that of increase of pressure and potential energy, the difference being due to energy expended in eddy formation. Assuming for the moment that the surface curve could be continued through the point, we should have the rate of increase of potential and pressure energy greater than that of decrease of kinetic energy, and hence should have an actual increase in total energy, a state of affairs which is manifestly impossible.

This can only be overcome by a sudden change in the distribution of pressure over the section of the stream, the effect being almost identical with that produced by the introduction of a solid obstacle in the path of the stream. As a consequence of the shock thus produced there is a sudden loss of energy in eddy production, the velocity of flow of necessary falls, and a corresponding rise of surface ensues.

After rising to the level, \( h_2 \), where
\[
\frac{2}{f} \sqrt{\frac{2}{g}} < H,
\]
we have the state of affairs considered in Case 1 (b).
Case 1 (b). Here \( \frac{2i}{f} < 1 \), while \( h \) is greater than \( \sqrt[3]{\frac{2i}{f}} \). \( H \) and is
less than \( H \).

In equation (6) the numerator is now negative, while the denominator
is positive, so that \( \frac{dh}{dl} \) is negative, or the depth \( h \) diminishes down-
stream.

As the velocity increases and \( h \) diminishes, the denominator of the
fraction \( \frac{h^3 - H^3}{h^3 - \frac{2i}{f} H^3} \) vanishes before the numerator, so that \( \frac{dh}{dl} \) tends to
a limiting value \(-\infty\) where \( h = \sqrt[3]{\frac{2i}{f}} \cdot H \).

At this point the surface curve becomes vertical as shown in Fig. 137.
If produced by a sudden drop in the bed of a stream, as shown in this
figure, \( h \) increases up-stream, and
approaches more nearly to \( H \) as this
distance increases, the surface curve
being asymptotic to the line \( II' \).

Such a drop in the bed may cause an appreciable increase in the velo-
city of flow for a considerable dis-
tance up-stream and may thus affect
the foundations of structures (bridges, etc.) which may be placed
up-stream, besides causing serious erosion of the bed.

In the case of the sluice (Fig. 135), the water after rising to the
height \( h_2 \) is governed by this second set of conditions, so that the level
again falls until \( h = \sqrt[3]{\frac{2i}{f}} \cdot H \). Inertia then causes the level to fall
below this, when we have the conditions of Case 1 (a) repeated. Thus a
series of stationary waves are produced, the level alternately rising and
falling above and below that given by \( h = \sqrt[3]{\frac{2i}{f}} \cdot H \). At each
successive jump a loss of energy occurs, and the velocity energy after
the jump is therefore diminished. It follows that the value of \( h_2 \) must
be greater after each successive jump, and ultimately will become equal
to \( H \), after which steady flow occurs.

The same reasoning applies to the stream after passing the drop in the
bed (Fig. 137) the depth ultimately settling down to \( II \).

The state of affairs outlined in this second case may be met with where
A flume having a slight inclination delivers water to a penstock from which one or more turbines or water wheels are fed. It then becomes important that the proportions of the flume should be such as to prevent the water level from falling below a certain minimum. This problem will afterwards be considered in detail (p. 324).

**Case 1 (c).**

Let \[ \frac{2i}{f} < 1. \]

\[ h > H. \]

Here the surface is everywhere above the line \( R R' \) (Fig. 138). Both the numerator and denominator in the fraction

\[ i \left( \frac{h^3 - H^3}{h^3 - \frac{2i}{f}H^3} \right) \text{ (equation 6) are positive} \]

\[ \therefore \frac{dh}{dl} \text{ is positive.} \]

Down-stream then the depth increases, and \( \frac{h^3 - H^3}{h^3 - \frac{2i}{f}H^3} \) is ultimately

\[ \frac{h^3 - H^3}{h^3} \]

tends to the limit unity, i.e., \( \frac{dh}{dl} \) tends to the limit \( i \), the slope of the bed. It follows that the down-stream surface tends to become horizontal.

Up-stream \( h \) tends to the limit \( H \), and \( \frac{dh}{dl} \) to the limit zero, so that the curve tends to become asymptotic to the line \( R R' \).

This is the form of surface curve produced by a weir or dam in a stream of small slope, and is of importance since the introduction of such a dam causes what may be a serious raising of the backwater level for some considerable distance up-stream.

Next let \( \frac{2i}{f} \) be greater than unity, the state of affairs usually existing.

**Case 2 (a).**

Let \[ \frac{2i}{f} > 1. \]

\[ h < H. \]
FLOW IN OPEN CHANNELS

\[
\frac{dh}{dl} \quad \text{(equation 6)} \quad \text{is now positive and the depth increases down-stream.}
\]

Since, as \( h \) increases, the numerator vanishes before the denominator \( \frac{dh}{dl} = 0 \) in the limit, i.e., the surface curve tends to become asymptotic to the line \( RR' \) (Fig. 139).

This state of affairs is attained at a sluice in a stream having a slope greater than \( \frac{f}{2} \).

**Case 2 (b).**

Let \( \frac{f}{j} > 1 \), and let \( h \) be greater than \( II \) and less than \( \sqrt[3]{\frac{2i}{f}} \cdot II \).

Here \( \frac{dh}{dl} \) (equation 6) is negative and the depth diminishes down-stream. As \( h \) diminishes the numerator vanishes before the denominator and in the limit \( \frac{dh}{dl} = 0 \), or the curve becomes asymptotic to the line \( RR' \), and the stream settles down to the uniform depth \( II \). This state of affairs is realised where an obstacle in a river bed may have caused the level to rise to within the required limits. The surface curve is then as shown in Fig. 140. Up-stream, as \( h \) increases it finally reaches the value \( \sqrt[3]{\frac{2i}{f}} \cdot II \). Here \( \frac{dh}{dl} \) is \(-\infty\), and the curve becomes perpendicular to the bed of the stream. As \( h \) increases still further the state of affairs considered in Case 2 (c) is attained. This vertical front is seen when a sudden rush of water, such as may be produced by the bursting of an embankment, is caused in a channel of fairly rapid slope. It is also seen in the bores which occur at certain states of the tide in the Seine between Havre and Rouen, and in various other rivers and contracted channels.

**Case 2 (c).**

\[
\begin{cases}
2i > 1 \\
h > \sqrt[3]{\frac{3i}{f}} II
\end{cases}
\]
HYDRAULICS AND ITS APPLICATIONS

Here \( \frac{dh}{dl} \) is positive \( \therefore h \) increases down-stream. Down-stream \( h - H^2 \) tends to the limiting value unity, so that in this direction
\[
h^3 - \frac{2}{j} H^3
\]
the limiting value of \( \frac{dh}{dl} \) is \( i \), or the surface tends to become horizontal (Fig. 141).

Up-stream, as the depth diminishes, we reach a point where
\[
h = \sqrt[3]{\frac{2}{j} i H^2},
\]
and, for this value of \( h \), \( \frac{dh}{dl} = \infty \) or the surface curve here becomes perpendicular to the bed of the stream, a standing wave being produced.

This is the curve obtained where an under-water obstruction such as a dam or broad-crested weir is placed across a stream of rapid slope. Since the possibility of \( \frac{dh}{dl} \) becoming infinite, depends on \( \frac{2}{j} i \) being greater than unity, the production of a standing wave under these circumstances is only possible where this latter condition is satisfied.

In practice the two most important cases are those represented in 1 (b) and 1 (c). In the first of these, the effect of a sudden drop in the bed of a stream may, as already explained, be serious, while the case of the reduction in level in a fore-bay feeding a power plant, used by the sudden demand for energy by the turbines also comes under this heading. In the second, the effect of a dam in increasing the surface elevation at points further up-stream is important. The investigation of each of these cases resolves itself into determining, from a solution of equation (6), the value of \( h \) corresponding to any point at a distance \( l \) from some datum, since when this is known, the rise or fall from normal, and consequently the change in velocity, can be determined.

To obtain a solution for the equation, we have
\[
\frac{dh}{dl} = \frac{\left( \frac{h^3}{H^3} - \frac{2}{j} \right)}{\left( \frac{h^3}{H^3} - 1 \right)}.
\]

Fig. 141.
BACKWATER FUNCTION

Writing \( \frac{h}{H} = m \), so that \( \frac{d}{dl} \frac{h}{m} = \frac{H}{l} \), this becomes

\[
\frac{H}{l} \frac{d}{dl} \frac{m}{l} = i \left( \frac{m^2 - 1}{m^2 - \frac{2i}{f}} \right)
\]

\[
\therefore \frac{d}{dl} \left( \frac{1}{m^2 - \frac{2i}{f}} \right) = \frac{i}{H} d l.
\]

Integrating this expression,

\[
\int \frac{d}{dl} \frac{m}{m^2 - 1} = m + \left( 1 - \frac{2i}{f} \right) \int \frac{d}{dl} \frac{m}{m^2 - 1} + \text{const.}
\]

while

\[
\int \frac{d}{dl} \frac{m}{m^2 - 1} = \frac{1}{2} \int \left( \frac{m + 1}{m^2 + m + 1} \right) d m
\]

\[
= -\frac{1}{2} \left( \frac{1}{3} \log \frac{m^2 + m + 1}{(m - 1)^2} + \frac{2}{\sqrt{3}} \tan^{-1} \frac{2m + 1}{\sqrt{3}} \right)
\]

\[
\therefore \frac{i}{H} l = m - \left( 1 - \frac{2i}{f} \right) \int \left[ \frac{1}{6} \log \frac{m^2 + m + 1}{(m - 1)^2} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2m + 1}{\sqrt{3}} \right] + C.
\]

The expression

\[
\left[ \frac{1}{6} \log \frac{m^2 + m + 1}{(m - 1)^2} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2m + 1}{\sqrt{3}} \right]
\]

is often termed the "backwater function." Writing this as \( \phi(m) \), the equation becomes

\[
\frac{i}{H} l = m - \left( 1 - \frac{2i}{f} \right) \phi(m) + C
\]

\[
\therefore i l = h - H \left( 1 - \frac{2i}{f} \right) \phi(m) + C
\]

\[
\therefore h = i l + H \left( 1 - \frac{2i}{f} \right) \phi(m) + C
\]

Thus if \( h_1 \) and \( h_2 \) are the depths at points distant \( l_1 \) and \( l_2 \) from the datum, we have

\[
h_1 - h_2 = i (l_1 - l_2) + H \left( 1 - \frac{2i}{f} \right) \left\{ \phi(m_1) - \phi(m_2) \right\}
\]

The following table gives values of \( \phi(m) \) for different values of \( \frac{h}{H} \) in the case of a dam, where \( h \) is always greater than \( H \).
HYDRAULICS AND ITS APPLICATIONS

From these values a curve may be constructed if required for use, and intermediate values obtained by interpolation.

<table>
<thead>
<tr>
<th>$\frac{h}{H}$</th>
<th>$\phi \ (m)$</th>
<th>$\frac{h}{H}$</th>
<th>$\phi \ (m)$</th>
<th>$\frac{h}{H}$</th>
<th>$\phi \ (m)$</th>
<th>$\frac{h}{H}$</th>
<th>$\phi \ (m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>$\infty$</td>
<td>1.020</td>
<td>2.098</td>
<td>1.10</td>
<td>1.587</td>
<td>2.20</td>
<td>1.015</td>
</tr>
<tr>
<td>1.001</td>
<td>3.090</td>
<td>1.025</td>
<td>2.025</td>
<td>1.15</td>
<td>1.468</td>
<td>2.50</td>
<td>0.989</td>
</tr>
<tr>
<td>1.002</td>
<td>2.860</td>
<td>1.030</td>
<td>1.966</td>
<td>1.20</td>
<td>1.387</td>
<td>3.0</td>
<td>0.963</td>
</tr>
<tr>
<td>1.003</td>
<td>2.725</td>
<td>1.036</td>
<td>1.938</td>
<td>1.30</td>
<td>1.280</td>
<td>4.0</td>
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</tr>
<tr>
<td>1.004</td>
<td>2.629</td>
<td>1.044</td>
<td>1.813</td>
<td>1.40</td>
<td>1.211</td>
<td>5.0</td>
<td>0.927</td>
</tr>
<tr>
<td>1.005</td>
<td>2.555</td>
<td>1.050</td>
<td>1.803</td>
<td>1.50</td>
<td>1.162</td>
<td>7.0</td>
<td>0.915</td>
</tr>
<tr>
<td>1.007</td>
<td>2.415</td>
<td>1.056</td>
<td>1.763</td>
<td>1.60</td>
<td>1.125</td>
<td>10.0</td>
<td>0.911</td>
</tr>
<tr>
<td>1.010</td>
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<td>1.060</td>
<td>1.745</td>
<td>1.70</td>
<td>1.096</td>
<td>15.0</td>
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<td>1.80</td>
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<td>20.0</td>
<td>0.908</td>
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<td>1.015</td>
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<td>1.656</td>
<td>2.00</td>
<td>1.089</td>
<td>50.0</td>
<td>0.907</td>
</tr>
</tbody>
</table>

In the case of a fall down-stream, $\frac{h}{H}$ is always less than unity. The following table gives values of $(\phi \ m)$ for this case.

