and eclipses, matters in which the Hindus had a practical ceremonial interest, reached a remarkable degree of approximation (much above Graeco-Arab computations) to the figures in Laplace’s Tables, which can only be explained by the circumstance that in the case of these constants the Hindus carried out for more than a thousand years a systematic process of verification and correction by comparison of the computed with the observed results (like the navigator’s correction of the course of the ship at sea), a process which was termed स्रग्मण्डवक. In Zoology, the enumeration of the species of Vermes, Insecta, Reptilia, Batrachia, Aves, etc., makes a fair beginning, but the classification proceeds on external characters and habits of life, and not on an anatomical basis. In Botany, the observation was mainly in the interests of Materia Medica, and the classification was as superficial as possible. (Vide my paper on the Hindu Classification of Plants and Animals).

Experiments:—Experiments were of course conducted for purposes of chemical operations in relation to the arts and manufactures, e.g., Metallurgy, Pharmacy, Dyeing, Perfumery and Cosmetics, Horticulture, the making and polishing of glass (lenses and mirrors of various kinds are mentioned, the spherical and oval दण and चतुत्त, being well-known—Pliny indeed mentions that the best glass ever made was Indian glass). And the
results of such experiments were freely drawn upon for building up scientific hypotheses and generalisations. But of experiment as an independent method of proof or discovery, the instances recorded in books are rare. I may note one interesting example in Udayana’s Kiranávali, relating to the weight of air. Udayana argues that air must be a distinct and independent Bhūta, for if air were a form of the Earth-Bhūta, it would have weight, and it has none. To prove the absence of weight, he refers to an experiment. A small bladder made of a thin membrane, filled with air, will not cause a greater descent in the scale than the same bladder weighed empty. Hence the air possesses no weight. Then Udayana makes an interesting statement. It may be objected, he says, by one who accepts the weight of air—that this argument is inconclusive. For a counter-experiment may be suggested. The balloon filled with smoke (or gas, घ्रेण) rises in the air, whereas the air-filled balloon comes down. This would go to show that air has weight. Udayana replies that this would only show that both smoke (or gas, घ्रेण) and air have no weight. The Hindus appear to have been ignorant of the principle of Archimedes. Vallabhácháryya in the Lilávati, it is true, speaks of a peculiar resistance to sinking (or gravity) exercised by water, which explains the tendency in certain objects to float.
or to come up to the surface of the water, but
the description shows that he had no clear ideas
on the subject.]

Mathuránátha, again, states that the determination
of the degree of purity (the carat) of gold by
rubbing against the assaying stone and observing
the character of the yellowish streak against the
black smooth background, is only an indirect means
of ascertaining weight (गुन्धवेद्यत्स प्रतिलिंगम् lit. specific gravity)
— which seems to suggest that there was a more
direct means of arriving at the latter. Probably
this refers to the common Indian method of
comparing the lengths of wires of uniform thickness
that can be formed by drawing different pieces
of gold of equal weight through the same
diamond bore. I think it may be regarded as fairly
certain that the Hindus were ignorant of Archi-
medes’s discovery, an ignorance which, at any
rate, they could not have well borrowed from
the Greeks, no more than they could have thus
borrowed their knowledge of things unknown to the
Greeks themselves. [ Cf. Udayana, Kiranávali :—

वायुगिनिकनुम्न, किं स (बाधी: ) प्रविषोले कुबलमारि सत्।

ग य वशस्यापूरितको वचारवृक्षः धार्मिक सत्यमन्मः।

कनौनाम्य घूम्पूरितिम श्रेष्ठ समेद्यालिनः दति चेत्।

गः सबं धूम्पद्वां।

नः अति गर्वनिवेशे लोकषोभन्तिशविश्वम् चपोतीति।

—cf. Vallabha-

चार्यया, Līlāvatī—गुन्धवेद्यत्स प्रतिलिंगम् पदार्थानामुपास्तविव¯

वांकन विभववत् चक्रपवतुचता सांबंधित पतनीतपत्तनः निवृत्ति भविष्यतीति।
Fallacies of Observation—Mal-observation and Non-observation:—These were carefully studied in relation to errors of observation, and Hallucination (बम, बमास, बारोक),—which were ascribed to three causes:—

(a) Dosha, दोष, defect of sense organ, as of the eye in jaundice, or of the skin in certain forms of leprosy (leading to tactile insensibility, cf. Susruta), or defect of necessary stimulus, e.g., too faint light, or undue distance or nearness, in vision; (b) Samprayoga, सम्प्रयोग, presentation of a part or an aspect instead of the whole; and (c) Sanskāra, संस्कार, the disturbing influence of mental predisposition, e.g. expectation, memory, habit, prejudice, etc.

The Doctrine of Inference:—Anumāna (Inference) is the process of ascertaining, not by perception or direct observation, but through the instrumentality or medium of a mark, that a thing possesses a certain character. Inference is therefore based on the establishment of an invariable concomitance (Vyāpti, व्याप्ति) between the mark, and the character inferred. The Hindu Inference (Anumāna) is therefore neither merely formal nor merely material, but a combined Formal-Material
Deductive-Inductive Process. It is neither the Aristotelian Syllogism (Formal-Deductive Process), nor Mill’s Induction (Material-Inductive Process), but the real inference which must combine formal validity with material truth, inductive generalisation with deductive particularisation.

An inference admits of a rigorous formal statement,—in the shape of five propositions, for dialectical purposes (i.e. in demonstrating to others),—or of three propositions when the inference is for oneself (प्रमाणभवन):—(1) the probandum, the statement of the proposition to be established (प्रलोक, शास्त्रिक, e.g., yonder mountain is fiery (say, an active volcano); (2) the reason, the ascription of the mark (विन्यास, रूपवर्ग),—e.g., for it smokes; (3) now, the general proposition, stating the invariable concomitance which is the ground of the inference,—clenched by an example bringing home the responsibility of the reasoner to establish a real relation, e.g., whatever smokes is fiery, as an oven; (विन्यास);—(4) next, the application, the ascertainment of the existence of the mark in the present case (व्यवस्था) e.g., yonder mountain smokes;—(5) finally, the conclusion, the probandum proved (विश्लेष), e.g., yonder mountain is fiery.

1. Yonder mountain is fiery.
2. For it smokes.
3. Whatever smokes is fiery, as an oven.
4. Yonder mountain does smoke.
5. Therefore, yonder mountain is fiery.

For inference for oneself, only the first three or the last three propositions, are held to be sufficient.

The Hindu Anumána, it will be seen, anticipates J. S. Mill's analysis of the syllogism as a material inference, but is more comprehensive;—for the Hindu Udáharana, the third or general proposition with an example, combines and harmonises Mill's view of the major premise as a brief Memorandum of like instances already observed, fortified by a recommendation to extend its application to unobserved cases, with the Aristotelian view of it as a universal proposition which is the formal ground of the inference. This Formal-Material Deductive-Inductive process thus turns on one thing—the establishment of the invariable concomitance (आश्रित) between the mark and the character inferred,—in other words, an inductive generalisation. The question is—what is our warrant for taking the leap from the observed to unobserved cases? Under what conditions are we justified to assert a Universal Real Proposition on the basis of our necessarily limited observation?

The Chárváka view:—Among the Chárvákas there were two classes, the cruder school of materialists who accepted perception (प्रेक्ष) as a valid source of knowledge, as well as the reality of
Natural Law (ब्रजाव), and the finer school of sceptics, who impugned all kinds of knowledge, immediate as well as mediate, and all evidence, Perception as well as Inference, vide Jayanta's reference in the Nyāya-Manjari to सूरितविजयांशिपका; also यानेंकृपूर्तसं प्रतिपत्तया रतिप्रतिपत्तया प्रमाणविवेक संबंधनात्मकारयोगवक्त्र आयातकम्—A'hnika I, Manjari.

The Chārvākas hold that the principle of causality, which the Buddhists assume to be a ground of an induction (ब्रह्म) is itself an induction (a case of Vyápti), which amounts to reasoning in a circle (प्रक्षेप); that every inference is based on an unconditional invariable concomitance which itself must be inferred, as universal propositions cannot be established by our limited perceptions, and thus there is a regressus ad infinitum (ब्रह्मवर्धय); and that the nexus between cause and effect, or between the sign and the thing signified (e.g., smoke and fire) is only a mental step or subjective association based on former perception, a mental step which by accident is found justified by the result in a number of cases.

