BOARD OF SECONDARY EDUCATION,
WEST BENGAL

Higher Secondary Examination Papers

1960

1. (a) Prove that the radian is a constant angle. Find its value in degrees, minutes etc. \[ \pi = \frac{180}{\pi} \]

(b) The angles of a triangle are in Arithmetical Progression and the number of degrees in the least is to the number of radians in the greatest as 60 to \( \pi \). Find the angles in degrees.

2. (a) If \( A, B, A + B \) are all acute angles, prove (geometrically) that \[ \cos (A + B) = \cos A \cos B - \sin A \sin B. \]

(b) Find the value of \[ \sin^2 60^\circ + \cos^2 150^\circ + \tan^2 120^\circ + \cos 180^\circ - \tan 135^\circ. \]

3. (a) Find the values of \( \theta \) between \( 0^\circ \) and \( 360^\circ \) which satisfy the equation \[ 2 \sin^2 \theta + 3 \cos \theta = 0. \]

(b) If \( A + B = 90^\circ \), prove that \[ \frac{\cos 2B - \cos 2A}{\sin 2A} = \tan A - \tan B. \]

4. (a) In a triangle \( ABC \), prove that \( a = b \cos C + c \cos B \).

(b) In a triangle, the angles are to one another as \( 1 : 2 : 3 \); prove that the corresponding sides are as \( 1 : \sqrt{3} : 2 \).

5. Two vertical pillars, the height of one of which is double that of the other, are at a distance of 150 ft. from each other. At a point between the pillars and on the line joining their feet the angular elevations of the tops of the taller and the shorter pillar are found to be \( 60^\circ \) and \( 30^\circ \) respectively. Find the heights of the pillars and the position of the point.

6. Draw the graph of \( \sin x \) between the values \( x = -\pi \) and \( x = \pi \) and find, from the graph, the value of \( \sin 120^\circ \).

1960 (Compartmental)

1. (a) The difference between the two acute angles of a right-angled triangle is \( \frac{\pi}{3} \) radians; express these angles in degrees.

(b) If \( s \) is the length of the arc of a circle whose radius is \( r \) and \( \theta \) is the radian measure of the angle at the centre, standing on the arc, prove that
2. (a) If $A$ and $B$ are both acute angles and $A$ is greater than $B$, prove (geometrically) that

$$\sin (A - B) = \sin A \cos B - \cos A \sin B.$$ 

(b) If $\sin A = \frac{1}{2}$ and $\cos B = \frac{1}{2}$, where $A$ and $B$ are acute angles, find the value of

$$\frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

3. (a) Find the values of $\theta$ between $0^\circ$ and $360^\circ$ which satisfy the equation

$$\sin^2 \theta - 2 \cos \theta + \frac{1}{2} = 0.$$

(b) If $A + B + C = 180^\circ$, prove that

$$\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}.$$

4. In a triangle $ABC$, prove that

(i) $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$,

(ii) $a \cos \frac{B-C}{2} = (b+c) \sin \frac{A}{2}$.

5. The upper part of a straight tree broken over by the wind, but not completely separated, makes an angle of $30^\circ$ with the ground, and the distance from the root to the point where the top of the tree touches the ground is 50 feet. What was the height of the tree?

6. Draw the graph of $\cos x$ between the values of $x = -\pi$ and $x = \pi$ and read off from the graph, the value of $\cos 150^\circ$.

1961

1. (a) The radius of a circle is 10 cm.; find the angle, in degrees and minutes, subtended at its centre by an arc 6 cm. in length. [ $\pi = \frac{180}{\pi}$ ]

(b) The angles of a triangle are in Arithmetical Progression. If the number of degrees in the greatest angle is the same as the number of grades in the least, find the angles in degrees.

2. (a) If $A$, $B$ and $A - B$ are positive acute angles, prove geometrically that

$$\sin (A - B) = \sin A \cos B - \cos A \sin B.$$ 

(b) Find the value of

$$\sin 330^\circ + \tan 45^\circ - 4 \sin^2 120^\circ + 2 \cos^2 180^\circ + \sec^2 180^\circ.$$

3. (a) Find the values of $\theta$ between $0^\circ$ and $360^\circ$ which satisfy the equation

$$\sqrt{3} \sin \theta + \cos \theta = 1.$$

(b) If $A + B + C = 180^\circ$, prove that

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C.$$

4. In a triangle $ABC$, prove that

(a) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

(b) $a \sin (B-C) + b \sin (C-A) + c \sin (A-B) = 0$
5. On a straight coast there are three objects $A$, $B$ and $C$ such that $AB = BC = 4$ miles. A steamer approaches $B$ in a line perpendicular to the coast and at a certain point $AC$ is found to subtend an angle of $60^\circ$; after sailing in the same direction for ten minutes, $AC$ is found to subtend an angle of $120^\circ$; find the rate at which the steamer is going.