<table>
<thead>
<tr>
<th>$\frac{h}{H}$</th>
<th>$\phi \ (m)$</th>
<th>$\frac{h}{H}$</th>
<th>$\phi \ (m)$</th>
<th>$\frac{h}{H}$</th>
<th>$\phi \ (m)$</th>
<th>$\frac{h}{H}$</th>
<th>$\phi \ (m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>$\infty$</td>
<td>2.183</td>
<td>.850</td>
<td>1.367</td>
<td>.100</td>
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<td>.650</td>
<td>1.006</td>
<td>.200</td>
<td>5.08</td>
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<tr>
<td>.995</td>
<td>2.552</td>
<td>1.769</td>
<td>.600</td>
<td>.989</td>
<td>.150</td>
<td>4.53</td>
<td></td>
</tr>
<tr>
<td>.994</td>
<td>2.491</td>
<td>1.705</td>
<td>.550</td>
<td>.977</td>
<td>.100</td>
<td>4.02</td>
<td></td>
</tr>
<tr>
<td>.992</td>
<td>2.395</td>
<td>1.602</td>
<td>.500</td>
<td>.819</td>
<td>.050</td>
<td>3.52</td>
<td></td>
</tr>
<tr>
<td>.990</td>
<td>2.319</td>
<td>1.522</td>
<td>.450</td>
<td>.763</td>
<td>.000</td>
<td>3.02</td>
<td></td>
</tr>
</tbody>
</table>

As an example of the use of these tables, calculate the rise in level at a point ½ mile up stream, produced by a dam arranged so as to raise the level at its crest through 8 feet. The original depth of the stream, supposed uniform, was 2 feet, the slope of the bed 1 in 500 (= 0.002), and the value of $f = 0.006$.

The necessary height of the dam may be calculated by an application of
of equation (1), p. 168. Assuming this to be done, we have, in the preceding formulae

\[ H = 2 \cdot \left(1 - \frac{2i}{j}\right) = 1 - \frac{2}{3} = \frac{1}{3}. \]

Let the suffix (1) refer to a point just above the dam, and the suffix (2) to the given point. Then since the positive direction of \( l \) is down-stream, we have in equation (10), \( h_1 = 10 \); \( l_1 = 0 \); \( l_2 = -2640 \), and from the tables we get \( \phi \left( \frac{h_1}{2} \right) = 0.927 \), so that this equation becomes

\[ 10 - h_2 = 0.002 \cdot (2640) + 0.06 \left(9.27 - \phi \left( \frac{h_2}{2} \right) \right) \]

\[ \therefore h_2 - \frac{2}{3} \phi \left( \frac{h_2}{2} \right) = 4.102. \]

This equation can only be solved by trial.

Let

\[ h_2 - \frac{2}{3} \phi \left( \frac{h_2}{2} \right) - 4.102 = y \]

Then if

\[ h_2 = 5, y = 5 - 0.6593 - 4.102 = 0.2987 \]

\[ h_2 = 4, y = 4 - 0.6927 - 4.102 = -0.7947 \]

For a solution of the equation, \( y \) must equal 0, and since the value of a continuous function such as \( y \) cannot change from + to - without passing through the value zero, it follows that for some value of \( h_2 \) between 4 and 5, \( y = 0 \).

Evidently, too, the correct value of \( h_2 \) is nearer 5 than 4. Try \( h_2 = 4.75 \).

If \( h_2 = 4.75, y = 4.75 - 0.6656 - 4.102 = -0.0176 \). The value of \( h_2 \) is then between 4.75 and 5.0.

A close approximation to the correct result can then be obtained by drawing a curve connecting these values of \( y \) and of \( h_2 \) already found. Where this curve intersects the axis of \( h_2 \) we shall have the value of \( h_2 \) which makes \( y = 0 \), and therefore which satisfies the equation. In the problem, \( h_2 = 4.78 \) provides a very close approximation to the correct value.

At a distance up-stream equal to 4,000 feet the value of \( h \), determined in the same way, is 2.34 feet. Since the slope is 0.002 the height of the bed at this latter point, above that at the dam, is 0.002 \times 4,000 = 8.0 feet.

The surface at this latter point is therefore 84 feet higher than at the dam. With the dam removed and the flow per minute unchanged, the flow being uniform and the depth of channel equal to \( H \), the difference of level instead of being 84 feet would be 8.0 feet.

Figure 142 illustrates the form of backwater curve observed by Η.Α.
D'Aubuisson on the Wesser. Here the mean slope of the bed was 2'33 feet per mile = 0'00141, and the depth before introducing the dam was 2'46 feet. The effect of the dam in raising the surface level was apparent for 4'33 miles up-stream.

In a second series of observations on the Werra, the following results were obtained.

Mean depth $H = 1'7$ feet, width = 80 feet; fall = 3'88 feet per mile.

![Fig. 142.](image)

A dam 15'66 feet in height was placed across the stream, and the height of water over the sill was found to be 1'13 feet.

The following table indicates the observed and calculated depths, and rises in surface level at points above the dam.

<table>
<thead>
<tr>
<th>Distance of observed point from Dam.</th>
<th>0</th>
<th>¾ Miles</th>
<th>1'5 Miles</th>
<th>2'5 Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of water in feet</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>observed</td>
<td>16'79</td>
<td>12'30</td>
<td>11'28</td>
<td>10'09</td>
</tr>
<tr>
<td>calculated</td>
<td>—</td>
<td>19'50</td>
<td>11'35</td>
<td>3'20</td>
</tr>
<tr>
<td>Rise of level in feet</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>observed</td>
<td>15'09</td>
<td>10'60</td>
<td>10'58</td>
<td>1'31</td>
</tr>
<tr>
<td>calculated</td>
<td>—</td>
<td>11'80</td>
<td>10'65</td>
<td>1'50</td>
</tr>
</tbody>
</table>

In these calculations the value of $f$ has been taken as $0'2$.

As an example of the effect of a drop in the bed of a stream in producing an increased velocity at points up stream, consider the same stream as before to have a fall at some point in its length (Case 1 (b)), and suppose that this causes a lowering of the surface just above the fall through a depth of 6 inches.
Then as before \( H = 2 ; \ 1 - \frac{2i}{f} = \frac{1}{3} \).

Also in equation (10), we have

\[
h_1 = 1.5 ; \ l_1 = 0 ; \ \phi (m_1) = \phi \left( \frac{1.5}{2} \right) = 1.159 \ \text{(from table)}.
\]

To determine the depth at a point 50 feet up stream we have

\[
l_2 = -50 \ \text{feet}, \ \text{so that the equation becomes}
\]

\[
1.50 - h_2 = -0.02 \ (50) + \frac{2}{3} \left[ 1.159 - \phi \left( \frac{h_2}{2} \right) \right]
\]

\[
\therefore \ h_2 - \frac{2}{3} \phi \left( \frac{h_2}{2} \right) = 1.50 - 10 - 0.773 = 0.27.
\]

Putting \( h_2 = \frac{2}{3} \phi \left( \frac{h_2}{2} \right) = 0.27 \) we get

if \( h_2 = 1.8, \ y = 1.8 - 1.0146 - 0.627 = +0.1584 \)

,, \( h_2 = 1.9, \ y = 1.9 - 1.1793 - 0.627 = +0.1037 \)

,, \( h_2 = 1.95, \ y = 1.95 - 1.3393 - 0.627 = -0.0163 \)

On plotting values of \( h_2 \) and \( y \), the curve shows that \( y_2 \) is zero when \( h_2 = 1.946 \) (approximately), and this, therefore, gives the depth at the given point.

At a point 20 feet up stream we have

\[
h_2 - \frac{2}{3} \phi \left( \frac{h_2}{2} \right) = 0.667
\]

On solving this equation in the same way, we find that the depth here is 1.932 feet (approximately). Since the breadth is constant, the mean velocity at any point is inversely proportional to the depth, so that at the two points 20 and 50 feet up stream the velocities are increased by the fall in the ratios \( \frac{2}{1.932} \) and \( \frac{2}{1.946} \) respectively.

This action becomes increasingly important as the slope is diminished. For example, in the previous case, if the slope were diminished to 0.01, the velocities at the same two points would be increased by the fall in the ratios \( \frac{2}{1.56} \) and \( \frac{2}{1.72} \) respectively.

As a further example of the use of these formulae, consider the case of a flume of rectangular section, feeding a forebay from which a turbine is to be supplied.

The breadth of flume is 20 feet, the slope 1 in 1,000, the length 1,000 yards, and the value of \( f = 0.008 \). The discharge required is 500 cubic feet per second.
HYDRAULICS AND ITS APPLICATIONS

The value of $H$, the uniform depth necessary to give this supply is given by

$$H = \sqrt{\frac{3}{g}} \frac{\phi^2 \rho^2}{\theta^2 \gamma^2} = \sqrt{\frac{3}{25 \times 25 \times 10,000}} \frac{61.4 \times 400 \times 0.001}{3.08 \text{ feet.}}$$

If at the upper end the depth of water is greater than this, say 4 feet, we have (Case 1 (c)) and everywhere $h > H$. The depth of water thus increases down stream.

Applying equation (10), we now have

$$\begin{cases} h \text{ (at entrance to flume)} = 4.0 \quad l_1 = 0 \\ H = 3.08 \quad l_2 = 3,000 \text{ feet} \\ \frac{2}{i} = 0.002 \quad \frac{2}{i} = 0.003 = 2 \\ \phi \left( \frac{h_1}{H} \right) = \phi \left( \frac{4}{3.08} \right) = \phi (1.3) = 1.280 \text{ (tables) } \end{cases}$$

$$\therefore 4 - h_2 = -0.001 (3,000) + \frac{3.08}{3} \left[ 1.280 - \phi \left( \frac{h_2}{H} \right) \right]$$

$$\therefore 7 - 1.315 = h_2 - 1.027 \phi \left( \frac{h_2}{H} \right)$$

If $h_2 = 6.70 \quad y = 1.015 - 1.027 (1.017) = -0.030$

,, $h_2 = 6.75 \quad y = 1.065 - 1.027 (1.015) = -0.022$.

The correct value of $h_2$, the depth in the forebay, is approximately 6.78 feet. The depth of water in the flume at different points in its length can be calculated in the same way, and the necessary height of side ascertainment.

If, at the upper end, the depth of water is equal to $H$, this will remain constant throughout, while if less than $H$ and greater than $\sqrt{\frac{3}{g}} \frac{2}{i} H$ or 871 $H$, i.e., between 3.06 feet and 2.68 feet, we get Case 1 (b). The height will now decrease down stream until it reaches the value $\sqrt{\frac{2}{g}} \frac{2}{i}$, after which a series of waves will be produced, and the depth will, as explained in Case 1 (b), finally settle down to 3.08 feet.

The critical point is found by putting $h_2 = \sqrt{\frac{2}{g}} \frac{2}{i} H = 2.68 \frac{2}{i}$ in equation (10). Let $h_1 = 3.0$ feet.

Then we have at the critical point

$$3 - 2.68 = 0.001 (-h_2) + \frac{3.08}{3} \left\{ \phi \left( \frac{3.0}{3.08} \right) - \phi \left( \frac{2.68}{3.08} \right) \right\}$$
\[ \therefore 0.32 = 0.001 (- l_2) - 1.027 \{ 1.996 - 1.429 \} \]
\[ \therefore 262 = 0.001 l_2 \]
\[ \therefore l_2 = 262 \text{ feet}. \]

If, at the entrance, the depth is less than 2.68 feet, as might occur if this flume were fed from a sluice, we have Case 1 (a) repeated, and the depth would finally increase up to 3.08 feet. With an open channel leading directly out of a reservoir, the required discharge could only be obtained by having the depth at entrance equal to or greater than \( H \).