The Buddhists—their Analysis and Vindication of Inference:—The Buddhists, however, take their stand on the principle of the Uniformity of Nature (प्रतिपत्तया संवभाव प्रतिपत्तया, Nyaya-Vindu). This uniformity, for scientific purposes, has to be divided into two different relations,—(1) the uniformity of succession in the relation of cause and effect,
of smoke to fire (कार्यकारणभाव, सहस्रवति, Nyaya-Vindu); (2) the uniformity of co-existence (in the form of co-inherence in the same substrate) in the relation of genus and species, e.g., the relation of invariable concomitance expressed in the proposition,—all Sinsapās are trees,—which is not a relation of causality, but of co-existence or co-inherence in the same substrate (i.e., the co-inherence of the generic qualities of a tree with the specific characters of a Sisu tree, in this particular individual before me, a Sisu tree),—a relation which may be termed essential identity (तत्तत्त्व, सम्बन्ध—Nyaya-Vindu). To these two, the Buddhists add a third ground of inference, non-perception of the perceptible (अभ्यावलय, अभ्यावलय), which is employed in inferring the absence (नलिकाधि) of a thing from the non-perception of something else. In all cases of inference based on the Uniformity of Nature, the relation is that of inseperableness or non-disjunction between the mark and the character inferred. The question is—how is this inseperableness (विन्यासावधि) ascertained, and what is the warrant of our belief in it, in these cases?

Ascertainment of Inseperableness or Non-disjunction: Buddhist Account:—First take the case of causation. The cause is the invariable antecedent of the effect. What is meant is that the specific effect (with all the distinctive and relevant accompaniments, कार्यविशेष) is invariably preceded by a
specific cause (कारणविशिष्ट). It is not that clouds always lead to rain, or that floods in the river valley always imply rain in the hills higher up. But this particular conjunction of antecedent circumstances (e.g. the appearance of a particular kind of clouds accompanied with flashes of lightning, the roll of thunder and flights of Valákás—driven by the wind from a particular quarter of the horizon, and ascending in black masses, etc.) is as a rule the precursor of a particular assemblage of rain effects (rain with particular accompaniments). Again, this particular kind of flood (overflowing of the river banks accompanied with muddy discoloration of the water, rapid currents, the bearing down of tree trunks, etc.) is always preceded by rain in the hills higher up (though, no doubt, other cases of floods in a river may be due to a breach in an embankment or the melting of the snows). In other words, the Buddhists (and the earlier Nyáya Schools) avoid the difficulty arising from the plurality of causes by taking into consideration the accompanying phenomena, which, if properly marked, would always point to a specific cause of a specific effect.

I quote Nyáya authorities, but this device to obviate the plurality of causes is common to the early Nyáya and the Buddhistic systems.
In other words, a single condition called a cause is not invariably succeeded by the effect, nor does the effect phenomenon in general point to any particular cause as antecedent, for there may be a plurality of causes of a general effect. The skilful observer will therefore select the full complement of causal conditions, which is invariably succeeded by the effect,—and also the specific effect (e.g., धूमविशिष्ट) which points to a specific causal antecedent. Compare also Jayanta:—we infer an effect from a specific assemblage of causes—ग च वाष्पसाधनम् केतुलं हुमी वेयाल्य अभिप्रायः सातु। छवि च विभिन्नदीय वार्ष्य वेदुः। ग च वाष्पसाधनो दुर्जयः। संधीर्षितासाधनादिप्रियः सीता—

Similarly we infer a specific cause from a specific assemblage of effects:—वेयाल्यनिति गच्छ वार्ष्य
A specific assemblage of causes, therefore, has only one specific assemblage of effects, and vice versa. Of course, the observer is to find out the essential or relevant features (as distinguished from the irrelevant ones) which, being included, will enable him to specify the particular cause of the particular effect.

Now this being premised to be the exact meaning of the inseparableness or non-disjunction in the case of cause and effect, we come to the question with which we started—how is this relation to be ascertained or established between two phenomena or assemblages of phenomena? Obviously, mere observation of their agreement in presence (प्रविष्ट) and their agreement in absence (विनिर्भ) is no help in the matter. Take a concrete example. The ass is customarily employed to bring the fuel with which fire is lighted. In a hundred cases you have observed the ass among the antecedents of smoke. In a hundred cases you may have observed that when there is no ass, there is no smoke. This is no
warrant for concluding a relation of cause and effect between an ass and smoke. It may be that you happen to have never observed smoke without an antecedent ass, or an ass without smoke following. Even this is of no avail. It is not agreement (unbroken and uniform though it be) in presence, or in absence, or in both, that can settle the matter. There is one and only one way of ascertaining the causal relation. Suppose \( A \) with certain accompaniments is found to precede \( B \) immediately. Now if \( A \) disappearing, \( B \) disappears, even though all other antecedents remain and there is no other change in the case, then and then only can the causal relation be ascertained. It is not a mere table of positive instances or negative instances (चढ़ गाया न दध गाया);—it is this method which we may term the Method of Subtraction (the Method of Difference in its negative aspect) that is the only exact and rigorous scientific Method. Such was the statement of the earlier Buddhists (cf. Udyotakara's and Váchaspati's report of the Buddhist Doctrine of Inference—इह प्रतिवर्गी न दशमावावधीयोऽ।

मदवान् बाधाय: ॥ वाजहातु प्रतिविरुध्दवधनम् एव प्रतिवर्गोऽ। बहादु वाजहातु
माधवा श्रमाधव सिंहासनात। बिवासाधवनीववंभव गाय न दशमान्त
—a Buddhist Karika quoted in Váchaspati, Udayana, Sríharsha, Mádhava, etc.—वाजहातु प्रतिविरुध्दवधनम् प्रदन—चर्चण
क्ति भवति—हर्वागुप्ति गवि तदन्दव्यु भविष्यति न भवति—एवत्मातार: ॥

... (एवं तादात्मानि निपथं शाख:...
But the canon in this form is not sufficiently safe-guarded against possible abuse. Two points have to be emphasised: -(1) It must be carefully observed that no other condition is changed, (2) that the appearance and disappearance of $A$ must immediately precede the appearance and disappearance of $B$. The definition of a cause is based on two fundamental characters, (1) the unconditional invariableness of the antecedence, and (2) the immediateness of the antecedence. The canon of the Method of Difference must therefore be stated in such a form as to emphasise each of these aspects. And one main difficulty in the practical application of the canon is that along with the introduction or sublation of an antecedent, some other phenomenon may be introduced or sublated unobserved. As a safe-guard against this radical vitiation of the Method, the later Buddhists formulated the canon of a modified Method, termed the Panchakáraní, a Joint Method of Difference, which combines the positive and the negative Methods of Difference (the Method of Addition and the Method of Subtraction) in a series of five steps, and which equally emphasises the unconditionality and the immediateness of the antecedence as essential moments of the causal relation. This is
neither agreement in presence, nor agreement in presence as well as absence (the foundation of J. S. Mill's Joint Method of Agreement), but the Joint Method of Difference. The Panchakárani runs thus:—

The following changes being observed, everything else remaining constant, the relation of cause and effect is rigorously established:—

First step—The 'cause' and the 'effect' phenomena are both unperceived.
Second step—Then the 'cause' phenomenon is perceived.
Third step—Then in immediate succession, the 'effect' phenomenon is perceived.
Fourth step—Then the 'cause' phenomenon is sublated or disappears.
Fifth step—Then in immediate succession, the 'effect' phenomenon disappears.

Throughout, of course, it is assumed that the other circumstances remain the same (at least the relevant or material circumstances).

This Panchakárani, the Joint Method of Difference, has some advantages over J. S. Mill's Method of Difference, or what is identical therewith, the earlier Buddhist Method; and the form of the canon bringing out in prominent relief the unconditionality and the immediateness of the antecedence, is as superior from a theoretical point of view to J. S. Mill's canon,
and is as much more consonant than the latter to the practice of every experimenter, as the Hindu analysis of Anumána as a Formal-Material Deductive-Inductive Inference is more comprehensive and more scientific than Aristotle’s or Mill’s analysis of the Syllogism (or Mediate Inference).

But even the Panchakárani is no sufficient answer to the question with which we started. The Panchakárani is only a method; it shows only how in a particular case the relation of cause and effect is to be established (प्रतिक्रियाविद्या).

But we want more than this—we require a warrant for the process. The Buddhists therefore supply the following proof of the Method:—Doubt is legitimate, but there is a limit to doubt. When doubt lands you in a contradiction of a fundamental ground of practice, and would thus annul all practical exercise of the will, the doubt must cease; else the doubt would be suicidal or sophistical. In this particular case, when the Panchakárani is satisfied, the antecedent in question must be the cause, for there is no other antecedent to serve as cause; the proof is indirect but rigid. If this be not the cause, there is no cause of the phenomenon. It was not, and it begins to be, without a cause; which would be a contradiction of the rational ground of all practice, for all volitional activity proceeds by implication on the principle of causality. If things could happen
without a cause, all our motives to action would be baffled. The link between a presentation and the instinctive volitional reaction would snap, and the circuit of consciousness would be left incomplete. In fact, the Buddhists go further; they hold causal efficiency (अंतरिक्ष) to be of the essence of empirical (relative) Reality. The proof of the Joint Method of Difference, then, lies in a strict application of the principle of causality in its negative form (viz., there can be no phenomenon without a cause) and the truth of this last is guaranteed by the same ultimate criterion of empirical (relative) Reality as the truth of Perception itself, viz., the correspondence between the rational and the practical activity of the self.