6. Draw the graph of $\sin \alpha$ between the values of $\alpha = 0^\circ$ and $\alpha = 360^\circ$ and read off from the graph, the value of $\sin 240^\circ$.

1961 (Compartmental)

1. (a) Define a radian. Taking $\pi = 3.1416$, show that a radian contains 206265 seconds approximately.

(b) One angle of a triangle is $\frac{\pi}{2}$ grades and another is $\frac{\pi}{2}$ degrees, whilst the third is $\frac{\pi}{75}$ radians; express them all in degrees.

2. (a) If $A$, $B$ and $A - B$ are all positive acute angles, prove geometrically that

$$\cos (A - B) = \cos A \cos B + \sin A \sin B.$$

(b) Find the value of

$$\frac{2 \tan^2 30^\circ}{1 - \tan^2 30^\circ} + (\sec^2 45^\circ - \cot^2 45^\circ) - (\cos^2 60^\circ + \sin^2 120^\circ).$$

3. (a) Prove that

$$\cos 3A = 4 \cos^3 A - 3 \cos A.$$

(b) If $A + B + C = 180^\circ$, prove that

$$\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C.$$

4. In a triangle $ABC$, prove

(a) $c = a \cos B + b \cos A$.

(b) $b - c \cos \frac{A}{2} = a \sin \frac{B + C}{2}$.

5. Two vertical poles are 190 feet apart and the height of one is double that of the other. From the middle point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary. Find their heights.

6. Draw the graph of $\cos \alpha$ between the values $\alpha = 0^\circ$ and $\alpha = 360^\circ$ and read off from the graph, the value of $\cos 300^\circ$. 
IMPORTANT FORMULÆ AND RESULTS

Solid Geometry (Mensuration)

1. *Rectangular parallelopiped* (or cuboid).
   If \( a, b, c \) be its length, breadth and height
   (i) Area of the surface = \( 2(bc + ca + ab) \).
   (ii) Volume = \( abc \).
   (iii) Surface area of a cube of side \( a = 6a^2 \).
   (iv) Volume = \( a^3 \).

2. *Right Pyramid on any regular base*
   (i) Slant surface = \( \frac{1}{2} \text{(Perimeter of base)} \times \text{slant height} \).
   (ii) Volume = \( \frac{1}{3} \text{(area of base)} \times \text{height} \).

   Volume = \( \frac{1}{3} \text{(area of base)} \times \text{height} \).

   (i) Lateral surface = \( \text{(perimeter of base)} \times \text{height} \).
   (ii) Volume = \( \text{(area of base)} \times \text{height} \).

5. *Right circular cylinder.*
   If \( r \) is the radius of the base and \( h \) the height of the cylinder,
   (i) Area of the curved surface
       = \( \text{(circumference of base)} \times \text{height} \)
       = \( 2\pi rh \).
   (ii) Area of the *whole surface*
       = \( 2\pi rh + 2\pi r^2 = 2\pi (h + r) \).
   (iii) Volume = \( \text{(area of base)} \times \text{height} = \pi r^2 h \)

   If \( r \) is the radius of the base, \( h \) the height, and \( \theta \) the semi-vertical angle of the cone.
\[ \frac{\partial}{\partial \theta} \]

(i) Area of curved surface
\[ = \frac{1}{2}(\text{circumference of base}) \times \text{slant side} \]
\[ = \frac{1}{2} \cdot 2\pi r \cdot l = \pi rl \]
\[ = \pi r \sqrt{h^2 + r^2} = \pi r^2 \csc \alpha. \]

(ii) Area of the whole surface \(= \pi r(l + r). \)

(iii) Volume \(= \frac{1}{3}(\text{area of base}) \times \text{height} \]
\[ = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi h \tan \theta. \]

7. **Sphere.**

If \( r \) be the radius of the sphere,

(i) Area of curved surface \(= 4\pi r^2. \)

(ii) Volume \(= \frac{4}{3} \pi r^3. \)

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**Co-ordinate Geometry**

1. Distance \( PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

Distance \( OP = r = \sqrt{x^2 + y^2}. \)

2. Point dividing the line joining two given points in a given ratio:

\[ x = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \quad y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}. \]

Middle point \( \frac{1}{2}(x_1 + x_2), \frac{1}{2}(y_1 + y_2). \)

3. Area of a triangle with given vertices

\[ \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|. \]

4. General equation of a straight line

\[ ax + by + c = 0 \text{ (} a \text{ and } b \text{ both } \neq 0). \]

Every first degree equation in \( x, \) \( y \) represents a straight line.