**Art. 90. — Channel with Horizontal Bed.**

Here \( i = 0 \) is zero, so that equation (6) of the last article ceases to apply. Making \( i = 0 \) in equation (3), p. 310, we get

\[
\frac{d h}{d l} = \frac{2 g A}{P} \left( 1 - \frac{v^2}{g h} \right) \quad (1)
\]

Writing \( v^2 = \frac{Q^2}{A} \), we have, if the channel is rectangular and broad, so that \( \frac{P}{A} = \frac{1}{h} \) (sensibly),

\[
\frac{d h}{d l} = \frac{f Q^2}{2 g \frac{v^2}{h} h^3} \left( 1 - \frac{2 g \frac{v^2}{h} h^3}{f Q^2} \right)
\]

\[ \therefore d h \left( \frac{2 g \frac{v^2}{h} h^3}{f Q^2} - \frac{2}{f} \right) = d l. \quad (2) \]

Integrating between the limits \( l_1 \) and \( l_2 \) we get

\[ l_1 - l_2 = \int \frac{Q^2}{g \frac{v^2}{h} h^3} \left( h_1^4 - h_2^4 \right) - 2 \left( h_1 - h_2 \right) \quad (3) \]

from which the difference in level \( h_1 - h_2 \) at any two points distant \( l_1 - l_2 \) from each other, may be calculated when a given quantity \( Q \) cubic feet per second is flowing along the channel.

Expressing (2) as

\[
\frac{d h}{d l} = \frac{1}{f} \left( \frac{g \frac{v^2}{h} h^3}{Q^2} - 1 \right) \quad (4)
\]

we have, for \( \frac{d h}{d l} \) to be infinite, the condition

\[ g \frac{v^2}{h} h^3 = Q^2 = v^2 \frac{v^2}{h} h^3 \]

\[ \therefore v^2 = g h, \]
so that if by any means such as drawing off a considerable amount of water suddenly by opening a lock gate, the velocity can be made equal to $\sqrt{\frac{g}{h}}$, a wave with vertical crest will be produced.

Art. 91.—Change of Level in a Stream Produced by Bridge, Piers, etc.

Where a series of piers are placed across the bed of a stream, the effect is to raise the up-stream level exactly as if a dam were placed in the stream, and the form of the surface curve will depend on whether the stream satisfies the conditions of Case 1 (c) or 2 (c) (pp. 316 and 817).

The height will be a maximum at the up-stream end of the pier. On arriving at the contracted section of the stream, the velocity will be increased, the increase in kinetic energy necessitating a corresponding loss of potential energy, and the depth is diminished. On again arriving at the open channel the velocity diminishes and the depth increases (Fig. 143).

Neglecting losses by friction between the sections (1) and (2), if $b_1$ and $b_2$ are the effective breadths and $h_1$, $h_2$ the depths at these points, we have

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g}$$

assuming the kinetic energy to be that given by $\frac{v_1^2}{2g}$ and $\frac{v_2^2}{2g}$ where $v_1$ and $v_2$ are the mean velocities at the two sections, an assumption which within narrow limits is justified by experiment.

$$v_2^2 = v_1^2 + 2gyx,$$

where $x = h_1 - b_2$.

If the discharge through the contracted area at (2) is given by $Q$, $h_1$, $h_2$, $v_2$, we have

$$v_2 = \frac{Q}{c b_1 h_1}$$
EFFECT OF BRIDGE PIERS

Also

\[ v_1 = \frac{Q}{b_1 h_1} \]

\[ \therefore \frac{Q^2}{(c b_2 h_2)^2} = \frac{Q^2}{(b_1 h_1)^2} + 2 g x \]

\[ \therefore x = \frac{Q^2}{2g} \left( \frac{1}{c^2 b_2^2 h_2^2} - \frac{1}{b_1^2 h_1^2} \right) \]

or

\[ x = \frac{Q^2}{2g} \left( \frac{1}{c^2 b_2^2 (h_1 - x)^2} - \frac{1}{b_1^2 h_1^2} \right). \]

The value of \( c \) varies with the form of pier, but with pointed cutwaters is about ‘95 (Eytelwein), diminishing to ‘85 for a bridge having square or rectangular piers. By considering the problem as one of flow through a weir or notch having a submerged crest (this crest being level with the bed of the stream), and under an effective head \( x \), we may obtain a second expression for \( Q \) in terms of \( x \), and by equating these two expressions the value of \( h_1 \) may be obtained in terms of \( Q \) and of \( b_1 \) and \( b_2 \). From this, an application of equation (9) (p. 319) will give the depth at any point up stream, and the entire up-stream profile may then be plotted. On the down-stream side of the obstacle there is a gradual rise of the surface level as the depth increases to a uniform value \( H \).

Art. 32.

With radial outward flow over a horizontal bed, such as occurs when a vertical stream impinges on such a surface, we have, if \( h \) is the depth at a radius \( r \), \( Q \) the quantity per second, and \( r \) the velocity at radius \( r \),

\[ Q = r \times 2\pi rh = \text{const.} \]

\[ \therefore v r \frac{dr}{dr} + r h \frac{dr}{dr} + r h = 0 \]

\[ \therefore \frac{dv}{dr} = -\left( \frac{r \frac{dh}{dr} + \frac{v}{r}}{h \frac{dr}{dr}} \right) \quad (1) \]

Substituting this value in equation (2) (p. 309), we have

\[ i - \frac{dh}{dr} = -\left( \frac{v^2}{g h} \frac{dh}{dl} + \frac{v^2}{g r} \right) + \frac{fv^2}{2g} \cdot \frac{P}{A}. \quad (2) \]

But with \( q \) horizontal bed \( i = n \).

Also

\[ \frac{P}{A} = \frac{2\pi r}{2\pi rh} = \frac{1}{h}, \text{ and } l = r \]

\[ \therefore (2) \text{ becomes } \frac{dh}{dr} = \frac{v^2}{g h} \frac{1 - \frac{fv^2}{2g h}}{1 - \frac{v^2}{g h}}. \quad (3) \]
This becomes infinite if \( v^2 = gh \),

i.e., if

\[
\frac{\pi^2 \cdot \sqrt{\frac{g}{h}}}{4} = \frac{h^3}{\sqrt{\frac{g}{h}}}.
\]

if

\[
h^3 = \frac{\pi^2}{4} \cdot \frac{1}{g}.
\]

(4)

At the radius at which this relation holds, a standing wave will be formed.

The height of this wave may be estimated, as in the case of the wave produced in a rectangular channel.

If \( v_1 \) and \( v_2 \) are the velocities and \( h_1 \) and \( h_2 \) the depths immediately before and after the rise, so that \( h_2 - h_1 = x \), we have, as on p. 313,

\[
\frac{v_1^2}{g} = \frac{(h_2 + h_1)h_2}{2h_1}
\]

from which

\[
h_2 - h_1 = x = \sqrt{\frac{h_1^3}{4} + \frac{2h_1v_1^2}{g}} - \frac{3}{2}h_1.
\]

Art. 93.—Change of Level Produced by the Passage of a Boat through a Narrow Canal with Horizontal Bed.

Let \( A = \) cross sectional area of canal.

\( a = \) sectional area of vessel amidships, beneath water line, by a plane perpendicular to its axis.

Let \( v = \) velocity of vessel.

Here the state of affairs may be simplified if we first consider the water to be a perfect fluid. As the vessel moves along through this fluid, the volume displaced by the forepart passes along backwards between the vessel and the sides and bottom of the canal to fill the space vacated in the rear. In this case we get a backward current extending from the prow to the stern of the boat, its velocity increasing as the effective area of the channel diminishes, and having a maximum value \( v \left( \frac{a}{A - a} \right) \) at the amidships section.

To produce this current a surface gradient is necessary, the surface falling from its normal level at the prow to a minimum at the amidships section, and from this point rising to its normal level at the stern.

* Due to the adhesion and viscosity of the fluid, however, a mass of water is dragged along with the boat, forming a current confined mainly to the centre and surface of the canal.

* Since, for permanence of the régime, the backward flow across any section of the canal must equal the corresponding forward flow, the back-
ward bottom current must now be sufficiently great to supply an additional mass of water equal to this, so that its velocity at the minimum section will be \( K + \frac{a'}{A - a'} \) \( v \), its velocity at the bows being \( K v \). Here 
\[ a' = a + a_1 \] where \( a_1 \) is the area of the channel occupied by the forward current at the amidships section. \( A - a' \) is then the effective area of the backward current.

But to produce a backward velocity of flow at the bows of the boat the surface level at the bows must be less than at some distance ahead, and will thus be below the normal. The result is that the water level in the canal falls as the boat approaches, has its minimum value near the amidships section, and then rises to attain its normal value. The effect of the bow waves in modifying the level at the bows is here neglected.

By applying equation (3) (p. 325), the difference of level at any two points in advance of the boat may be deduced in terms of \( K v \), since \( Q = K v b h_1 \), where \( h_1 \) is the depth of water at the bows.

Fig. 144 shows the surface curves for a perfect fluid (dotted lines) and for water.\(^1\)

If the boat is nearer to one side of the channel the velocities of flow are greater on this side of the boat, the pressures, particularly abaft its beam are consequently less, with the result that it tends to sheer off towards the further side.

**Art. 93a.—Suction Effect between Passing Ships.**

Even in open water, where one boat is overtaking another in moderately close proximity on a parallel path their mutual action, due to interference of currents between their hulls may have serious effects: The mass of water displaced by the forepart of the leading boat, returning to fill the space vacated by its stern causes a continual influx of water towards the

---

\(^1\) For the further investigation of the change of level round a moving vessel, a paper in *The Engineer*, vol. 63, p. 202, may be consulted.
stern of the latter, and this is increased by the influx necessary to provide the water thrown astern by the propeller. Further astern the impact of the streams converging from the two sides of the hull produces a region of outflowing currents.

The bows of the overtaking boat first come within the influence of this region which tends to produce a slight outward sheer. As it creeps further ahead the bows come within the influence of the infalling currents while the stern is still being repelled, with a resultant tendency to inward sheer which has often led to serious collisions. This effect is a maximum where the bows of the follower are about one-third of its length aft the bows of the leading ship. As the follower draws further ahead the tendency to sheer diminishes and is replaced by a tendency to bodily inward drift, while when almost abreast, the bows of the follower become exposed to the outflowing currents from the leader's bows while its stern is still being attracted, with a consequent tendency to an outward sheer.\(^1\)

The effect depends largely on the sizes, speeds, and relative speeds of the vessels, increasing with the size of the leader, and with the common speed, and diminishing as the relative speed increases.

**Art. 91.**—Flow round River Bends.

A river flowing through an alluvial plain always tends to gradually increase any winding which may occur in its course, until finally a new channel is cut through the narrow neck of land thus formed. The following explanation of this scouring of the outer bank of a bend and the deposition of detritus on the inner bank has been given by Professor James Thompson.\(^2\)

In consequence of the centrifugal force, the pressure at any level in a transverse section of the stream increases outwards, so that the level of the free surface is highest near the outer bank. Near the bottom, however, the resistance of the bed reduces the velocity and consequently the centrifugal force of the water, which now becomes insufficient to overcome the tendency to inward flow produced by the higher level of the free surface at the outside of the curve. The water near the bottom, then has a tendency to flow inwards and to carry with it gravel and other detritus which is left at the inner bank. Experiments in a model river

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1 For a full discussion of the phenomena of interaction an article in "Bedrock," Vol. 1, No. 1, pp. 66—87, may be consulted.

FLOW ROUND RIVER BENDS

bend, in which the direction of flow was indicated by coloured stream lines and by the behaviour of threads tied to pins fixed in the bed of the stream, as well as by floating particles of matter, indicated a state of affairs as represented in Fig. 115. Here the dotted line $A B$ indicates the path of a particle floating in the surface, while the curves shown in full represent the motion near the bed of the stream.

As indicated, a counter-current flows from inner to outer bank over the upper portion of the stream, but since the same volume of water is moved by the two currents, and since the sectional area of the outer current is comparatively very large, its effect in carrying suspended matter to the outer bank is negligible.

While this theory undoubtedly accounts for a portion of the erosion, and for the deposition of detritus at the inner bank, it is probable that the impact of the stream on the concave bank is a more potent factor in actually causing erosion, and more particularly is this the case when the stream is in flood and when in consequence the erosive effects are most serious. Under such circumstances observation shows that the surface velocity is a maximum near the outer, and not the inner, bank.

Art. 94A.—Loss of Head Produced by Bends in an Open Channel.

Very little experimental evidence is available regarding the loss of head due to bends in an open channel. Experiments on a cement lined semicircular conduit, 9.8 feet in diameter, divided into four consecutive sections $A B C D$, showed the following results.\(^1\)

Section $A$ is a tangent 640 feet long.

" $B$ is 120 feet long and includes a curve of 100 feet radius.

\(^1\) *Engineering Records*, Oct. 21st, 1911. By E. G. Hobson.
Section C is 230 feet long and includes two 50 feet radius reverse curves.

"D is 1075 feet long and is practically straight.