But invariable concomitance (or non-disjunction), the Buddhists argue, has another form, e.g., the relation of the genus to the species. We may have perceived a hundred instances of the association of certain characters with certain others; we may also have never perceived the former when the latter are absent; but this would not enable us to generalise and establish invariable and unconditional co-existence. We must be first satisfied that there is identity of essence (तदायत्त अभ्यास). It is only when we perceive that the characters of a Sinsapá are co-inherent with the generic characters of a tree in the same individual object (a Sinsapá tree before me), and when we further
perceive that the characters are held together by
the relation of identity of essence, that we can say
that all Sinsapás are trees. For as there is identity
of essence, a Sinsapá would not be a Sinsapá, if
it were not a tree. It would lose its self-identity
which is a contradiction. Hence the relation of
identity of essence (तादात्म्य समावेष), as in the
relation of the species to the genus, is the sole
ground for establishing uniformity of co-existence
(परिवर्तन, भवान्यविवर्तन).

(For the Buddhist Method of Induction, in
its later form, the Panchakárani, vide Sarvadarsana
Sangraha—Buddhist reply to the Chárváka attack
on Inference:—

१४० यद्यपायेः अविनाशाभी दुर्भेष्ट श्रति तद्भाषीयः।
तदात्म्यात्मदुन्-
परिवर्तनसमावेष सुकुमारलात्।
तदुन्—काव्याकारसमावेष वा
समावेष वा सिद्धान्तकात्।
अविनाशाविविधानीन्द्रहं भागम् न दृष्ट सा नात्।
As for the Nyáya view अविनाशाविविधानी अविनाशाविविधानीको
प्रति:—the Buddhist objects—नमु एवे साधारणविद्यामित्री
हृदयार्थः भवेन। भूतं, भविष्यति, वर्तमाने जनविषयमात्रे तथा
विभागः अविनाशाय अविनाश्यात्:—the Nyáya retorts नमु
तान्त्रिकः भविष्यति नन्दे अविनाशाय रुपविविधः वति:—the Buddhist
answers—नैव श्चेष्ट: विनाशति। कारणं कार्यं क्रियमदुपच्छन्ते
प्रेममपाय: अविनाशतः विविधाकात्। तदात् विविधतः
तथाकारेण विनाशायात् अविनाशाया:। नातवर्यं:।
तदुन् काव्याकार
dृष्ट:।
तदात् तदन्त्वतिविविधेन अविनाशाय निकृष्टेन
तदन्त्वतिविविधः कार्यं दिलोः। प्रत्येकं विविधं अविनाशाय निकृष्टेन:।
The Nyāya Doctrine of Inference:—The Nyāya easily demolishes the Buddhist contention about identity of essence. The Nyāya writers, being realists, do not impugn the reality of the genus (त्रावण) like the nominalists or the nominalistic conceptualists in the Mimansa Schools (मिंमाण्ड: and मार्माण्ड:) ; but they point out that the inseparableness (or non-disjunction) in such cases can only be established by the experience of unbroken uniformity (अभावचारित्व i.e., by अभाववारित्व, द्वन्द्व नाइति). Uniform agreement in presence with uniform agreement in absence,—not the mysterious identity of essence irresistibly perceived in any individual case or cases—is the only basis for constituting genera and species in Natural Classification. Indeed some of the later Nyāya writers point out that individuals do not always possess in Nature all the characters that go to form the definition of the class to which they are referred.
Similarly, as regards the relation of cause and effect, a nexus is sometimes fancied to be perceived, a power in the cause to produce the effect (अभिविध), or an ultimate form (जाति सौरण), which is supposed to be present, whenever the effect (quality or substance) is produced (cf. Bacon’s view of the ‘Forms’ of Simple Qualities). All this is neither a matter of observation nor of legitimate hypothesis. There is nothing except the invariable time-relation (antecedence and sequence) between the cause and the effect. But the mere invariableness of an antecedent does not suffice to constitute it the cause of what succeeds; it must be an unconditional antecedent as well (समयस्वीकरण निपटपूर्व वस्नंत, being the definition of वायुसूकारमाच). For example, the essential or adventitious accompaniments of an invariable antecedent, may also be invariable antecedents, but they are not unconditional but only collateral and indirect, in other words their antecedence is conditional on something else (व स्वातंत्र्य). The potter’s stick is an unconditional invariable antecedent of the jar, but the colour of a stick, or its texture or size or any other accompaniment or accident, which does not contribute to the work done (so far as we are considering it) is not an unconditional antecedent, and must not therefore be regarded as a cause. Similarly, the co-effects of the invariable antecedents, or what enters into the production of these
-co-effects, may themselves be invariable antecedents but they are not unconditional, being themselves conditioned by those of the antecedents of which they are effects. For example, the sound produced by the stick, or by the potter’s wheel, invariably precedes the jar, but it is a co-effect,—and Ākāsa (ether) as the substrate, and Vayu (air) as the vehicle, of the sound, enter into the production of this co-effect, but these are not ‘unconditional’ antecedents, and must therefore be rejected in an enumeration of conditions or causes of the jar. Again, the conditions of the conditions, the invariable antecedents of the invariable antecedents, are not unconditional. The potter’s father is an invariable antecedent of the potter who is an invariable antecedent of the jar, but the potter’s father does not stand in a causal relation to the potter’s handiwork. In fact, the antecedence must not only be unconditionally invariable, but must also be immediate (चालनकालपूर्वभावभाव: एव कार्यभाव एव अवैधात्मिकताः).

Finally, all seemingly invariable antecedents which may be dispensed with or left out, are ipso facto not unconditional, and cannot therefore be regarded as causal conditions; in short, nothing that is unnecessary is unconditional; for this class, vide Visvanātha: —निरतात्मकपूर्वभावभाव: एव अवैधात्मिकताः एव वाच्यविवेदी।

Visvanātha, Siddhānta Muktāvalī, on Sloka 20. For example, it is the custom to point to spatial position or direction with the
fingers, but finger-pointing, though invariably present, is not causally related to the perception of direction or spatial position, because we can imagine such perception without finger-pointing, (भवानिविश्वरत्वा पञ्चुः। निर्देश विद्यावन्धेन न भावाभिंक्षः सङ्कल्पः, Vāchaspati, Tātparyya-Tikā, Chap. I, A'hnika 1, Sutra 5,—this shows that the doctrine of भवानिविश्वरत्वा was long anterior to Gangesa).

[Visvanātha in the Bhāsha-Parichchheda mentions five kinds of भवानिविश्वरत्वा, conditional antecedents—(1) श्रेण वच्चुःवर्णाभासः, (2) वल्ला वा भार्ष-नादाय (पूर्वभाषः), (3) कारण पूर्वभावे भावे वच्चुःवर्णाभविभागभा, (4) जगच्छ धन्ति पूर्ववर्णिताम् परिवर्तः यथा (पूर्ववर्णितां) न गर्जने, and (5) प्रति यथा प्रति प्रवर्तिताम् परिवर्तः यथा (पूर्ववर्णितां) न गर्जने, and (5) भवानि पद्म निद्मावर्णख्यानुभाविनः परिवर्तः भवेत् (प्रतः पद्म प्रामाण्यवर्णक्षी—Slokas 19, and 20), यथा वाच्यवर्ण भवानिविश्वरतीको ग स: कितु लक्ष्यवाध्यायेव भवानिविश्वरतीको भवेत्ते नहः, Siddhánta Muktávali, loc. cit.—The Dinakari points out that the first two cases are comprehended under the formula प्रतिनिधित्वपरिवर्त: प्रथमान्विश्वरतीकाशिल्भ। There are several classifications of these irrelevant antecedents (भवानिविश्वरत्वा; I quote one of the best known).

The unconditional (भवानिविश्वरत्वाण्य) as interpreted in this comprehensive sense is a far more fruitful conception than Mill’s, and is well adapted to its work—the elimination of the irrelevant factors in the situation. In the end, the discrimination of what is necessary to complete the sum of causes, from what is de-
pended, collateral, secondary, superfluous or inert, 
(i.e. of the relevant from the irrelevant factors) 
must depend on the test of expenditure of energy. 
This test the Nyáya would accept only in the sense 
of an operation analysable into molar or molecular 
( परिख्यान एवं भोलिके व्यापारः करोय्यां—اجटालियालु 
व्यापारी वालि, Jayanta, Manjari, A’hnika I); but would 
emphatically reject, if it is advanced in support of 
the notion of a mysterious causal power or efficiency 
( तत्तति). This is a peculiarity of the Nyáya—it 
insisting that the effect is only the sum or resultant 
of the operations of the different causal conditions—
that these operations are kinetic, being of the 
nature of motion, in other words holding firmly to 
the view that causation is a case of expenditure of 
energy, in the kinetic form,—but at the same time 
absolutely repudiating the Sánkhya conception 
of power or productive efficiency as metaphysical 
or transcendental ( पविप्रिव ), and finding nothing in 
the cause other than an unconditional invariable 
complement of operative conditions ( कारधारवो ) 
and nothing in the effect other than the consequent 
phenomenon which results from the joint operations 
of the antecedent conditions ( श्रविज्ञकवारकरणकर 
परिख्यान एव—Jayanta—बामशो बार्ते—कालिकार्यते तु कार्य- 
वन्मा जिनमें रायसिन्हा। बक्खा—Jayanta, ibid, A’hnika, 1, 
वलिनिरकरणम्—it may be noted that the Nyáya, 
while repudiating transcendental power (Sakti) in 
the mechanism of nature and natural causation,
does not deny the existence of metaphysical conditions like merit (सूत्र,) which constitute a system of moral ends that fulfil themselves in and through the mechanical system and order of Nature—vide Jayanta, अतोन्निवर्ण वाचनिक कारण अधितानक वाचनिक—A'hnika i, विद्यमानादयम्).