5. Transfer of the origin (directions of axes remaining unchanged) from \((0, 0)\) to \((a, b)\)

\[ x = X + a, \quad y = Y + b. \]

6. Straight line parallel to the \( x \)-axis : \( y = b. \)

Straight line parallel to the \( y \)-axis : \( x = a. \)
7. Equations of straight lines in standard forms:

(i) Intercept form: \( \frac{x}{a} + \frac{y}{b} = 1 \).

(ii) 'm' form: \( y = mx + c \).

(iii) Form through a given point:

\[ y - y_1 = m(x - x_1), \quad \text{or} \quad \frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} \]

(iv) Normal (or perpendicular) form:

\( x \cos a + y \sin a = p \).

(v) Two points form: \( y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \).

8. Point of Intersection of the two lines

\[ a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0 \]

\[ x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \quad y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1} \]

9. Condition for concurrence of the three given lines

\[ a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0, \quad a_3x + b_3y + c_3 = 0 \]

\[ a_1(b_2c_3 - b_3c_2) + a_2(b_3c_1 - b_1c_3) + a_3(b_1c_2 - b_2c_1) = 0 \]

10. Condition for collinearity of the three given points

\((x_1, y_1), (x_2, y_2), (x_3, y_3)\), is

\[ x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0 \]

11. Angle between two given lines:

(i) When the lines are \( y = m_1x + c_1, \ y = m_2x + c_2 \)

\[ \tan \phi = \frac{m_1 - m_2}{1 + m_1m_2} \]

(ii) When the lines are

\[ a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0 \]

\[ \tan \phi = \frac{a_1b_2 - a_2b_1}{a_1a_2 + b_1b_2} \]

12. Conditions for

(a) parallel lines, \( \quad (i) \ m_1 = m_2, \quad (ii) \frac{a_1}{a_2} = \frac{b_1}{b_2} \)
(b) perpendicular lines,  
(i) \( m_1 m_2 = -1 \),  
(ii) \( a_1 a_2 + b_1 b_2 = 0 \).

18. Length of the perpendicular from the point \((x_1, y_1)\) upon the line \( ax + by + c = 0 \) is
\[
\pm \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}.
\]

14. Equations of the bisectors of the angle between the lines \( a_1 x + b_1 y + c_1 = 0 \), \( a_2 x + b_2 y + c_2 = 0 \) are
\[
\frac{a_1 x + b_1 y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2 x + b_2 y + c_2}{\sqrt{a_2^2 + b_2^2}}.
\]

15. Equation of the circle

(i) Standard form: \( x^2 + y^2 = a^2 \)
   centre: \((0, 0)\); radius \(a\).

(ii) general form: \( x^2 + y^2 + 2gx + 2fy + c = 0 \)
   centre: \((−g, −f)\), radius: \( \sqrt{g^2 + f^2 − c} \).

16. Circle with the given points \((x_1, y_1)\) and \((x_2, y_2)\) as extremities of a diameter
\[
(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0.
\]

17. Equation of the tangent to the circle at \((x_1, y_1)\)

(i) for standard form: \( xx_1 + yy_1 = a^2 \),
(ii) for general form:
\[
x x_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.
\]

18. Equation of the normal to the circle at \((x_1, y_1)\)

(i) for standard form: \( \frac{x}{x_1} = \frac{y}{y_1} \).
(ii) for general form: \( x(y_1 + f) − y(x_1 + g) = fx_1 − gy_1 \).

19. Length of the chord of the circle \( x^2 + y^2 = a^2 \) intercepted by the line \( y = mx + c \) is
\[
2 \frac{\sqrt{a^2(1 + m^2) − c^2}}{a}.
\]
20. Condition of tangency: condition that the line
\( y = mx + c \) may touch the circle \( x^2 + y^2 = a^2 \) is
\[ c = \pm a \sqrt{1 + m^2} \]
\( y = mx + a \sqrt{1 + m^2} \) is a tangent to the circle \( x^2 + y^2 = a^2 \)
for all values of \( m \), and in that case the point contact is
\[ -\frac{am}{\sqrt{1 + m^2}}, \frac{a}{\sqrt{1 + m^2}} \.

21. Length of the tangent from an external point \((x_1, y_1)\)
to the circle \( x^2 + y^2 + 2gx + 2fy + c = 0 \) is
\[ \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} \.

22. Standard forms of the equations of conics.
   (a) Parabola
      (i) \( y^2 = 4a(x - a) \) *(with axis and directrix as axes of co-ordinates).
      (ii) \( y^2 = 4ax \) (Standard form),
           (with the vertex as origin and the axis and the tangent at the vertex as axes of co-ordinates).
   (b) Ellipse
       \[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \] (Standard form).
       (with centre as origin, and major and minor axes as axes of co-ordinates).
   (c) Hyperbola
       \[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \] (standard form)
       (with centre as origin and transverse and conjugate axes as axes of co-ordinates).