The horizontal curves were approximated to in construction by 10 feet tangents. The mean depth throughout was approximately 4 feet and the hydraulic mean radius 2·14 feet.

<table>
<thead>
<tr>
<th>Section</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean velocity</td>
<td>7·10</td>
<td>6·86</td>
<td>6·94</td>
<td>7·15</td>
</tr>
<tr>
<td>Co-efficient C</td>
<td>129</td>
<td>114</td>
<td>90</td>
<td>119</td>
</tr>
<tr>
<td>Kutter's n</td>
<td>0·0132</td>
<td>0·0149</td>
<td>0·0189</td>
<td>0·0142</td>
</tr>
</tbody>
</table>

The low value of the coefficient C in D as compared with that in A is doubtless due to the loss produced in this section during the redistribution of velocities produced by bend C and ought strictly to be debited to that bend, while similarly, a certain portion of the loss really due to bend B would appear due to the bends in the next section C.

Probably, the following values of C would be more approximately correct.

<table>
<thead>
<tr>
<th></th>
<th>Value of C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. and D. Straight channel</td>
<td>128</td>
</tr>
<tr>
<td>B. Curve of 100 ft. radius</td>
<td>118</td>
</tr>
<tr>
<td>C. Reverse curves of 50 ft. radius</td>
<td>88</td>
</tr>
</tbody>
</table>

Art. 95.—Distribution of Velocity in an Open Channel.

Depression of the Filament of Maximum Velocity.

As in the case of a closed pipe, the resistance introduced by the solid boundaries causes the velocity to diminish in the neighbourhood of the
sides and bottom of an open channel, and from analogy with pipe flow it might be expected that the maximum velocity in any cross section would be found in the surface astf at the centre of the stream.

While this is commonly the case in a broad, rapid, and shallow stream in any other case the filament of maximum velocity is usually found below the surface even with a down-stream wind. Its depth varies with the direction of the wind, with the depth and physical characteristics of the stream, and with the velocity of flow. On a calm day it usually lies at a depth between \( \cdot 1 \, h \) and \( \cdot 4 \, h \) (where \( h \) is the depth of the stream) and for depths above 5 feet, has a mean depth of about \( \cdot 3 \, h \).

Fig 146, taken from a gauging by Darcy of a rectangular channel \( \cdot 25 \) metre deep and \( \cdot 8 \) metre wide, shows the general distribution of velocity over various vertical sections of a rectangular channel and also the equivalent velocity contours in a cross section, while Figs. 147 and 148, show
the results of gaugings on the experimental channel of the Cornell University.¹

This is of rectangular section, with concrete sides and bottom having a slope of 1 in 500, and has a width of 16 feet. Velocity measurements were made in eight verticals in a cross-section by means of current meters.

The curves in Figs. 147 and 148 show the variations of velocity in a vertical plane in typical of those experiments, each plotted point giving the mean of all eight observations at that depth in the cross-section.

The effect of a large ratio of width to depth in raising the filament of maximum velocity is evident from a comparison of the curves of Fig. 147 and of Fig. 148, while the effect of an increased velocity of flow in raising the filament is evident from a comparison of the several curves of Fig. 148.²

The depression of this filament of maximum velocity is mainly due to the action of the sides of the channel. Frictional losses at the sides reduce the energy and thus the head of the water in their neighbourhood, with the result that the surface level at the sides is lower than near the centre of the stream, and the cross-sectional profile of the water surface is a curve concave to the bed. Owing to this super-elevation of the water near the centre and to its tendency to find its own level, transverse currents are set up which travel downwards near the centre of the stream; outwards along the bottom to either bank, upwards along the sides, and, for permanence of régime, inwards along and near the surface.

¹ U. S. Geological Survey. Water Supply and Irrigation Papers, No. 95, pp. 76 and 77.
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Since the inward surface drifts consist of water which has travelled up the sides and has come from the region of minimum velocity, they will evidently have the effect of reducing the surface velocity and of depressing the filament of maximum velocity.

The sketches in Fig. 149 a and b show respectively the directions of the transverse currents, and of the resultant motion of the stream, the full lines $a'a'$, $aa$, in Fig. 149 b representing the direction of the surface currents, and the dotted lines $b'b'$, $bb$, those of the bottom currents. The presence of such currents in channels of various sizes has been experimentally demonstrated by the author.¹

In addition to this action of the sides, any retardation of the flow being more marked over the central and swifter portions of the stream tends to increase the super-elevation of the central surface and the formation of these transverse currents, while an acceleration of the flow

tends to reduce them. The author's experiments however indicate that for any such non-uniform flow as is likely to be experienced at a gauging station, the influence of the sides is the all-important factor.

This theory explains why, as is found in practice, the depth of the filament of maximum velocity, and, as will be seen later, also that of mean velocity in any vertical,

(a) is greater as the influence of the sides increases and hence as the ratio of depth to width of the stream increases.

(b) is less as the roughness of the bottom increases, since this roughness retards the transverse current without having any compensating effect.

(c) in the case of a rectangular channel, is greater nearer the sides.

It also explains why, on measuring the velocities across a horizontal in a stream, two points of maximum velocity are often found, those being one on each side of the centre as shown in Fig. 150, which is taken from a gauging of the Cornell channel.¹

The effect of the wind on the curve of velocities in a vertical is indicated in Fig. 151, which shows the curves

(a) with a strong up-stream wind.

(b) with no wind.

(c) with a strong down-stream wind.

It is found that although both the magnitude and position of the filament of maximum velocity is affected, that of the filament of mean velocity (m in Fig. 151) is sensibly independent of the state of the wind. The probable explanation of this is that an up-stream wind banks up the head waters and so increases the surface gradient of the stream, thereby increasing the velocity of flow over its lower portions to an extent which compensates for the reduced velocity of the surface layers.

From U. S. Geol. Survey, Water Supply Papers, No. 95, pp. 73 and 74.
**Mean Velocity in a Vertical.**

Actually the position of the filament of maximum velocity in a vertical is not of great importance* That of the filament of mean velocity, which is at a greater depth, is however very important in stream gauging, since, if it be known, the operation of gauging reduces itself to the measurement of a single velocity at this depth in each of a series of verticals distributed across the stream.

The depth of the latter filament varies from about \(0.5\) to \(0.7\), having the former value in a wide and shallow stream of depth less than about 2½ feet, and the latter value in a smooth wooden or cement channel whose depth is approximately one-half the width. In the great majority of cases in practice it lies between \(0.55\) and \(0.65\) increasing with the depth and diminishing with the roughness of the channel. An examination of a large number of river and canal gaugings by members of the U.S. Geological Survey ¹ leads to the following as the most probable values of its depth.

<table>
<thead>
<tr>
<th>Condition of bed.</th>
<th>Very rough with boulders</th>
<th>Large gravel and small boulders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of stream, feet</td>
<td>0 to 2</td>
<td>2 to 4</td>
</tr>
<tr>
<td>Ratio depth of filament of mean vel.</td>
<td>(0.50)</td>
<td>(0.56)</td>
</tr>
<tr>
<td></td>
<td>(0.54)</td>
<td>(0.58)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Condition of bed.</th>
<th>Small gravel and sand.</th>
<th>Very smooth wood or cement.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of stream, feet</td>
<td>0 to 2</td>
<td>2 to 4</td>
</tr>
<tr>
<td>Ratio depth of filament of mean vel.</td>
<td>(0.57)</td>
<td>(0.60)</td>
</tr>
<tr>
<td></td>
<td>(0.61)</td>
<td>(0.65)</td>
</tr>
</tbody>
</table>

Generally speaking, the velocity at six-tenths depth in any vertical will give the mean velocity in that vertical within 5 per cent. except in


H.A.
abnormal cases, while the mean of the velocities at two-tenths and at eight-tenths of the depth may also be relied upon as giving the mean velocity within narrow limits.

The velocity in a vertical is least at the bottom, the ratio of bottom to mean velocity ranging from 0.6 to 0.9. It usually lies between 0.75 and 0.85, but varies widely in a short interval in the same stream.

The ratio of the mean to the surface velocity in a vertical also varies within somewhat wide limits and depends largely on the direction and force of the wind. On a calm day it lies between 0.80 and 0.90, diminishing with the velocity of flow, with the roughness of the channel, and with the ratio of breadth to depth. While usually inadvisable to use the surface velocity in computing the discharge of a stream, it is sometimes impossible, in time of flood to make any other measurements. Under such circumstances the surface velocity, multiplied by 0.85 will give the mean velocity in a vertical with a fair degree of approximation.

The ratio of the mean velocity over the whole section to the maximum surface velocity varies considerably with the depth and state of the channel and with the direction of the wind. On a calm day it usually lies between 0.60 and 0.85, increasing with the depth of the stream. In a gauging of the Rhine—depth 6 to 19 feet—its value was 0.73 while Harlacher obtained the same value in gauging the Elbe—depth 4—7 feet. Gaugings of the Eger at Falkenstein (62 to 1.1 feet deep) gave a value of 0.58. The following values of this ratio are deduced from Bazin’s formula (p. 342), by writing $v_{\text{mean}} = C \sqrt{m \sin \theta}$ and by giving $C$ its appropriate values.

<table>
<thead>
<tr>
<th>Depth of Section</th>
<th>Material of Bed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planed Planks, Cement, etc.</td>
<td>Brickwork.</td>
</tr>
<tr>
<td>1.0 feet</td>
<td>0.85</td>
</tr>
<tr>
<td>2.0 &quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>3.0 &quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>4.0 &quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>5.0 &quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>6.0 &quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>10.0 &quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>20.0 &quot;</td>
<td>&quot;</td>
</tr>
</tbody>
</table>
In a very shallow stream the vertical velocity curves approximate to straight lines, and in this case the means of surface and bottom velocities give a close approximation to the mean velocity.

The main results of a large number of river gaugings by members of the U.S. Geological Survey are given below.¹

Here the symbol \( S \) means sandy; \( G \) gravelly; \( R \) rocky; \( B \) boulders.

<table>
<thead>
<tr>
<th>River</th>
<th>Approximate With Feet</th>
<th>Range of depth, Feet</th>
<th>Coefficient for reducing to mean velocity in any vertical plane, observed at following points,</th>
<th>Per cent. of depth at which mean veloc. is found</th>
<th>Character of bottom</th>
</tr>
</thead>
</table>
| Appomattox, Va.        | 150                   | 2.5 - 1.9            | 1.00 1.11 0.81 0.615 1
| James, Va.             | 850                   | 3.0 - 4.1            | 1.01 1.08 0.88 0.506 S 1
| Roanoke, Va.           | 110                   | 1.8 - 3.3            | 0.99 1.01 0.81 0.609 S and R 1
| Staunton, N.C.         | 100                   | 2.0 - 6.0            | 0.96 1.12 0.95 0.667 G and S 1
| Dan, N.C.              | 600                   | 1.4 - 3.7            | 0.99 1.01 0.86 0.616 S 1
| Dan, Va.               | 350                   | 2.5 - 9.8            | 0.98 1.02 0.90 0.636 S 1
| Reddie, N.C.           | 150                   | 1.7 - 2.5            | 0.97 1.03 0.83 0.777 R 1
| Yadkin, N.C.           | 150                   | 2.7 - 5.3            | 0.96 1.12 0.91 0.665 S and B 1
| Yadkin, N.C.           | 150                   | 1.10 - 11.3          | 1.02 1.05 0.81 0.632 S and R 1
| Catawba, N.C.          | 200                   | 1.8 - 5.0            | 1.00 1.01 0.82 0.666 S and G 1
| Catawba, N.C.          | 200                   | 6.3 - 7.0            | 1.08 0.98 0.88 0.565 M 1
| Catawba, N.C.          | 110                   | 3.5                  | 1.01 0.95 0.85 0.560 S 1
| Wateree, S.C.          | 300                   | 12.3 - 17.7          | 1.01 1.03 0.93 0.584 M 1
| Broad, S.C.            | 300                   | 5.0 - 8.9            | 0.99 1.02 0.92 0.624 M and S 1
| Saluda, S.C.           | 300                   | 3.0 - 8.0            | 0.97 0.95 0.92 0.622 S 1
| Little Tennessee N.C.  | 600                   | 34.6 - 69.0          | 1.02 1.06 0.82 0.638 B 1
| Nolichucky, Tennessee  | 300                   | 14.6 - 51.0          | 1.02 1.02 0.90 0.591 B 1
| Frickill, N.Y.         | 90                    | 2.5                  | 1.036 0.79 0.587 G 1
| Wallkill, N.Y.         | 130                   | 3.17                 | 0.98 0.83 0.637 S 1
| Forks Flume, Cal.      | 104                   | 7.98                 | 0.95 1.25 1.09 0.760 Wood 1
| Cornell Canal          | 160                   | 7.2 - 8.3            | — 0.99 0.637 Concrete 1
|                        | 100                   | 5.5 - 1.9            | — 0.99 0.643 1

The ratio of the velocity at mid depth to the mean velocity in any vertical appears to vary very little, its usual value ranging from 1.02 to 1.06, increasing with the depth of the stream. A value of 1.04 may be relied upon as giving the mean velocity within 3 per cent. for all normal sections and velocities of flow.