The causal relation, then, like the relation of genus to species, is a natural relation of concomitance (वाचन: सामायिक: सूत्र:—Váchaspati) which can be ascertained only by the uniform and uninterrupted experience of agreement in presence and agreement in absence, and not by deduction from a certain a priori principle like that of Causality or Identity of Essence.

Nyáya objection to the Buddhist Method of Difference as a means of ascertaining causality.

Take for example the Buddhist deduction of Causality in any particular conjunction by means of the negative Method of Difference, or of the Panchakárani. The ascertainment of the causal relation by these Methods is open to the following objections:—(1) The unconditionality of the antecedent cannot possibly be ascertained. As the Chárváka rightly points out, the Methods enable you to eliminate irrelevant antecedents that are or can be perceived; but the introduction or sublation of latent or undetected antecedents can be imagined against which the Method of Difference is powerless. In the case of the production of smoke, for
example, by fire,—what if I say that an invisible demon intervenes in every case between the fire and the smoke, that this demon (विद्वा) is the immediate antecedent and real cause of the latter, and that the fire is an accident which, in every such case, is brought about by its own causal antecedents;—in saying this, I do not go counter to the principle of causality and am landed in no contradiction (आपात) such as strikes at the very roots of all practice, or baffles the completed circuit of consciousness, however much I may violate probability.

(2) In the second place, even supposing that the fire, in this particular case (which satisfies the Method of Difference rigidly) is ascertained to be the cause of the smoke, how can I know that fire is the cause in other cases, or that there is no other cause. You will perhaps argue that if there were an indefinite number of causes of the same specific phenomenal effect, it would violate the principle that phenomena are all conditioned, i.e., exist only under certain conditions (सायणजन),—which is more comprehensive than the principle of causality, and the contradiction of which equally overthrows all rational practical activity. Yes, I accept the conditionality of phenomena, but this is not violated by supposing that one specific assemblage of phenomena has more than one cause. It is true that if you suppose such plurality of causes, you cannot establish the invariableness of
the particular conjunction (green-wood fire and smoke) which your Method of Difference fixes upon as a case of cause and effect; in other words, with your special principle of Causality so restricted, and without any general principle of Uniformity of Nature to fall back upon, you cannot ascertain, from the present case, or from any number of similar cases that you may have observed, that all green-wood fires are followed by smoke; i.e., you are helpless in demonstrating with apodictic certainty (or ascertaining indubitably) the relation of cause and effect. But this is an objection against your own position, not mine. Why not admit at once that certain phenomena are naturally connected (as invariable concomitants or antecedents) with other phenomena, and take your stand on observed concomitance (uniform and uninterrupted experience of agreement in presence as well as absence) without assuming causality as an a priori principle and making deductions therefrom, and without the trouble of ascertaining the relation of cause and effect in every individual case. I am free to admit that theoretical objections of irresistible force (like those of the Chárváka Sceptics) can be urged against this ascertainment of universal invariable and unconditional concomitance (आत्मीयिकवाद) on the basis of mere observation. Doubts of this kind can no more be laid by my view of the matter than by your canons of causality and essential
identity (तदन्वर्तन and ताधामा). Ultimately we all have to fall back on the rational practice of thinking persons (वेषा: ), and such persons are always content to act on practical certitude instead of hankering after an unattainable apodictic certainty in the affairs of life (प्रामाणिकवेषा: वाक्यम् न ग्रहनीयम् ). This same practical certitude is also the ultimate warrant of the Deductive-Inductive Inference by which we ascertain the characters of things without direct perception and through the medium or instrumentality of a mark.

[To the earlier Buddhist canon of the Method of Subtraction, i.e., the negative Method of Difference, Udyotakara and Vachaspati of the Nyāya School pertinently and acutely object as follows:—

चबं यत्विशिष्ट राजित सत्यं तत्र वयं प्रतिभः, ताहित: तस्मि सत्

गनके तव धिति अनुभागितम्। क एव तु प्रतिभेनं ग तावत् तद्वपत्तम्।

यथा तत्। का पुनर्वियते तद्वपत्तम्। धूम्यं किं जगान्तरं भावः। क

ताहित: अंजं राजसवापि। तत्तप्रियिहीर्विध धूमं स्थित। यथा तद्वपत्तम्

वमिन। न च राजसवापि भद्रं शमिवी तद्वपत्तम् भूवति, तथाविन विनि

विनि पञ्चवं चरं संबंधे तद्वपत्तम्। अवति पञ्चवं तञ्चक्कां सम्बंधबंध

विनि तद्वपत्तम्। कव वयं भूवं विनिविनिविनिविनिविनिविनिविनिविनि

तथाविन इशारावाद्विवी तद्वपत्तम् ज्ञातान।। तथा द्वि भूमो भूमो

राजसे इत्य धूमदद्वि:।। तद्वपत्तम् च ज्ञात:।। न च तद्वपत्तम्।। तद्वपत्तम्

वेष्य पुनरं दासुः तथा तद्वपत्तम् धिति विनि तद्वपत्तम्।। एवं भविति पञ्चवं विनिविनि

ज्ञाता धूमं, जिथिनेव तद्वपत्तम् एव राजसवापि विनिविनिविनिविनि

विनिविनि विनि विनिविनि, इति अवानं वर्णि साधू मन्यामां विनिविनिविनि।। न च द्वि
Buddhist reply:—आजानि। यी दी धुनीर्द, ह सर्बं लक्ष्मणे
आग्रहसाधित वक्रान्तकलित, न विज्ञानकर्तर। य एव वाकातिकृत्या
निमित्तपेषनाथी ददनकर्तिन सम्बन्धित तद्विद्य भवतीत्वमिचार निमित्तेन
करित, न तु प्रत्येकत्वमिचार रासमाति। नाधी सर्वव्यापकबन्धनां
वाहितः। यथा न तरिकानिन जन्मात् विमानविन सत्वं ज्ञातृ घूली न तप
नभये। यथा एव निमित्त (विषाखः) सर्वोत्तम विनिरूपित: तथा रति ज्ञातन
वन्तं जरं विशिष्टमि न जड़ं। वाकात्यं अेकां समानेन घूमभाव अष्टात।
Now the phenomenon that is contingent (कायाविकृत) cannot be uncaused—पक्षारभ स कायेकं निर्विकाि कािं
अवकं सा स्वात्: अमरपृच्छात् न कादाविनिर्विलम्। And with the
limitation of a specific effect, there must be only
one specific cause, for an unrestricted plurality of
causes would amount to the denial of uniformity in
causality (i.e., of the unconditional invariable
antecedence)—

नाधी निमित्त कार्यं, वाकातिकृत्या वाहित:। वक्रान्तकलितम् साधित नाधी
गुणसंविचारविवर्तम्। यथा बृहु सत्त्वमा सत्त्वमार, न एव कार्यार्थ: स्वात्,
विद्वतं न वल्ल: कायेनू। एवम् ज्ञातिकृत न कार्येः। न इत्य ज्ञातिकृत: अन्तितः,
विशिष्टमि सत्त्वा सामान्तं, सत्त्व स्थितं घूमं: स्वात्। तथा एव
कादाविनिर्विलमाविहितः।

Nyāya rejoinder:—हवम्। सत्त्वमि विना भर्जः नीपवक्त्री
नूरः, सत्त्वमि विनाशायास्तेयनवहिते तेषामुपनववः, सत्त्वार्थिपि विशाख—
The Nyāya proceeds to point out that what is contingent need not necessarily be produced by a cause. It is enough if there is natural connection with something else, a relation of antecedence and succession (or concomitance), without any element of productive efficiency or causation.