23. Parabola:
   (i) Standard form \( y^2 = 4ax \).
   (ii) Latus rectum = 4a ; focus is \((a, 0)\) ; extremities of
        the latus rectum are \((a, \pm 2a)\) ; directrix is \( x = -a \).
(iii) Equation of the tangent at \((x_1, y_1)\) is 
\[ yy_1 = 2a(x + x_1). \]

(iv) Normal at \((x_1, y_1)\) is 
\[ y - y_1 = -\frac{y_1}{2a}(x - x_1). \]

(v) Length of the chord intercepted by the straight line 
\[ y = mx + c \] is 
\[ 4 \sqrt{a(a - mc)(1 + m^2)}. \]

(vi) Condition that \(y = mx + c\) may touch the parabola is 
\[ c = \frac{a}{m} \] \((m \neq 0)\).

The line \(y = mx + \frac{a}{m}\) is a tangent to the parabola for all values of \(m\) (except zero), the point of contact being \(\left(\frac{a}{m}, \frac{2a}{m}\right)\).

(vii) Parametric representation: \(x = at^2, y = 2at\).

(viii) Equation of the diameter: \(y = \frac{2a}{m}\).

24. Ellipse

(i) Standard form 
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

(ii) Latus rectum \(= 2a(1 - e^2) = 2 \frac{b^2}{a}\).

(iii) Eccentricity: \(b^2 = a^2(1 - e^2)\) or \(e^2 = \frac{a^2 - b^2}{a^2}\).

(iv) Focal distances of \(P(x_1, y_1)\):
\[ SP = a - ex_1, \quad S'P = a + ex_1; \quad SP + S'P = 2a. \]

(v) Tangent at \((x_1, y_1)\): 
\[ \frac{x_1}{a^2} + \frac{y_1}{b^2} = 1. \]

(vi) Normal at \((x_1, y_1)\): 
\[ \frac{x - x_1}{a^2} = \frac{y - y_1}{b^2}. \]
(vii) Length of the chord intercepted by the line 
\[ y = mx + c \] on the ellipse
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]
\[ = \frac{2ab \sqrt{1 + m^2} \sqrt{a^2m^2 + b^2} - c^2}{a^2m^2 + b^2} \]

(viii) Condition of tangency:
The line \( y = mx + c \) is a tangent to the ellipse if
\[ c = \pm \sqrt{a^2m^2 + b^2}. \]
The line \( y = mx + \sqrt{a^2m^2 + b^2} \) is a tangent to the ellipse for all values of \( m \), and the point of contact is
\[ \frac{a^2m}{\sqrt{a^2m^2 + b^2}}, \frac{b^2}{\sqrt{a^2m^2 + b^2}}. \]

(ix) Auxiliary circle: \( x^2 + y^2 = a^2 \).

(x) Parametric representation: \( x = a \cos \theta, y = b \sin \theta \).

(xi) Diameter \( y = -\frac{b^2}{a^2m} x \).

(xii) Director circle \( x^2 + y^2 = a^2 + b^2 \).

25. Hyperbola

(i) Standard equation: \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \).

(ii) Latus rectum: \( 2a(e^2 - 1) = 2\frac{b^2}{a} \).

(iii) Eccentricity: \( b^2 = a^2(e^2 - 1) \) or \( e^2 = \frac{a^2 + b^2}{a^2} \).

For rectangular (or equilateral) hyperbola
\[ a = b, e = \sqrt{2}. \]

(iv) Focal distances of \( P(x_1, y_1) \)
\[ SP = ex_1 - a, \quad S'P = ex_1 + a \]
\[ S'P - SP = 2a. \]
(v) Equation of the tangent at \((x_1, y_1)\)

\[ \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1. \]

(vi) Equation of the normal at \((x_1, y_1)\) is

\[ \frac{x-x_1}{y_1} = \frac{y-y_1}{-b^2}. \]

1) Length of the chord of the hyperbola intercepted by \(y = mx + c\) is

\[ 2ab \sqrt{1 + m^2} \sqrt{c^2 - a^2m^2 + b^2}. \]

\[ \frac{a^2m^2 - b^2}{a^2m^2 - b^2}. \]

(viii) Condition of tangency:

The line \(y = mx + c\) will be a tangent to the hyperbola if \(c = \pm \sqrt{a^2m^2 - b^2}\).

The line \(y = mx + \sqrt{a^2m^2 - b^2}\) is a tangent to the hyperbola for all values of \(m\), the point of contact being \(\sqrt{a^2m^2 - b^2} - \frac{a^2m^2 - b^2}{a^2m^2 - b^2}\).

(ix) Equation of the diameter is \(y = \frac{a^2}{a^2m^2} x\).

(x) Equation of the asymptotes: \(y = \pm \frac{a^2}{a} x\).