Art. 96.—Distribution of Velocity over a Vertical through the Centre of the Stream.

In spite of very many experiments which have been carried out to determine the distribution of velocity, and of many attempts to formalise the results of such investigations, so many and so varied are the factors

¹ U. S. Geol. Survey, Water Supply and Irrigation Papers, No. 95, pp. 150—158.
which influence this distribution, that, as might be expected from the nature of the case, the formulae so far collected can only be considered as giving useful approximations to the required result, and this is more particularly the case where the flow in a natural channel of irregular section is under consideration.

By making one or two assumptions as to the circumstances governing the flow in an open channel, a theoretical formula may be deduced, which, while only applying so far as these assumptions are justified, may still serve as the rational basis of a more exact empirical formula, for giving the distribution of velocity. Such a formula will now be considered.

Suppose the stream to be sensibly parallel; of width which is great in comparison with its depth; flowing steadily; and that the resistance to flow is due entirely to simple viscous shear, a state of affairs never exactly realised in practice.

Let \( y \) be the vertical distance from the surface, of a stratum of the fluid, \( \delta y \) the thickness, \( \delta l \) the length, and \( b \) the breadth of the stratum (Fig. 152).

The weight of this element of fluid = \( W \) \( b \) \( \delta y \) \( \delta l \).

The resolved part of this weight in the direction of motion = \( W \) \( b \) \( \delta y \) \( \delta l \) \( \sin \theta \).

The difference of tractive force on the upper and lower faces of the stratum = \( \mu \frac{d^2 u}{d y^2} \) \( b \) \( \delta l \) \( \delta y \), where \( \mu \) is the coefficient of viscosity (p. 67) and where \( u \) is the velocity of flow in the direction of the stream. The pressures on the two ends of the stratum are equal since these are at the same depth and are of the same area. Also since the stream is wide, the variation of shear on the two vertical sides of the stratum may be neglected, as explained on p. 66.

\[
\begin{align*}
W \cdot b \cdot \delta y \cdot \delta l \cdot \sin \theta &= -\mu \frac{d^2 u}{d y^2} \cdot b \cdot \delta l \cdot \delta y \\
\therefore \frac{W \sin \theta}{\mu} &= -\frac{d^2 u}{d y^2}
\end{align*}
\]

the negative sign denoting that the resultant shear force acting upon the element is in opposite direction to the force \( W \sin \theta \).

Integrating this expression twice, we get

\[
v = C + By - \frac{W \sin \theta}{2 \mu} y^2.
\]
DISTRIBUTION OF VELOCITY IN OPEN CHANNEL

If \( v_s = \) surface velocity, i.e., where \( y = 0 \), \( C = v_s \)

\[
\therefore \quad v = v_s + B y - \frac{W \sin \theta}{2 \mu} y^2
\]

\[
= v_s - \frac{W \sin \theta}{2 \mu} \left( y - \frac{B \mu}{W \sin \theta} \right)^2 + \frac{B^2 \mu}{2 W \sin \theta}
\]

\[
\therefore \quad v = v_s + \frac{B^2 \mu}{2 W \sin \theta} \left( y - \frac{B \mu}{W \sin \theta} \right)^2
\]

the equation to a parabola having a horizontal axis at a depth

\[
\frac{B \mu}{W \sin \theta} = y_1.
\]

Since

\[
\frac{d}{dy} v = \frac{W \sin \theta}{\mu} \left( y - \frac{B \mu}{W \sin \theta} \right)
\]

we have \( \frac{d}{dy} v = 0 \), i.e., the velocity is a maximum where

\[
y = \frac{B \mu}{W \sin \theta} \text{ i.e., at a depth } y_1
\]

\[
\therefore \quad r_{max} = v_s + \frac{B^2 \mu}{2 W \sin \theta} \]

\[
= v_s + \frac{W \sin \theta}{2 \mu} y_1^2.
\]

\[
\therefore \quad \text{from (2) and (4) we have } r_{max} - v = \frac{W \sin \theta}{2 \mu} (y_1 - y)^2.
\]

If \( v_b = \) bottom velocity, where \( y = h \)

\[
\begin{align*}
  r_b &= v_s + B h - \frac{W \sin \theta}{2 \mu} h^2 \text{ from (2)} \\
  \therefore \quad B &= \frac{r_b - v_s}{h} + \frac{W \sin \theta}{2 \mu} h \\
  \therefore \quad v = v_s + \frac{r_b - v_s}{h} \cdot y + \frac{W \sin \theta}{2 \mu} (h y - y^2).
\end{align*}
\]

Also

\[
\begin{align*}
v - v_b &= B (y - h) + \frac{W \sin \theta}{2 \mu} (h^2 - y^2) \\
\therefore \quad v_{max} - v_b &= \frac{W \sin \theta}{2 \mu} \left[ 2 y_1 (y - h) + (h^2 - y^2) \right] \\
\therefore \quad v_{max} - v_b &= \frac{W \sin \theta}{2 \mu} \left[ h - y_1 \right]^2.
\end{align*}
\]

While different observers have deduced different forms for the vertical velocity curve, the parabola with its vertex either in or below the surface appears to fit the majority of cases fairly well.

The foregoing analysis is therefore interesting, as giving the correct form of curve.
Razin, experimenting on a stream having the maximum velocity in the surface, obtained

\[ v_{s(max)} - v_{\text{mean}} = 25.4 \sqrt{m \sin \theta} \]
\[ v_{\text{mean}} - v_b = 10.87 \sqrt{m \sin \theta} \]
\[ v_{s(max)} - v_b = 36.27 \sqrt{m \sin \theta} \]

the general equation being

\[ v = (v_b)_{max} - 36.27 \sqrt{m \sin \theta} \left( \frac{y}{h} \right)^2 \]

Here \( v_{\text{mean}} \) is the mean velocity over the whole section, and \( m \) is the hydraulic mean depth, the dimensions being taken in feet and the velocities in feet per second. He also states that wherever the position of maximum velocity, the relation

\[ v_{\text{max}} - v_b = 36.27 \sqrt{m \sin \theta} \]

holds true. In the vertical plane containing the filament of maximum velocity, we have from equation (7)

\[ 36.27 \sqrt{m \sin \theta} = \frac{W \sin \theta}{2 \mu} \left[ h - y_1 \right]^2 \]

Substituting the value of \( \frac{W \sin \theta}{2 \mu} \) thus found, in equation (5), when the maximum velocity is below the surface

\[ v_{\text{max}} - v = 36.27 \sqrt{m \sin \theta} \left( \frac{y - y_1}{h - y_1} \right)^2 \]

a formula which gives fairly accurate results in practice.

Rankine states that the maximum, mean, and bottom velocities may be taken as being in the ratio 5 : 4 : 3 in ordinary cases, and in the ratio 4 : 3 : 2 in very slow currents, and these ratios may be taken as being approximately correct for streams and rivers of moderate size.

**Velocity at Mid-depth.**

From equation (5) of this article, we may obtain the mean velocity over any vertical by integrating the sum of such terms as \( v \delta y \) over the vertical, and by dividing this sum by its length \( h \). Thus

\[ \bar{v} = \frac{\int_0^h \left[ v_{\text{max}} - \frac{W \sin \theta}{2 \mu} (y_1 - y)^2 \right] \delta y}{h} \]

\[ = v_{\text{max}} - \frac{W \sin \theta}{2 \mu} \left[ \frac{h^2}{3} - h y_1 + y_1^2 \right] \]

while from (5) we may obtain the velocity at mid-depth, i.e., where

\[ y = \frac{h}{3}, \]

\[ y = \frac{h}{3}. \]
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Thus

\[ v_h = \frac{v_{max}}{2} - \frac{W \sin \theta}{2 \mu} \left( \frac{h^2}{4} - h y_1 + y_1^2 \right) \]

\[ \therefore \frac{v_h}{2} - \bar{v} = \frac{W \sin \theta h^2}{24 \mu} \]

from which, by determination of the mid-depth velocity, the mean velocity may be determined.

In general the mid-depth velocity is from 1·02 to 1·06 times the mean, and from 0·94 to 0·98 of the maximum.

If \( \bar{y} \) be the depth at the point having a velocity equal to the mean, we have, from equations (5) and (9)

\[ \frac{h^2}{8} - h y_1 + y_1^2 = \left( y_1 - \bar{y} \right)^2 \]

\[ \therefore \bar{y} = y_1 \pm \sqrt{\frac{h^2}{8} - h y_1 + y_1^2}. \]

Putting \( y_1 = 2 \ h \) this gives \( \bar{y} = 62 \ h. \)

" \( y_1 = 3 \ h \) " \( \bar{y} = 65 \ h. \)

Art. 97.—Permissible Velocity in Open Channels; Erosion and Deposition of Silt.

Water in motion exerts an erosive or scouring action on the bed and sides of the containing channel, and the maximum permissible velocity thus depends on the nature of the bed.

Particles of matter once disturbed, may be transported either by being rolled along the bed of the stream or by being carried in suspension, and for each material a certain critical velocity must be attained, depending on its size and specific gravity, before this is set in motion. Once in motion, however, the velocity may be reduced somewhat below this critical value before the material is again deposited, as is indicated (p. 344) by the results of experiments by Du Buat\(^1\) on transportation in small wooden channels.

While the erosive power of water varies as the square of its velocity, its transporting power, or the power to move boulders, etc., which may lie in its path, varies approximately as \( v^2 \). This may be seen if we consider that the force exerted by the stream on any body is equal to the change of momentum produced in the stream passing the body, and since the area of that portion of the stream affected is proportional to the sectional area, \( a^2 \), of the body, this force will be equal to \( K v^2 a^2 \) lbs. The force resisting motion is that of the friction of the body on the

\(^1\) *Principes d'Hydraulique*, Dubuat, Paris, 1816.
bottom of the stream and is proportional to its weight, and therefore to its volume $a^3$.

<table>
<thead>
<tr>
<th>Material</th>
<th>Bottom Velocities, in ft. per sec., at which</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Transportation Begins.</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>1.07</td>
</tr>
<tr>
<td>Gravel—</td>
<td></td>
</tr>
<tr>
<td>Size of pea</td>
<td>0.71</td>
</tr>
<tr>
<td>&quot; small bean</td>
<td>1.56</td>
</tr>
<tr>
<td>Shingle—rounded, one inch or more in diameter</td>
<td>3.2</td>
</tr>
<tr>
<td>Flints—Size of hen’s egg</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Equating these forces, we have $K v^2 a^2 = c a^3$

\[
\therefore a \propto v^2 \\
\therefore a^3 \propto v^6
\]

i.e., the weight of the solid moved is proportional to the sixth power of the velocity. Obviously this only holds so long as the bodies are similar, the velocity necessary to move a sphere being much less than that to move a cubical block of the same weight.

The size of particle moved by a stream over a smooth sandy bed is given approximately by $d = \frac{45 v^2}{w - 0.4}$ inches, where $w$ is its density in lbs. per cubic foot, and $v$ is the velocity in feet per second.$^1$

A stream which carries a certain amount of fine material in suspension has a greater capacity for transporting larger material than one which carries only the larger material. Experiments show that a stream will carry more than four times the weight of sand of 4 to 5 mm. diameter in the presence of a certain weight of sand of 3 mm. diameter than in its absence.$^2$

While an excessive velocity of flow leads to erosion of the channel, a too

---

$^1$ Dr. G. S. Owens. *Engineer*, May 15, 1908, p. 511.

sluggish flow favours the growth of aquatic plants, while any change of velocity from high to low in a stream carrying material in suspension, causes a deposition of a portion of the material, and a consequent silting up of the channel. In order to prevent deposit in small sewers or drains, a mean velocity of not less than 3 feet per second is necessary. For sewers from 12 to 24 inches diameter the velocity should not be less than 2·5 feet per second, while with larger sizes than this the velocity may be reduced to 2 feet per second.

Mr. R. G. Kennedy, from observations on a large number of Indian irrigation canals, concludes that there is a certain critical velocity at which a long canal will maintain its channel in silty equilibrium. This velocity is given by 

\[ v_0 = c \cdot 0.04 \text{ feet per second} \]

where \( c \) has the following values:

- Light sandy soil \( c = 0.82 \)
- Coarse sandy soil \( c = 0.90 \)
- Sandy loam \( c = 0.99 \)
- Coarse silt \( c = 1.07 \)

Where a main canal supplies or is supplied by feeders, the various depths and velocities of flow should be adjusted to suit this relationship in order that there may be no silting or erosion in the main or feeder canals.