Vāchaspati, Tātparya-ya-Tīkā, Chap. I, A’hnika 1, Sutra 5, वाचस्पति, तत्पर्यया-टिका, Chap. I, A’hnika 1, Sutra 5, वाचस्पति, तत्पर्ययात्मक, अर्थात् ॥]
ferent varieties of irrelevant antecedents (अन्तर्मूच्छु),
we have already noticed that co-effects of the same
cause are apt to be confounded as cause and effect.
In some cases, the co-effects may be simultaneous,
s. g., the case of the ascending and the descending
scale in a balance, which are co-effects of gravity
(vide Pártha-sáráthí Misra on Kumárila, Sloka-Várti-
tika—नकासमोनोभसव्योऽस न्यून: न्यायचक्रवर्तस्य, भावीरिका
क्षमान—sloka 157, Súnya-Váda). In other cases,
the co-effects may be successive effects of the same
cause, and here the risk is great of mistaking the
antecedent co-effect to be the cause of the suc-
cceeding co-effect; s. g., the case of ants moving in a
line to carry their eggs upward, which is observed
before the summer rains;—where the movement of
ants and the rains are not cause and effect, but
successive effects of the same cause, viz., the heat
(थङ्ङा), which disturbs the elements, viz., the earth
and the atmosphere (महाभौमचोऽ); the ants being
affected by this heat earlier than the atmospheric
movements which bring the clouds and the
rain.

(Udyotakara, and Váchaspati—नच्च विवेचिकाग्न
मन्नरेः वर्षस्य कारणकुपकालनुग्रान्तः; अन्तर्यापि तत्तत
वर्षीतपसः। वर्षेनान्तकारणस्य स महाभौमविभाग्यकाळस्य
पूर्वकार्यम्।) इहमाना: खश्च विवेचिका भौतिकीयश: लानि खशानि
भौतिकि भार्तिन्द्रानि—Tátparyya—Tiká II, 2, Sutra
37).
Synchronousness of Cause and Effect.—This is resolved into a case of simultaneous co-effects of the same ultimate cause, e.g., the ascent of one scale and the descent of the other in the balance, which are not related as cause and effect, but are simultaneous effects of gravity. In other cases, the synchronousness is only apparent, the interval between the antecedent and the consequent being too small (सृजनाल) to be apprehended (वैद्यवाल्लभ) e.g., in the case of the needle piercing a hundred soft lotus petals laid one upon another, where the steps are really successive; or the illumination of the whole room by the light of a lamp, where the succession is unperceived owing to the inconceivable velocity of light (cf. Kumārilā, Sloka-Vartika,—हेतुहेतुसमव विभ योगपद-निदर्शनम्,—दर प्रदीपप्रभारतत, सृजनाल्लोकित सत्त नः। हित्वला यथा वेषः प्रदीपप्रभातत वथः। Sunyavāda, Sloka 156-157.—I quote Mimansa authorities, but the view is common to the Mimansa and the Nyaya-Vaiseshika).

The time-relation in a chain of causes and effects:—A careful study of the time-relation in a chain of causes and effects is a peculiarity of the Vaiseshika system (and the later Nyaya). A moment (ultimate unit of time, Kshana, अक्ष) is defined to be the time-interval between the completion of the sum of conditions and the appearance of the effect. The Vaiseshika conceives the unit to be determined by reference to the division
of one atom from another (सिद्धान्तवाच्यंविचारं विद्यते; कारणः कारणः—Sapta Padârthâ, Sivarâditya;—i.e., the ultimate unit of time is the time during which motion exists in an atom prior to its division from another atom, in a case of division due to motion). The Sâńkhya, we have already seen, determines this ultimate unit by reference to the motion of a Tanmâtra.

The number of such units will determine the time-interval between a given set of physical conditions and a particular effect, for between a so-called sum of causes and a so-called sum of effects, there intervenes a series of atomic (or molecular) motions, with conjunctions and disjunctions which form the causal chain. However crude in the practical application, the fundamental idea is, in connection with the principle of work and energy (for which both the Sâńkhya and the Nyâya-Vaiseshika furnish a rudimentary basis), immensely suggestive of a possible Time Calculus.

Plurality of Causes:—This will be discussed, when we consider the relation of Vyâpti to the principle of Causality.

The Nyâya Ground of Inference—Vyâpti (नियोजनाः—Vâchaspâti):—Inference, then, in the Nyâya, depends on the ascertainment, not of the causal relation, nor of the relation of genus to species, but of a natural relation, between two phenomena, of invariable and unconditional concomitance
(तौतिकिपुर: भास्कर: शंकर: आरि:—Udyotakara and Váchaspati). Of the two phenomena so connected, one is called the Vyápya or Gamaka (the sign, mark or indicator), and the other Vyápaka or Gamya (the thing signified, marked, or indicated). In the relation of fire and smoke, for example, smoke is the Vyápya or Gamaka (sign or mark); and fire, the Vyápaka or Gamya (the thing signified or marked). Now the relation of Vyápti between $A$ and $B$ may be either unequal or equipollent (विषयवादित्व or समवादित्व). When $A$ is the sign of $B$, but $B$ is not the sign of $A$, the Vyápti is one-sided or unequal, and here a Vyapti is said to exist between $A$ and $B$, but not between $B$ and $A$. For example, smoke is a sign of fire, but fire is not universally a sign of smoke. When, therefore, the relation of Vyápti is an unequal one, as between smoke and fire, it is expressed in the proposition:—Wherever the Vyápya (sign or mark, e.g., smoke) exists, the Vyápaka (the thing signified or marked, e.g., fire) also exists. From this it follows by necessary implication (a sort of प्रशीधांत) that whenever the Vyápaka (e.g., fire) is absent, the Vyápya (e.g., smoke) is also absent (व्याप्तिमात्र व्याप्तिमात्र). Again, the Vyápti may be a mutual or equipollent one, i.e., $A$ and $B$ may be signs of each other, e.g., green-wood fire and smoke. Here each in turn is Vyápya and Vyápaka, and this is expressed in.
the two propositions: — (1) Wherever there is smoke, there is green-wood fire, and (2) wherever there is green-wood fire, there is smoke. By necessary implication it follows—(1) where there is no green-wood fire, there is no smoke; (2) where there is no smoke, there is no green-wood fire. We have seen that a Vyāpti exists between smoke and fire, for wherever there is smoke, there is fire. But we cannot say that a Vyāpti exists between fire and smoke, for we cannot say that wherever there is a fire, there is smoke. The combustion of an iron-ball (अग्निगृह), for example, is a case of fire without smoke. But it would be correct to say that a Vyāpti exists between green-wood fire and smoke, as well as between smoke and green wood fire. The question, therefore, is—What is the relation between fire and smoke? The relation between fire and smoke is a conditional relation; i.e., on condition that the fire is green-wood fire, it would be a sign of smoke. In other words, a Vyāpti implies unconditional invariable concomitance, and the relation between fire and smoke is not therefore a Vyapti (natural unconditional concomitance), for fire requires a ‘condition,’ Upādhi, viz., green-wood, to be followed by smoke. Smoke, on the other hand, requires no ‘condition’ to indicate fire. For the purposes of Inference, therefore, relations between phenomena may
be considered as of two kinds:—(1) Contingent conditional relations, holding good on the fulfilment of a certain condition or Upádhi, and (2) Vyápti, or unconditional invariable relation, between a mark and that which it marks, a relation without any Upádhi or determining condition (धातविषये रसनः). It is this latter kind of relation that serves as the ground of Inference. If we can ascertain that a Vyápti exists between $A$ and $B$, then $A$ is a sign of $B$, and an inference of the presence of $B$ from the presence of $A$, and of the absence of $A$ from the absence of $B$, would be warranted. The question, therefore, is—how to ascertain the relation of Vyápti between two phenomena.

Ascertainment of Vyápti according to the early Nyáya (न्यायविशेष).—Briefly speaking, the observation of agreement in presence (सम्भव) as well as agreement in absence (अस्तितरिक), between two phenomena, with the non-observance of the contrary (अस्मितास्मादम्), is the foundation of our knowledge of Vyápti (इदानिम् ने—Váchaspati). This suggests a natural relation (सांविकालिकम्—Váchaspati) of invariable concomitance (विस्तुलीकरणम्—Váchaspati) between the phenomena, which is fortified by our non-observance (परमाण) of the contrary (अभिषार). But this does not establish the unconditionality of the concomitance (धातविषये रसनः), which is essential to a Vyápti. We have therefore to examine the cases carefully.
to see if there is any determining condition (Upádhi—i.e. some hidden or undetected but really operative or indispensable accompaniment) which conditions the relation between the supposed sign or mark (Gamaka) and the supposed signate (thing signified, Gamya). Now let us consider what constitutes an Upádhi. It is a circumstance which always accompanies, and is always accompanied by, the supposed signate (the thing signified, Gamya), but does not invariably accompany the supposed sign or mark (Gamaka). If, therefore, in the set of positive instances where both the sign and the signate are present, nothing else is constantly present, there can be no Upádhi. Or, again, if in the set of negative instances where both the sign and the signate are absent, no other material circumstance is constantly absent, there is no Upádhi. This follows from the very definition of an Upádhi. It is impracticable to fulfil these requirements rigorously. Still, every one of the accompanying circumstances (of course the likely ones) may be taken successively, and it may be shown that the concomitance continues even when the suspected Upádhi (शब्दनेरापावि) is absent, and therefore it cannot be the Upádhi. And this is to be fortified by the observance of uniform and uninterrupted agreement in absence (Vyatireka) between the two concomitant phenomena. In this way, when we have disproved all
suspected Upādhis, we conclude by establishing the Vyāpti. It is true that we may still go on doubting. But doubt has a certain limit for the ‘experimenter’ and the thinking person (परीक्षाक, प्रेषाणु). When doubt overthrows the foundation of all rational practice (ग्रामाविवलथीकयाब), or leads to a stoppage or arrest of all practical activity (क्षाकवभूत), it stands ipso facto condemned, and must be abandoned (आपाततविधिरास्ता). Sṛharsha and Udayana—आगाण is mentioned by Vāchaspati). Thus it is that Vyāpti is ascertained. In this way, we observe innumerable instances of Vyāpti. Now by means of repeated observations of this kind (सूत्रविच्छन्न), we have established the principle of the Uniformity of Nature (सत्वप्रतिवेद), and also of Causality; and these two principles thus ascertained may be made use of in their turn as the basis of an argumentation or deduction (Tarka, U'ha, तर्क) to confirm a particular Vyāpti in a particular case. Tarka or U'ha, then, is the verification and vindication of particular inductions by the application of the general principles of Uniformity of Nature and of Causality, principles which are themselves based on repeated observation (सूत्रविच्छन्न) and the ascertainment of innumerable particular inductions of uniformity or causality (सूत्रविच्छन्न जननितर्कारकत्वम् रक्षितवेद सामाविज्ञानशास्त्री—Vāchaspati). Thus Tarka also helps in dispelling doubt (विद्यां). Sṛharsha,
however, questions the validity of this verification, —cf. the well-known couplet ending तत्त्व: ।

It will be seen that the process of disproving all suspected Upádhis (उपाधिगत्याविवाह), in the early Nyáya, answers exactly as a process to Mill’s Method of Agreement. In fact, the disproof of a suspected Upádhi by pointing to instances of Agreement in presence (चन्द्र) even in the absence of the Upádhi, fortified as this is by the instances of agreement in absence (वतितिः), virtually amounts to Mill’s Joint Method of Agreement. But the fundamental difference is this:—Mill’s Method of Agreement is formulated in view of the phenomena of causation (including co-effects, etc.) and, as usually enunciated, confessedly breaks down in dealing with cases of Uniformities of Co-existence unconnected with Causation; the Nyáya Method based on the disproof of suspected Upádhis is a more daring and original attempt, and is far more comprehensive in scope, being applicable to all Uniformities of Co-existence and of Causation alike. And this the Nyáya successfully accomplished, by introducing the mark of unconditionality (वपाधिविभारता) into the relation of Vyápti (Concomitance), even as the same mark of unconditionality (वप्पाधितिशृंखला) had been previously introduced into the definition of Causality (साययंकारस्या). The difference between the early Nyáya and the Buddhist systems
may be briefly put thus:—The former relied on empirical induction based on uniform and uninterrupted agreement in Nature, and accordingly regarded the Method of Agreement as the fundamental Method of Scientific Induction, founding Inference on Vyápti to which they subordinated Causality in the doctrine of Method; the latter assumed two a priori principles, viz., causality and identity of essence, deduced the canon of the Method of Difference by an indirect proof from the principle of causality, and made this Method the foundation of all scientific Induction of Causality, just as they based all natural classification of Genera and Species on their a priori principle of Identity of Essence.

[Texts from the early Nyáya:—
Method of Agreement and the Joint Method without the device of the Upádhi:—

Cf. Jayanta on सङ्क्रमण—तत्त्वालं क्षेत्र अवसं न विना
मध्यं तत्। चयनकोविविधामां निम्न: क्रष्णारिता। 'तत्त्वालं क्षेत्र'
'only this remaining throughout', while others change—implying the Method of Agreement. The set of positive instances, in which this antecedent alone is constant, must be supplemented by a set of negative instances (agreement in absence):—अतरंिक्षिणवसलकृष्ण
परिवर्तनवर्गातुपने। निम्नवर्ग अधिते वर्त तत्त्वाल। वर्ष तत्त्वाल। वर्ष वर्ष इति।
विना न अवसं इति नीतिः कृष्णारिते। तत्त तत्त्वाल। वर्ष वर्ष इति।
 Doctrine of the Upádhi:—Unconditional concomitance distinguished from conditional:—

The Buddhist objects—In Nature, everything is connected with everything else. Hence if there were no nexus of causality between antecedent and consequent, everything might follow from everything else. The Nyāya replies—You admit uniformities (of co-existence, etc.) other than causal;—so you confess that a natural fixed order can exist without the causal nexus......

An Upádhi, how established and how disproved:—
The Nyāya then proceeds to show how an Upádhi
is established, or how disproved by observation:—

General Method of Induction by exhaustion of the Upádhi, more comprehensive than Mill’s Joint Method:—

For the definition of an Upádhi, vide Śrīharsha and Udayana: abhāmārtvāt vac: mahāmahārtvāt: vacane vā varṇārthāt. Hence to avoid an Upádhi (which is mahāmahārtvāt), the constant presence of anything relevant other than the sign and the signate in the positive instances (of agreement in presence, abhā), and the constant absence of any such thing in the negative instances (of agreement, in absence, mahā) must be safe-guarded against. This amounts to Mill’s Joint Method.

Suspicion of non-perceptible Upádhi—Limits of legitimate Doubt:—

न च बहुअतारोऽर्थि स्मार्तमार्कस्मा भवकववासमाशाखानवित
संविनाशाय प्रथाय: सामाविधान प्रतिप्रभारित प्रति सानिस्य । अर्थात
-सम्य अविद्या सिद्धाय बिजवेत्ताभासार्थनि हन्तवाक्षाय खलु श्रवण
विदायलोकादिनिनिन मका विनिषो भवम्यया न कष्टिन्त नालोति नेयं
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Tarka, U'ha,—Deductive verification of particular inductions by applying the two fundamental inductions of Uniformity of Nature, and Causality:—Doubt finally dispelled:—

Now innumerable particular uniformities of this kind (Vyáptis) are observed, and as a result of this repeated observation, a belief in the Uniformity of Nature (कामात् प्रताधि), as well as in the principle of Causality (कार्यकारकमाह) is generated in the mind, a belief which has evidential value and validity. It is not intuition (न साधनम्) but a mental pre-disposition based on uniform and uninterrupted experience (भूषितवक्ष्यात्तिकाक्षरधितम् पञ्जीकेय समाविश्ववाच्चालिक, —वत्र भूषीतवक्ष्यात्तिकाक्षरधितम् पञ्जीकेयसात्तिकातम्). Then armed with these new resources, the belief in uniformity and in causality as general principles, we proceed to fortify our particular inductions (Vyáptis), whether of uniformity of nature or of causality, by indirect deduction from these general principles:—We argue if, under these observed circumstances, A were not the mark of B, the principle of uniformity of nature would be violated,—Nature would not be uni-
is established, or how disproved by observation:—

General Method of Induction by exhaustion of the Upâdhi, more comprehensive than Mill's Joint Method:—

For the definition of an Upâdhi, vide Śrīharsha and Udayana: अभासाधनी व: माध्यमवार्ताम्: सच्चते व धौरमित्रित्।

Hence to avoid an Upâdhi (which is माध्यमवार्ता:), the constant presence of anything relevant other than the sign and the signate in the positive instances (of agreement in presence, भवन:), and the constant absence of any such thing in the negative instances (of agreement in absence, असरीक) must be safe-guarded against. This amounts to Mill's Joint Method.