If \( v \) be any other velocity, and if \( q_0 \) and \( q \) be the amounts of silt carried respectively at \( v_0 \) and \( v \),

\[ q = q_0 \left( \frac{v}{v_0} \right)^6 \]

approximately.

Taking \( c = 0.84 \), the following table shows suitable mean velocities of maximum flow for equilibrium of such channels in sandy soil:

<table>
<thead>
<tr>
<th>Depth of channel (feet)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean velocity (feet per sec.)</td>
<td>1.3</td>
<td>1.7</td>
<td>2.0</td>
<td>2.4</td>
<td>2.65</td>
<td>2.9</td>
<td>3.2</td>
</tr>
</tbody>
</table>

*Ganynillet and Kutter* give the following as the safe bottom and mean velocities, but state that these are probably too small rather than too large:

<table>
<thead>
<tr>
<th>Material of Channel</th>
<th>Safe Bottom Velocity. Feet per Second.</th>
<th>Safe Mean Velocity. Feet per Second.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft brown earth</td>
<td>.25</td>
<td>.33</td>
</tr>
<tr>
<td>Soft loam</td>
<td>.50</td>
<td>.66</td>
</tr>
<tr>
<td>Sand</td>
<td>1.00</td>
<td>1.32</td>
</tr>
<tr>
<td>Gravel</td>
<td>2.00</td>
<td>2.44</td>
</tr>
<tr>
<td>Pebbles</td>
<td>3.00</td>
<td>3.94</td>
</tr>
<tr>
<td>Broken stone or flint</td>
<td>4.00</td>
<td>5.58</td>
</tr>
<tr>
<td>Conglomerate</td>
<td>5.00</td>
<td>6.56</td>
</tr>
<tr>
<td>Stratified rock</td>
<td>6.00</td>
<td>8.20</td>
</tr>
<tr>
<td>Hard rock</td>
<td>10.00</td>
<td>13.13</td>
</tr>
</tbody>
</table>

Actually, as indicated by Kennedy, the safe mean velocity depends also upon the depth. More recent work shows that at medium depths through firm loamy soil a mean velocity of 3 to 3.5 feet per second is safe, while with fine well-rammed gravel or loose rock this may be increased to from 5 to 7 feet per second. In a concrete-lined channel faced with cement, the maximum safe velocity with water which carries solid material in suspension is about 9 feet per second. A higher velocity wears and roughens the bottom until the roughness thus produced reduces the velocity sufficiently to prevent further erosion. With an ordinary brick or heavy dry-laid rubble channel, the velocity should not exceed 15 feet per second, any higher velocity necessitating a carefully-laid facing of heavy masonry with cemented joints.

**Art. 98.—Gauging of Flow in Streams and Open Channels.**

Many methods are available for obtaining the discharge of a stream, these differing widely in the accuracy of their results and the cost and difficulty of their application. The method to be adopted in any case depends largely on the degree of accuracy required and on the size of the stream.

The accuracy of a discharge measurement, whatever be the method adopted, depends greatly on the physical characteristics of the stream at the point of measurement. If possible this should lie on a straight reach and away from the influence of a bend, the bed should be permanent and not strewn with boulders, and the slope and wetted perimeter such that at all stages of the stream the velocity at all parts of the section may be easily measurable. The banks should be sufficiently
high to prevent overflow in times of flood, and the section should be outside the sphere of influence of bridge piers or mill dams.

Where a high degree of accuracy is demanded, as may be required in determining the flow of compensation water from the supply reservoir of a waterworks, the best method is to deflect the stream and catch the whole discharge for a given time in a graduated tank.

This can, however, only be carried out in small streams where a measuring tank of sufficient capacity to hold the discharge for about two minutes is available. In this case the error should not exceed 1 per cent.

For larger streams, the most accurate method is that of gauging the flow by placing a weir across the stream and allowing the whole flow to take place over this or over one or more triangular or rectangular notches, the former being used for small and the latter for fairly large streams. Where every precaution is taken as explained in Art. 59 this method gives results which may be relied upon as being accurate within about 5 per cent. As a temporary measuring contrivance, however, the weir is too cumbersome and costly to be applied to a river of even moderate dimensions, and where the discharge is great the only method of obtaining the discharge is to obtain as nearly as possible the mean velocity ($v$ feet per second) of the stream, to multiply this by the cross sectional area ($A$ square feet), and to get the discharge $Q$ by the relation.

$$Q = vA \text{ cubic feet per second.}$$

The value of $v$ may be approximated to in many ways.

(a) By using one or other of the formulae given in Art. 85, a suitable coefficient being applied to take into account the state of the channel. The slope of the stream must then be obtained by field observations. To do this a long straight reach of the river should be selected where possible, and the reduced levels of bench marks placed at each end of the reach should be obtained by levelling. The level of each bench mark should then be transferred to a hook gauge (Fig. 153) or measuring staff placed in a gauge pit communicating with the bed of the stream through a pipe which opens out at a point away from any disturbing influences likely to lead to eddy formation. The difference of surface level at each end of the reach can then be obtained.

![Fig. 153. Hook Gauge.](image-url)
The distance from end to end of the reach may be obtained by chaining a line running as nearly as possible parallel to the centre line of the river. Soundings should then be taken at short distances apart at several cross sections of the stream, and these cross sections plotted. From these a mean value of the wetted perimeter and of the sectional area, and thus of the hydraulic mean depth, may be obtained, and the formula may then be applied. During the whole observation period the stream should be in a state of steady flow, and neither rising nor falling.

The method has the disadvantage that it is extremely difficult to measure the slope of a river accurately. Captain Cunningham as a result of some hundreds of slope measurements on the Ganges Canal,\(^1\) found that the slope was very different at different points of a reach from one to two miles long, and varied by as much as 50 per cent. at different sides of the stream. An examination of the Mississippi\(^2\) showed that with the main body of water flowing south with a velocity of four to five miles per hour, the water near the shore may be moving north at a speed of one or two miles per hour. It was in fact not unusual to find a slope towards the south on one bank and towards the north on the opposite bank. The slope then is so uncertain an element that no great accuracy is to be expected for any such formula, except possibly in the case of an artificial channel of uniform section. Under any other circumstances the results cannot be relied upon as being accurate within 25 per cent., and may under specially unfavourable circumstances, even with the most skilful observers, be in error by as much as 100 per cent.

Wherever possible, then, the mean velocity should be obtained in some other manner. This may be done

\(b\) By using a current meter or Pitot tube to give the velocity at a point or series of points in a cross section, and by deducing the mean velocity from such observations;

\(c\) By using one or other type of \textit{float}, and by measuring the time necessary for a series of these to traverse a given length of the channel.

Before considering these methods in detail, a few general observations as to their relative advantages and disadvantages may be made. Experiment shows that the motion at any point in an open channel is never steady and uniform, but suffers a series of pulsations, the periodic time of which may vary from a few seconds to two or three minutes. These are due to a variety of causes. Eddies formed at the sides and bottom

---
drift away to every portion of the stream; snags and hollows in the bed, bends, and falls, all produce some (irregular) disturbance of the flow, with the result that the velocity at a point in the surface may vary by 20 per cent., and at a point near the bed by as much as 50 per cent. (Harlacher) in a short interval of time.

In experiments on the St. Clair River (1899) the velocity-time curve showed two sets of waves, small ones of 15 to 60 seconds amplitude and larger ones of 3 to 6 minutes amplitude. The range of velocities as found from the larger waves was in some cases 35 per cent. of the mean velocity shown by the meter reading taken over ten minutes. These experiments indicate that the pulsations are very limited in extent in a direction at right angles to the current. The whole depth of the river is affected by them, although their effect decreases from the bottom towards the surface.

It follows that a float, measuring as it does the velocity due to a single pulsation, may give results which are greatly in error, and the only chance of obtaining a fair estimate of the mean velocity over a single section of the stream is to take the mean of a large number—40 or 50—of the values given by floats. The complexity of the motion is very evident when floats are used. Of a series dropped into a stream at the same point, no two will trace out the same path, and as may be well understood when the multitude of observations necessary to give any pretensions to accuracy is remembered, this method though at first sight so simple, may easily prove the most expensive method of determining the discharge. With current meters, on the other hand, the mean velocity at any point may be obtained with great accuracy, provided the period of observation is sufficient to cover a series of the pulsations of velocity. Professor Unwin found that the mean time of successive 100 revolutions of such a meter in the Thames, when plotted, gave a very irregular curve, while the mean times of successive 500 revolutions gave an almost straight line. In general the time of a single observation should not be less than five minutes, a period of six to ten minutes being advisable.

This renders it essential that in order to avoid spending an excessive length of time in the field and thus running the risk of serious fluctuations in the water level, the discharge be found from single observations in comparatively few verticals, and that the ratio of the velocity at the depth chosen, to the mean velocity, be known from vertical velocity curves. This emphasises the importance of a thorough investigation of the relation between velocity and depth in a vertical longitudinal
plane, and the change in this relation with any change in the state of a river.

Art. 99.—Current Meters.

Meters in use at the present day may be divided into two classes: (1) those in which the revolving part carries a series of helicoidal vanes mounted on a horizontal axis, and (2) those in which a series of conical or hemispherical cups is mounted on arms, as in an anemometer, on a vertical axis. The former type is illustrated in Fig. 154, which shows the Amster meter and in Fig. 155, which shows the Haskell meter, while the latter is shown in Fig. 156, which illustrates the Price meter. The latter type of meter has some advantages over the former in that friction is usually less since it practically all comes on one point which is easily protected from any grit in the water, while in addition this type will start in a current of less velocity than will move the other, and yet will not revolve as rapidly under the same conditions of high velocity flow.

The meter is fitted with a guide vane which keeps its axis perpendicular to the direction of the current. The wheel may be either geared to a counter which records the revolutions directly and is put into and out

![Fig. 154.—Amster Current Meter.](image)

![Fig. 155.—Haskell Current Meter.](image)
to clog are obvious, and the mechanically operated meter is becoming obsolete.

The instrument is previously calibrated by towing at known velocities through still water, the number of revolutions corresponding to these velocities being recorded. It has the disadvantages that it cannot be used where floating grass or weed is prevalent, and that it requires rating at frequent intervals. Further, it cannot be used at very low velocities. The minimum permissible velocity depends on the type of meter, but in general varies from 3 to 6 inches per second.

There are two methods of using the meter. In the first, the "point"

![Fig. 156.—Price Current Meter.](image)

method, it is held successively at certain points in a cross section. In a shallow stream this may be done by clamping it to a staff which is carried by an observer in waders, and which is held vertically at the required points, with one end resting on the bed of the stream. In deeper streams it is attached to a heavy sinker and is suspended from a convenient bridge or cable placed across the stream where possible, or from an outrigger fixed to an anchored boat where the width precludes this. When the "point" method is used, the meter may either be held, (1) at several equidistant points in certain equidistant verticals, the
mean velocity being deduced from these readings as explained later; (2) at six-tenths, or at mid-depth in a series of equidistant verticals, the mean velocity in each of these verticals then being found by applying a factor; (3) at the surface, bottom, and mid-depth in a series of verticals; (4) at the surface and bottom only, or at two-tenths and eight-tenths of the depth in a series of verticals, in which case the mean of the two readings is taken as the mean velocity in the vertical. While the first method is likely to give the best results in a steady stream, yet, as previously indicated, the length of time necessary to obtain the many observations is a serious drawback in a stream of any considerable size.

In a large stream where it is impossible to see the bottom, owing to the impossibility of fixing the meter very near to the sides and bottom where the velocities are least, the results tend to be too high. To obviate this the meter should not be placed nearer to the surface than one foot.

The mid-depth point is used because the factor, about .96, which is used to obtain the mean velocity is more constant for it than for any other point on the vertical; while the six-tenths point gives very approximately the true mean velocity on the vertical. These factors are discussed at further length on pp. 337 and 339. Observations taken at either of these points are capable of giving excellent results.