Suspicion of non-perceptible Upâdhi—Limits of legitimate Doubt:—

ग च वहादाःशोधिति वर्णवाचर्कः वचनकाशास्त्राशास्त्राविश्व

संविशेष तत्त्वादि: सामाधिनालि: प्रतिप्रतापि: द्रव्यः दैत्यासमु:।

अधिकः भवन: अभिवर्ग्याः विदाशापाशादिति वर्णवाचः खलु इत्यस्

वचनकाशास्त्राविश्वेण द्रव्यः विदाशोऽवस्यां दैत्याक्ष: च जन्मित नानन्ति नेष
Tarka, U'ha,—Deductive verification of particular inductions by applying the two fundamental inductions of Uniformity of Nature, and Causality:—Doubt finally dispelled:—

Now innumerable particular uniformities of this kind (Vyáptis) are observed, and as a result of this repeated observation, a belief in the Uniformity of Nature (समाव प्रमाणस), as well as in the principle of Causality (कार्यसंस्कृतम) is generated in the mind, a belief which has evidential value and validity. It is not intuition (व साधनम्) but a mental pre-disposition based on uniform and uninterrupted experience (स्वयम्भूप्रमाणितसंस्कृतस्वयमनिष्ठान्वितम् प्रदन्तविषयः सामाविकतविप्रार्थिक, एवम स्वयम्भूप्रमाणस वसाधारणस प्रति प्रमाणादिः). Then armed with these new resources, the belief in uniformity and in causality as general principles, we proceed to fortify our particular inductions (Vyáptis), whether of uniformity of nature or of causality, by indirect deduction from these general principles:—We argue if, under these observed circumstances, A were not the mark of B, the principle of uniformity of nature would be violated,—Nature would not be uni-
form,—or, if under these observed circumstances $A$ were not the cause of $B$, the principle of causality would be violated, the phenomenon $B$ would be without a cause;—and such indirect proof (तक्ष्य, जाह) gives us the overwhelming probability which we call practical certitude, and on which every reasonable man (every thinking and judging person) proceeds to act in due natural course.—स्मार्तः प्रतिपादः स्थितः स्मार्तः, यदि स्मार्तस्ते भवेदुः स्मार्तेदिं प्रमाणेन्तरिति स्वसंबंधायः। वाचस्पाति I, I, Sutra 5; cf. also, I, I, Sutra 40—अतः कार्यास्पदः कार्यायकं चक्षुत्वम्। कार्यास्पदः कार्यायकं प्रक्षुतात।]

Instances of Vyápti (uniformity) not comprehended under Causality, or the relation of Genus and Species:—The Nyáya points out that the relations of cause and effect and of genus and species do not exhaust the grounds of Inference. There are cases of Inference based on Vyápti (i.e., on invariable and unconditional concomitance) which come neither under Causality (सहभिर्भिर्भिः) nor under Identity of Essence (तास्तात्क्ष). Váchspati notes that to-day’s sunrise and yesterday’s sunrise, the rise of the moon and the tide in the ocean, the relative positions of the stellar constellations, are instances of Vyápti (invariable concomitance) between phenomena which are neither related as cause and effect, nor as genus and species. Jayantá adds the conjunction of sunset with the appearance of the stars,
of ants moving in procession (with their eggs) with the approach of the rains,—of the rising of the constellation Agastya (Canopus) above the horizon with the drying up of rivers; of the spring tide with the full moon; and dismisses as sophistical and far-fetched the Buddhist attempt to explain all these cases by means of causality. We have seen that the Nyāya and the Mimāṃsā reduce most of these conjunctions to cases of co-effects of the same cause, co-effects which may be either simultaneous or successive.

Vyāpti between Cause and Effect:—Relation of causality to Vyāpti:

On the Buddhist (and early Nyāya) view that one specific assemblage of 'effect' phenomena has one specific assemblage of causal conditions, there would be two aggregates, the sum of causal conditions (कारणावस्था), and the sum of effects (कारणेश्वास्था). For example, fire requires green-wood to complete the sum of causal conditions to give rise to smoke with some particular marks (चूमनिविशेष)
between an effect and a single condition (termed a cause) there is a relation of Vyápti. The effect is Vyápya or Gamaka (the sign or mark); the cause (or condition) is Vyápaka or Gamya (the thing signified). In other words the presence of the effect indicates the presence of the causal condition, and the absence of the causal condition will by implication indicate the absence of the effect. Smoke of this particular kind is supposed to be an effect of which there is one and only one assemblage of causal conditions (fire and green-wood); hence where there is smoke, there is fire; and when there is no fire, there is no smoke.

Now introduce the complication of the plurality of causes:—Fire, for example, is the effect of several assemblages, e.g., (1) blowing on heated grass, (2) focussing rays through a lens on a combustible like paper or straw, (3) friction with the fire drill, etc. Here each assemblage is regarded as a sum of causes. But in this case there is no Vyápti between the effect ‘fire’ and any particular assemblage of causal conditions, say, of the lens or the fire drill. For the presence of fire does not indicate the presence of the lens or the fire drill assemblage, nor does the absence of either of the latter in particular, indicate the absence of fire.
Indeed in such a case, the effect 'fire' is not a mark or sign (Gamaka or Výápya) of any one in particular of the different possible causal assemblages, though each of these particular assemblages of causal conditions is a mark or sign (Gamaka or Výápya) of fire.

The plurality of causes requires a further consideration in the light of the definition of the causal relation. A cause is defined to be the unconditional invariable antecedent. From the unconditionality, it follows that the entire sum of conditions, and not one single condition, is, properly speaking, the cause. In view of the plurality of causes, an invariable antecedent must be taken to mean that any particular cause (i.e., assemblage of causal conditions) is invariably followed by the effect,—not that the effect is invariably preceded by any particular cause.

Popularly, a single condition, say the lens or the fire-drill, is said to be a cause of fire; but, in view of the plurality of causes, this is apt to be misleading, as there is no Vyápti in this case; the lens or the fire-drill is no more a mark of fire than fire is a mark of the lens or the fire-drill.

The plurality of causes strains the definition of a cause, and undermines the relation of Vyápti between an effect and a cause. Any particular cause (causal aggregate) still indicates the effect, but not *vice versa*. The earlier Nyáya (down to
Vāchaspāti and Jayanta) obviated the plurality, as we have seen, by introducing distinctive marks in the effect such as would indicate a single specific cause (कार्यविनवियः or कार्यवेद, indicating कार्यविनवियः or कार्यवेद). Some indeed went further and held that when the antecedent causal assemblages differ in kind, the effect phenomena, though apparently the same, do really differ specifically (or in kind)—कार्यवेदांतः कार्यवेदायम्। But the Nyāya discards this hypothesis; the fire is the same, though the possible causes (or causal aggregates) differ, e.g., the lens, the drill, etc. But the effect phenomenon to which we attend is not the only effect;—in the case of plurality of causes, we must carefully examine the accompaniments of the effect, i.e., the sum of effects, and the examination will show some distinctive or specific circumstance or accompaniment which will enable us to definitely determine the particular assemblage of causal conditions that must have preceded in the case under examination. This is the device of the earlier Nyāya as well as of the Buddhists, as we have seen; but the later Nyāya doubts the practicability as well as the theoretical validity of such a step on an unrestricted assumption of the plurality of causes, and feels troubled by the circumstance that no effect for which more than one cause (or causal aggregate) can be assigned, can be regarded as a mark or sign (Gamaka or Vyāpya) of any one of
the causes in particular. Accordingly, some adherents of the later Nyāya advanced the proposition that when more than one causal aggregate can be supposed for any effect, the latter is a mark or sign (Gamaka or Vyāpya), not of any one of the causal aggregates in particular, but of one or other of them; and the absence, not of one such cause, but of each and every one of them, alone indicates the absence of the effect. A cause therefore should be defined to mean one or other of the possible alternative aggregates which, being given, the effect follows invariably and unconditionally. If we ask what is the defining mark (or quiddity) of the cause (कार्यसङ्गीत्वम्), we are told that it is one-or-otherness (वनसलय), and nothing else: others cut the Gordian knot by assuming that the different possible causes of the same effect possess a common power or efficiency (प्रतिद्विगुणत्व), or a common 'form' (शाश्वतत्त्व), which accounts for the production of a common effect. The latter is therefore a sign or mark of this power (शक्ति), or this form (शास्त्रतत्त्व), which is manifested by each of the causal aggregates. This hypothesis (कथन), they hold, is simpler and more plausible than the hypothesis of specific differences latent in the apparently identical effect of a plurality of causes (कार्यसङ्गान्तः कार्यसङ्गान्तः).

(Cf. Dinakari on the Siddhanta Muktavali—विवेक विरि प्रति तथापूर्वकार-संधीगाहिना तथापूर्वकार-संधीगाहिनादेश्य)

The Scientific Methods already noticed, the Joint Method of Difference (the Panchakárani), and the Joint Method of Agreement (Vyáptigraha with Upádhisanká-nírasa and Tarka), are not the only methods of ascertaining causality or concomitance, or establishing a theory (विवेचन); nor are these Methods always practicable. Very often, we reach the explanation of a fact (व्यवस्थित) by means of a Hypothesis (विश्लेषण) properly tested and verified (विषयश्रमित). A legitimate Hypothesis must satisfy the following conditions:—(1) the hypothesis must explain the facts (व्यवस्थित, or व्यवस्थित); (2) the
hypothesis must not be in conflict with any observed facts or established generalisations (हरिद्वितीयि शब्द वाक्यति, न हरिद्वितीयि—Jayanta, Nyáya-Manjari, Ahnika 1); (3) no unobserved agent must be assumed, where it is possible to explain the facts satisfactorily by observed agencies (यदि प्रदूषणस्वरूप शब्दं न एवज्जतं भालमंकं कल्याताम्, परमपाप्तं तद्दृष्टं तिं किं तदद्यक्ष्यनेन, ibid); (4) when two rival hypotheses are in the field, a crucial fact or test (विनिगमणि, ratio sufficiens) is necessary; the absence of such a test (विनिगमणि विरह) is fatal to the establishment of either, (5) of two rival hypotheses, the simpler, i.e., that which assumes less, is to be preferred (celeris paribus) (कल्याताम अर्थं versus कल्याताम अर्थं); (6) of two rival hypotheses, that which is immediate or relevant to the subject-matter is to be preferred to that which is alien or remote (पञ्जीपञ्जितः); (7) a hypothesis that satisfies the above conditions must be capable of verification (विषयं) before it can be established as a theory (विलम्ब). The process of verification of a hypothesis consists in showing that it can be deduced as a corollary from (or is involved by implication in) some more general proposition which is already well-established (cf. Vatsyayana’s exposition and illustration of Verification, विषयं,—including both the Deductive Method and Colligation).