Method (3) was adopted by Moore in his gauging of the Thames. Assuming the vertical velocity curve to be a parabola, its area is given by the formula

\[ A = \frac{h}{6} \left( v_s + v_b + 4v_f \right) \]

h being the depth, \( v_s \) and \( v_b \) the surface and bottom velocities respectively. The discharge per second flowing between the two verticals is given by

\[ \Psi = \frac{d}{8} \left( \sum (A_1 + A_n) + 4 \sum (A_2 + A_4 + ...) + 2 \sum (A_3 + A_5 + ...) \right) \text{ c.f.s.} \]

where \( d \) is the breadth of the successive vertical strips; the first term is the sum of the areas of the first and last velocity curves; the second term is four times the sum of the even sections; and the last term is twice the sum of the odd sections excluding the first and last. The total discharge is then obtained by adding the small volume flowing between each end section and the shore. On account of the large variation in bottom velocity with a given mean flow this method is, however, not to be recommended.

Method (4), in which the surface and bottom velocities are measured is only advisable for very shallow streams. Experiments at Cornell University show that the results thus obtained agree closely with those given by a weir if the bed is smooth or gravelly, the depth from 4 to

1.0 feet and the velocities from 4 to 1.5 feet per second. For a gravelly bed the meter should be held with its centre from 3 to 4 inches above the bottom and about 2 inches below the surface, while with a smooth bed each distance should be about 2 inches. With depths between 5 and 10 feet, the mean of velocities 5 feet above the bottom and 5 feet below the surface gave results too low by as much as 30 per cent. In such cases the mean of readings at two-tenths and at eight-tenths the depth gives good results.

In the “integration” method, the meter is kept in motion during the whole period of its immersion. It may either be moved uniformly from the surface to the bottom of the stream in a series of vertical lines; diagonally across from one side to the other, at the same time being moved from the surface to the bottom several times; or across the stream at a given depth. The recorded velocity is then taken as the mean for the particular vertical or for the whole section as the case may be. Although an observation by this method can be carried out in considerably less time than by the point method, the results are not nearly so accurate. The velocity recorded being the resultant of the velocities of the meter and of the water is always higher than the true velocity, the error increasing with the speed of movement of the meter and also increasing as the velocity of flow diminishes. It is only to be recommended where a stream is rising or falling rapidly and where in consequence the speed with which the observations can be made is a great advantage.

Simultaneously with the velocity observations, soundings should be taken from which the cross section of the stream may be obtained. In a narrow stream these should be taken at intervals of from 2 to 5 feet, while where the breadth exceeds 100 feet, they should be taken at intervals of from 10 to 25 feet, depending on the roughness of the bed.

**Field Notes.**—The following shows the method adopted for entering up field observations and computing mean velocities in the case where velocities are measured at several points on a cross section.

[Gauging made January 23, 1904, by B.S.D. Meter No. 340, on Dan River, Madison, N.C. Gauge height : beginning 2.10 feet; end 2.26 feet; river rising.]
These observations are recorded for a series of verticals in the cross section. They are then plotted on squared paper, depths as ordinates and velocities as abscissae, and a smooth curve is drawn through the plotted points, care being taken to give them as nearly as possible equal weight if they do not all fall on a smooth curve. From this curve velocities are read off at top and bottom and at equal intervals of, say, each 5 foot, and are set down in order. Thus from the above curve we get—

\[
\begin{align*}
0 & \quad 0 \ldots \quad v_1 = 2.90 \\
0.5 & \quad 0 \ldots \quad v_1 = 2.88 \\
1.0 & \quad 0 \ldots \quad v_1 = 2.77 \\
1.5 & \quad 0 \ldots \quad v_1 = 2.58 \\
2 & \quad 0 \ldots \quad v_1 = 2.25 \\
2.5 & \quad 0 \ldots \quad v_1 = 1.88 \\
3 & \quad 0 \ldots \quad v_1 = 1.31 \\
3.5 & \quad 0 \ldots \quad v_1 = 1.00
\end{align*}
\]

The mean velocity in this vertical is then computed from the prismoidal formula for seven abscissae as follows:—

\[
v_m = \frac{1}{8} \left[ v_s + v_b + 4 (v_1 + v_3 + v_5) + 2 (v_2 + v_4) \right]
\]

In this case we have:

\[
\begin{align*}
v_s + v_b &= 2.90 + 1.31 = 4.21 \\
4 (v_1 + v_3 + v_5) &= 4 \left( 2.88 + 2.58 + 1.88 \right) = 29.36 \\
2 (v_2 + v_4) &= 2 \left( 2.77 + 2.25 \right) = 10.04
\end{align*}
\]

\[
\therefore \quad v_m = \frac{1}{8} \left\{ 4.21 + 29.36 + 10.04 \right\} = 2.42 \text{ f.s.}
\]

The cross section having been plotted, the areas of the various compartments having such verticals as their centre lines may be obtained, either by direct measurement by planimeter or by calculation, and the discharge calculated as follows:

<table>
<thead>
<tr>
<th>Compartment</th>
<th>Area of section square feet.</th>
<th>Mean velocity, feet per second.</th>
<th>Discharge c.f.s.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.1</td>
<td>1.32</td>
<td>19.9</td>
</tr>
<tr>
<td>2</td>
<td>28.2</td>
<td>1.97</td>
<td>55.6</td>
</tr>
<tr>
<td>3</td>
<td>36.5</td>
<td>2.12</td>
<td>78.3</td>
</tr>
<tr>
<td>4</td>
<td>32.1</td>
<td>2.56</td>
<td>82.1</td>
</tr>
<tr>
<td>5</td>
<td>28.7</td>
<td>1.99</td>
<td>47.2</td>
</tr>
<tr>
<td>6</td>
<td>13.5</td>
<td>1.33</td>
<td>17.9</td>
</tr>
</tbody>
</table>

| Total ... 311.0 c.f.s. |

When the vertical velocity curves have been obtained the discharge may.
be computed somewhat more accurately by considering the discharge between any two such verticals as being represented by the volume of the solid having these curves bounding opposite parallel sides as shown in Fig. 157. For example, the discharge between the verticals 2 and 3 in this figure is given by

$$\frac{2d_3}{3} \left[ \bar{r}_2 h_2 + \bar{r}_3 h_3 + \sqrt{\bar{r}_2 h_2 \bar{r}_3 h_3} \right] \text{ c.f.s.}$$

where \(\bar{r}_2\) and \(\bar{r}_3\) are mean velocities in the verticals 2 and 3, and where \(h_2\) and \(h_3\) are the corresponding soundings, \(2d_3\) being the distance between the verticals. The discharge between the two end soundings is then given by the sum of such terms as the above between these soundings.

To this must be added the discharge over these sections outside the end soundings, which is given by

$$\frac{1}{3} \left[ \bar{r}_1 h_1 \times 2d_1 + \bar{r}_5 h_5 \times 2d_5 \right] \text{ c.f.s.}$$

**Calibration of Current Meters.**—In rating a meter it is usually suspended from a car or a boat, and is towed with a uniform velocity through still water at a depth of 2 or 3 feet. The length of a run varies from 100 to 300 feet, with sufficient of a starting run to attain a steady velocity before entering the measured length. It is moved in either direction from end to end of the run to eliminate the effect of a current in either direction, and the time of the run and the number of revolutions of the meter are recorded by means of a chronograph.

The range of velocities employed in rating should be those for which the meter is to be used, and no attempt should be made to extend the rating table beyond its experimental limits.

When in use the meter may either be suspended from a cable, in which case its axis is free to move about both a vertical and a horizontal axis, or it may be fixed to a vertical rod in such a way as to remove the second degree of freedom. Experiment indicates that the same rating table is
not strictly accurate for the two cases, and that with a given velocity of flow the revolutions increase as the freedom of motion decreases. The difference depends on the type of meter, and for velocities of 1 foot per second is usually about 2 per cent.

A further source of inaccuracy, particularly with very low velocities of flow, is due to the fact that a rating carried out in still water is not quite accurate when applied to the same relative velocities in moving water.

Experiments indicate that the meter, particularly of the cup type, does not indicate so high a velocity when dragged through still water as when held in a current, the difference varying from 1 to 4 per cent. at a velocity of 1 foot per second with different types of meter. The difference is about 1 per cent. at a velocity of 2 feet per second.

In calibrating a current meter it is usual to plot the curve connecting velocity of current and number of revolutions of meter per second. In

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1 U. S. Water Supply and Irrigation Paper, No. 95, pp. 83—89, also p. 81.
the majority of cases this is found to be of the form shown in Fig. 158. At very low speeds the friction of the instrument varies between fairly wide limits, but diminishes as the speed increases. This causes the plotted points to lie more or less on a curved line, but renders this portion of the calibration unreliable. At a certain critical speed of the instrument the friction takes a fairly constant value, and the curve becomes very approximately a straight line of the form

\[ v = an + b. \]

It is not advisable to use the meter to register speeds below the critical. This depends on the instrument, but is usually from 3 to 6 inches per second, and in general it may be taken that on this account the meter is not a suitable instrument for the measurement of the discharge of a stream if the velocity over more than 15 per cent. of its area is less than 6 inches per second.

A form of current meter which is occasionally used consists of a flat circular plate which is rigidly attached by means of a horizontal arm to one end of a vertical wire, the other end of which is fixed. The wire is supported in bearings, and the free end carries a pointer which, working over a graduated disc, enables the angle of twist to be ascertained. In using the instrument, the pointer is adjusted to zero with the plate out of water and normal to the direction of flow of the stream. The plate is then submerged and the angle of twist necessary to bring it once more normal to the direction of flow is noted.

Then if \( P = \text{force on plate in lbs.} \),
\[ l = \text{length of arm from centre of plate to centre of wire.} \]
\[ A = \text{area of plate in square feet.} \]
\[ \theta = \text{angle of twist of wire.} \]
\[ v = \text{velocity of flow of stream.} \]

We have \( Pl = K\theta \), where \( K \) is a constant for any instrument and depends solely on the material, length, and radius of the wire.

Also \( P = 1.15 A r^2 \) (approx.) the constant depending on the size of the plate, from which \( r^2 = \frac{K}{1.15 A l} \theta. \)

Even with a constant velocity of flow, however, eddy formation at the rear of the plate causes the value of \( P \) to undergo periodic fluctuations, and the difficulty in obtaining a true mean value for \( \theta \), and in keeping the plate normal to the direction of the stream, prevent this method from having any pretensions to great accuracy. The instrument is now practically obsolete.
ART. 100.—ESTIMATION OF VELOCITY BY FLOATS.

These are liberated at a series of points in a long straight reach (Captain Cunningham, from experiments on the Ganges Canal\(^1\) recommends that this length should not be less than 200 feet) and the time occupied in covering a measured distance is noted.

Floats may be divided into four classes:—

1. Surface floats.
2. Sub-surface floats.
3. Twin floats.
4. Velocity rods.

1. Surface Floats.—These consist of any easily seen masses of light material, painted cork or discs of light wood for example, of small size so as to move along with, and register, the velocity of the surface filaments. A series of trials are necessary to get the maximum surface velocity \(c_1\) of the stream, from which the mean velocity of flow may be estimated from Bazin’s formulas (p. 342). It is preferable, however, to deduce the mean velocity in each of a number of sections of the stream from repeated observations of the surface velocity in each of these sections. The sections may be marked in a stream of moderate dimensions by ropes hanging from a bridge or temporary support.

In a large river, observations with the theodolite are necessary to determine the track of the float. This may be satisfactorily carried out as follows:—A base line \(AB\) (Fig. 159) is chained out parallel to the river for a length of about 250 feet, depending on the width of the river. At the two ends stakes are erected, while second stakes are erected in lines ranged perpendicularly to the base line, as at \(S_1, S_2\). An observer with a theodolite is stationed near the centre of the base line at \(C\), and an observer is stationed at each stake, \(S_1, S_2\). The float being liberated upstream, the theodolite observer keeps the line of collimation of his instrument on this. As it passes the line of sight \(S_1 A\), the observer at \(S_1\) gives an audible signal and the theodolite observer notes the angle \(A C P_1\). On passing the line \(S_2 B\) a second signal is given at the angle \(B C P_2\) noted. The line \(P_1 P_2\) can then be plotted. With a stream of moderate velocity the same observer may give the signals both at

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A and B. The effect of wind on the surface velocity, however, together with the tendency of the floats to follow every variable cross current and to be affected by every surface eddy, renders the results obtained by this method unreliable except as approximations to the truth.