This doctrine of Scientific Method, in Hindu Logic, is only a subsidiary discipline, being com-
prehended under the wider conception of Methodology, which aims at the ascertainmant of Truth whether scientific (Vijnana) or philosophical (Jnana) (सिद्ध रूपं ज्ञात विद्वानं विद्वानां; Amara-Kosha);——the latter being the ulterior aim. In the investigation of any subject, Hindu Methodology adopts the following procedure: (1) the proposition (or enumeration) of the subject-matter (Uddesa), (2) the ascertainment of the essential characters or marks, by Perception, Inference, the Inductive Methods, etc.,—resulting in definitions (by लच्छ) or descriptions (by अवलोकन); and (3) Examination and Verification (विधि and दर्शन). Ordinarily the first step, Uddesa, is held to include not mere Enumeration of topics, but Classification or Division proper (विभाग; चारुक्रमणायामु च द्वितीय एवादि। सामायिकिया कौन्तम सुभूत्व; प्रकारिकिर्या कौरं विभाग विशिष्टं—Jayanta, Manjari); but a few recognise the latter as a separate procedure coming after Definition or Description. Any truth established by this three-fold (or four-fold) procedure is called a Siddhānta (an established theory). Now the various Pramānas Proofs, i.e., sources of valid knowledge) in Hindu Logic, viz., Perception, Inference, Testimony. Mathematical Reasoning (सम्प्रेक्षण, including Probability in one view) are only operations subsidiary to the ascertainment of Truth (तत्त्वविज्ञान). And the Scientific Methods are merely ancillary to these Pramānas themselves.
I have explained the principles of the Hindu doctrine of Scientific Method, avoiding the technicalities of Logic as far as possible; and I cannot here enter upon the logical terminology or the logical apparatus and machinery, which would require a separate volume to themselves. For these, I would refer the reader to my paper on Hindu Logic,—as also for an account of the later Nyāya (न्याय), which, in spite of its arid dialectics, possesses a three-fold significance in the history of thought: (1) logical, in its conceptions of Avachchhedaka and Pratiyogi, being an attempt to introduce quantification on a connotative basis, in other words, to introduce quantitative notions of Universal and Particular, in both an affirmative and a negative aspect, into the Hindu theory of Inference and Proposition regarded connotatively as the establishment of relations among attributes or marks; (2) scientific, in its investigation of the varieties of Vyāpti and Upādhi, (and of अध्यात्मिक) being an elaboration of Scientific Method, in the attempt to eliminate the irrelevant; and (3) ontological and epistemological, in its classification and precise determinations of the various relations of Knowledge and Being, with even greater rigidity and minuteness than in Hegel's Logic of Being and Essence. I will conclude with a few observations on Applied Logic, i.e., the logic of the special sciences, which is such a
characteristic feature of Hindu scientific investigation. What is characteristic of the Hindu scientific mind is that without being content with the general concepts of Science and a general Methodology, it elaborated the fundamental categories and concepts of such of the special sciences as it cultivated with assiduity, and systematically adapted the general principles of Scientific Method to the requirements of the subject-matter in each case. The most signal example of applied logic (or Scientific Method) worked out with systematic carefulness is the Logic of Therapeutics in Charaka, a Logic which adapts the general concepts of cause, effect, energy, operation, etc., and the general methodology of science, to the special problems presented in the study of diseases, their causes, symptoms and remedies (vide Charaka, Vimá nasthána, Chap IV,—also Sutra Stháná,—vide my Paper on Hindu Logic).

Here I will give an illustration of Applied Methodology from the Science of Analytical and Descriptive Grammar. Patanjali, in the Mahábháshya, is very careful as regards Methodology. I take no note now of the philosophical presuppositions of his philology (vide my Paper on the Hindu Science of Language), but will confine myself to his presentation of the Applied Logic of Descriptive and Analytical Grammar. The sentence is the unit of speech, as every Hindu philologist contends, but the first business of Analytical
Grammar is to analyse the sentence into its significant parts and their coherent relations to one another. Assuming that articulate sounds are significant, the question is:—how is the sentence, which is the unit of articulate speech, broken up into significant words and their mutual relations? Patanjali answers that this is done by an instinctive use (cf. Vāchāspati’s वृद्धि व्यविश्राम स्थापना रचनाद्वयम्) of the Joint Method of Difference (combined Addition and Subtraction), fortified by the Joint Method of Agreement (अक्षर आत्मिकता). Patanjali starts with a simple case. Take the two sentence-units:—Pathati (he reads) and Pachati (he cooks). Suppose you start with the assumption that these sounds are significant, and that separate elements have separate meanings. Then you hear Pathati, and, at the same time, a man reading is pointed at. Then the assemblage of sounds Pathati (i.e., Path+ati) stands for the assemblage—‘one reads,’ (i.e. the action reading + an individual agent). Similarly by finger-pointing or other indication, you find that the assemblage Pachati (i.e., Pach+ati) stands for the assemblage ‘one cooks’ (i.e. the action cooking + an individual agent, say, the same agent as before).

Now look at the groupings:—

Path+ati = reading + one agent.

Pach+ati = cooking + one agent.
From repeated observation of similar groupings, one is led to conclude that Path is the invariable concomitant of the action 'reading', because the disappearance of the former (the other element remaining) leads to the disappearance of the latter (a rough Method of Difference by subtraction—होते); that Pach is the invariable concomitant of the action 'cooking' because the introduction of Pach (with nothing else added) leads to the introduction of the action 'cooking,' (a rough Method of Difference by addition,—वज्ञायते); and that Ati, which is the only 'common antecedent' (चन्द्रिति) is the invariable concomitant of 'one agent' which is the only common 'consequent' (कन्धि). In fact, the last should come first by the Method of Agreement, then the other two either by Residue, or by Joint Agreement in presence and absence (चन्द्रमयति). So far all is plain sailing, though only very simple and very rough applications of the Methods are given. But—and this is the point—throughout the argument, it is assumed that one sound is the concomitant of one idea, in other words, there is no plurality of causes to vitiate the application of the Method of Agreement in the above example. And now the objection is advanced that this basal assumption is untrue. The same sound is not the concomitant of the same idea. Different words (sounds) may have the same meaning, and the same word (sound) may have
different meanings. We cannot therefore by the Joint Method of Agreement (समयतदिहित) determine the meanings of words, or the separate functions of roots (stems) and inflections,—at least not so simply as is pretended above. Patanjali states this difficulty in the way of applying the Scientific Methods to the problems presented by Analytical Grammar, a difficulty arising from the plurality of causes, but does not state the solution. The solution, however, depends on the Method of Subtraction and Addition (Patanjali’s ऋषि and वयसाधन). For it will be found by extensive observation that the number of meanings of which a word (sound) may be capable is limited. So also is the number of words (sounds) expressive of a given meaning. Hence, by the Method of Difference, etc., the causal relations of words and meanings may be determined. Patanjali thus establishes the doctrine of Prakriti and Pratyaya (roots and inflections’), with their separate significance, which is of course a necessary postulate in the case of an inflectional language like Sanskrit. Isolating and Agglutinative languages offer less difficulty, whereas the difficulties are in some respects enhanced in the case of languages with a polysynthetic or incorporating (incapsulating) morphological structure.

चर्चा एवंशार्य सर्व प्रज्ञवर्गः पर्यंत प्रवचन वैति। प्रज्ञविषयितमाभासः।
कोदन्ति सभ्यः आतिरेको वा। इह परम्परा अधिक शब्दः यूथे,
Patanjali not only applies the Scientific Methods to the foundations of Grammatical Analysis, but also by their means establishes and elaborates the fundamental categories and concepts of Grammatical (and Philological) Science, e.g., the concepts of action (क्रिया), agent (कर्ता), instrumental cause (कारण वाचकतमस्), end (निष्ठान), origin (प्रमुख), limit (प्रतिफल), substance (चरakter), quality (रूप), and genus (जाति);—also of the fundamental relations (at the bottom of all thought and speech)—the relations of Time (कालक्षणम्), of Space (विस्तरणभाष्यम्), of Causality (कारणकारणम्), of inference (कलापय), of co-inherence (विस्तविष्कारभाष्यम्), of the sign and the signate (संकेरकतिभाष्यम्), of mutual dependence (प्रतिनिधित्वम्);—an entire grammatical (and philological) apparatus, which will serve as a point d'appui for generations of philologists and grammarians to come.