(2) Sub-surface Floats.—These consist of bodies having surfaces of large area, as illustrated for example in Fig. 160, attached to small surface floats for ease of observation, the length of connection being adjusted so as to allow the true float to remain at any given depth. The velocity of the float will then be approximately that of the current at the required depth. The figure shows the float used in the Connecticut River survey in 1874. The sub-surface float was a hollow annulus of tin 8½ inches high, 8½ inches outside diameter, and 7½ inches inside diameter. This was weighted with 28 oz. of lead. The surface float was an ellipsoid of tin 6 inches in diameter and 1.5 inches deep, the connecting cord being 0.36 inches in diameter. A series of such floats liberated at different points in the cross section of a stream and at different depths may be used to give by their mean velocity the mean of that of the stream, or by arranging a single row, the depth of each being ½ that of the stream at the point of introduction, these may be taken as giving the mean velocities in their respective sections. While this type is more reliable than the surface float it suffers from the disadvantage that it is impossible to determine the exact position or depth of the lower float, for while the position of the upper float may be known, that of the lower float varies with the direction and velocity of the wind and with the length of cord connecting the two floats. Also the upper float may either drag or be dragged by the lower, and the upper is on this account likely to retard the lower where the latter is above the filament of maximum velocity, and to accelerate it when it is at a greater depth than this. As this latter effect extends over a greater proportion of the depth than the former, it would tend to make the velocities of flow recorded by the floats too high. Experiments made by T. G. Ellis, 1874, on the Connecticut River (mean velocity 2.1 ft. per sec.) with current meters and with
double floats\textsuperscript{1} showed that the mean velocity as found by floats was from 6 to 26 per cent. greater than by meter, the difference increasing with the velocity. Marr—October, 1879, on the Mississippi,\textsuperscript{2} the width being about 2,000 feet and the depth 16'4 feet (mean velocity 2'6 ft. per sec.) found the mean float velocity to be about 3'5 per cent. greater than the meter velocity, while Henry—1869—on the St. Clair River—45 feet deep—and with a mean velocity of 3'4 feet per second, found the mean float velocity about 10 per cent. greater than the meter velocity. In this case the float velocity was less than the meter velocity to a depth of about 7 feet. Below that depth the float velocity was the greater, the difference increasing with the depth.

(9) Twin Floats. These consist of two masses of equal size, usually spheres coupled together by means of a wire, the lower of which is weighted so as to remain vertically below the upper, which floats at the surface. The velocity of the float then gives the mean of the velocities at the surface and at the depth of the lower mass. If this is adjusted so as to just clear the bottom, the velocity of the float will be approximately the mean velocity for the vertical in which the instrument floats.

(4) Velocity Rods. The velocity rod, or rod float, consists of a light wooden rod or tin tube about 1 inch in diameter, and made in adjustable lengths. The lower end of the bottom length is weighted and the length adjusted until the rod floats vertically with its lower end clearing the bottom by a few inches. In a large river and where these are not likely to interfere with navigation, logs of wood about 12 inches in diameter, having their lower ends weighted with iron and their upper ends painted white, may be used.

The velocity of the rod is approximately the same as the mean over its depth, and gives the mean velocity over the vertical in which it floats. The difficulty in using the rod lies in its tendency to drag over shoals and weeds, and to obviate this its lower end may be arranged to float at a height $h_1$ above the bed of the stream.

For such a case Francis gives the empirical formula

$$v_m = v_r \left( 1.012 - 0.116 \sqrt{\frac{h_1}{h}} \right)$$

giving the mean velocity in the vertical containing the rod in terms of the velocity of the rod ($v_r$), $h_1$, and $h$ the depth of the stream. Here $h_1$ should be less than 25 $h$.

\textsuperscript{1} Report Chief Eng., U.S.A., 1878, Appendix B.
\textsuperscript{3} "Journal Franklin Inst.," vol. 62, p. 322. e
Of all floats the velocity rod gives the best results, and for channels of moderate and uniform depth, encumbered with floating weeds and grass, this is probably the best method of obtaining the velocity. In a series of experiments on the Loch Katrine Aqueduct; concave bottom; width 9' 1"; radius of curvature 20' 10½"; hydraulic mean depth 2'87 feet; velocity rods 2' 2" long, gave results which agreed within 3 of 1 per cent. with results as obtained by weir measurement, while the velocity as obtained from the maximum surface velocity, and an application of the formula \( v = v_s - 25.4 \sqrt{\frac{w}{i}} \), was 18 per cent. too low.

An elaborate series of experiments was carried out in 1856 by J. B. Francis on the Lowell Canal to determine the relative accuracy of weir and rod float measurements, this canal being 27'75 feet wide where the first 63 experiments were made and 14 feet wide where the remaining 52 were made. The length of run was 70 feet, the floats being loaded tin tubes, 2 inches in diameter. From these it was found that the mean difference in the discharge as obtained by the two methods was less than 2 per cent. in all but three of the experiments, the mean difference being about 1 per cent. The mean velocity in these experiments varied from 0.5 to 5 feet per second.

Experiments in 1900 at Cornell University showed about the same degree of accuracy in the case of flow in a canal 16 feet wide and with depths of water ranging from 5 to 10 feet, and velocities of flow up to 2'07 feet per second. The immersed portions of the rods varied from 75 per cent. to 95 per cent. of the depth of the stream, and the length of measured run varied from 7 to 25 feet, depending on the velocity. In every case the float velocity was slightly less than that given by the weir.

Other Methods of Measuring the Velocity.

Art. 101.—Ripple Formation.

An ingenious method of obtaining the surface velocity at various points in the cross section of a stream was described by Mr. E. C. Thrupp ("Proc. Inst. C.E.," vol. 167, 1907, p. 217). This depends upon the fact that if a small obstruction cut the surface of a stream, ripples are formed if the velocity exceeds about 9 inches per second, while the angle of divergence of these ripples appears to bear a definite relation to the surface velocity. To overcome the difficulty of accurately measuring this angle Mr. Thrupp constructed a velocity meter consisting of two vertical

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1 Lowell, Hydraulic Experiments, p. 170.
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pegs (.2 iron nails) at a known distance $d$ inches apart, with a scale for measuring the distance from the base line of the point of intersection of the ripples formed. Calling this distance $l$ (Fig. 161), the following equations were found to give the surface velocity in feet per second.

For $d = 6''$, $v = .40 + .206 l$.

For $d = 4''$, $v = .40 + .280 l$.

With $d = 6''$ and with a velocity of 8 feet per second the value of $l$ is about 2 inches, while with a velocity of 3.5 feet per second, $l$ is 15 inches.

This method would appear to be capable of results at least as accurate as those obtained by the use of surface floats, and possibly more so because of the greater possibilities of accuracy in the determination of the area of the stream at one definite cross section.

**The Pitot Tube.**—The velocity at any point in the cross section may also be estimated by means of the Pitot tube (Art. 68), p. 217.

This method is not so well adapted for measuring such low velocities as usually occur in open channels as for higher velocities such as are more common in pipe flow.

For small, shallow, and rapid streams it is, however, capable of giving fairly good results. Fig. 162 shows a tube, with positive and negative pressure openings as used for the rating of such streams by the United States Geological Survey. This tube was rated in still water in a reservoir and in moving water, being placed for the latter rating at about 60 points in the cross section of a channel 1 foot wide and 6 feet deep. The former rating gave values of C in the formula $V = C \sqrt{2gh}$ ranging from 885 to 887, with a mean value of 886. The velocities as found by this tube from the still water rating, were invariably greater than were given by the moving water rating the average difference being 6.4 per cent. This great difference is, however, probably due partly to the comparatively large disturbing effect of the tube in such a small channel, and partly to the impossibility of taking measurements near the walls where the velocity is least:

The Hydrometric Pendulum.—Where flow takes place through an uniform channel of small dimensions, and where the velocity can be initially determined by some accurate method, as by a weir, the hydrometric pendulum may be calibrated so as to record this velocity at any future time. The instrument consists of a pendulum having a submerged spherical bob which is heavier than the water which it displaces, and which hangs vertically when the water is at rest. When in motion the pressure of the water causes the pendulum to take up an inclined position, the angle of inclination being a measure of the velocity, and from a previous calibration, this may be read off directly.

Measurement of Flow in a Parallel Conduit.—A method recently devised\(^1\) for measuring the discharge from a parallel conduit consists in the provision of a light but rigid apron of canvas over a framework of angle iron, suspended vertically from a light carriage which runs on rails fixed on either side of the conduit. When lowered, the apron fills the conduit with very little clearance and is carried along with the same velocity as the stream. Its velocity is then measured by a chronometer and electrical contacts. This method necessitates a conduit of 80 to 100 feet in length, and has been applied to conduits up to 20 feet wide and 12 feet deep, for the measurement of the water supplied to turbines under test.

FIG. 162.—Pitot Tube for Steam Rating.

Art. 102.—Stream Rating Tables.

The usual object of velocity measurements in a river is the construction of a rating table, which shall show the relation at a given point between the height of water, referred to some permanent bench mark, and the discharge of the river. In order to prepare such a table it is necessary to obtain the discharge at various stages of a stream, covering the usual

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\(^1\) By Prof. E. Anderson of Stockholm. See Zeitschrift des Vereins Deutscher Ingenieure, April 20, 1907.
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Range of fluctuation. Usually the data are embodied in the form of a rating curve, showing graphically the relationship between discharge and height of surface level. The length of time during which such a curve can be safely applied depends on the class of channel. Where the channel is constantly shifting it cannot be used for many months unless the soundings are frequently checked with reference to the original datum level.

Art. 103. Gauging of Ice-Covered Streams.

When a river is ice bound its flow becomes somewhat similar to that in a closed flume, the water now flowing under pressure. A series of measurements of such streams by members of the U.S. Geol. Survey lead to the following conclusions:

1. The maximum velocity occurs at a point between 35 per cent. and 40 per cent. of the depth measured from the underside of the ice. The ratio of mean to maximum velocity ranges from about .80 with a depth of 3 feet to .92 with a depth of 16 feet, having a mean value of .85.

2. There are two points of mean velocity on a vertical, the first lying between .08 and .013 of the depth, and the second between .68 and .74 of the depth.

3. The vertical velocity curve becomes more concave as the river rises, owing to the increased head.

4. In making gaugings of such streams the vertical velocity curve method, or the integration method, should be adopted in preference to any of the single-point and co-efficient methods.

Examples.

1. A canal whose depth is 4 feet, having slopes 2 to 1, has a bottom width of 10 feet. The bed is of earth (Kutter's N = .025), and the gradient is 1 foot per mile. Determine the discharge in cubic feet per second.

   \[ \text{Hydraulic mean depth} = 2.58 \text{ feet}. \]
   \[ C = 68.8. \]
   \[ \text{Discharge} = 109.5 \text{ cubic feet per second}. \]

2. A rectangular flume 4 feet wide and 2 feet deep is roughly constructed of unplanked timber, and is required to deliver 80 cubic feet of water per second. Determine the necessary gradient, and assuming it to supply water to a power station distant 5 miles from the supply reservoir,

\[ \text{Water Supply and Irrigation Paper. } \text{No. 95, p. 158.} \]
determine the percentage loss in transmission if the difference of level between the supply reservoir and tail race is 500 feet. \( C = 127.6 \).

Answer. Gradient 1 in 162.9.
Loss of energy = 32.4 per cent.

(8) A flat-bottomed channel is required to have a constant velocity of flow for all depths of water. The bottom breadth is 5 feet. Determine the depth at a section where the breadth is 20 feet, if the hydraulic mean depth is 1.25 feet. \( C = 60 \).

Answer. 4.35 feet.

(4) The original depth of a wide stream is 3 feet, and the slope of its bed is 1 in 1,000, the value of \( f \) being 0.0131, \( C = 70 \). A dam 10 feet high is erected across the stream. Determine the rise in the water level immediately behind the dam and at points \( \frac{1}{2} \) and 1 mile up stream. (Assume the coefficient of discharge for the dam to be 0.560.)

Answer. Rise behind dam = 9.28 feet.
" \( \frac{1}{2} \) mile up stream = 6.87 feet.
" 1 " " " = 4.04.

(5) It is required to excavate a canal out of rock to be of rectangular section and to bring 500 cubic feet of water per second from a distance of 4 miles with a velocity of 7.5 feet per second. Determine the gradient and the most suitable section for the canal. Take \( C = 150 \).

Answer. Section 11.54 feet wide, 5.77 feet deep.
Gradient 1 in 1,156.

(6) In carrying out field operations to determine the discharge of a river, a straight reach 500 feet long is available. The slope is approximately 1 in 1,000, and the levelling is possibly accurate within \( \frac{1}{3} \) foot. The possible error in determining the wetted perimeter is 8 per cent., and in determining the mean sectional area is 5 per cent. To what degree of accuracy are the final results likely to approximate.

Answer. Within about 13.5 per cent.

(7) The value of \( f \) for a stream having a slope of 1 in 1,000 is 0.0050. Normally, the stream is of depth 4 feet and breadth 60 feet, but is passed through a sluice having an opening 2.4 feet deep. Determine whether the conditions are such as to lead to the formation of a standing wave, and if so determine the probable height of the crest of this wave above the upper edge of the sluice.

Answer. Yes. 93 feet.