\[ R_A \times 30 = \frac{30 \times 1 \times 30}{2} + 3 \times 10 + 9 \times 20 \]
\[ R_A = 15 + 1 + 6 \]
\[ = 22 \text{ t.} \]
\[ R_B = 30 + 9 + 3 - 22 \]
\[ = 20 \text{ t.} \]

Let maximum dip of 3 m. be at distance \( x \) from \( A \). At this section, tension, in the cable will be horizontal and hence vertical component of tension in cable is zero.

Considering equilibrium of left portion of cable.
Equating sum of vertical forces to zero,
\[ 22 - w \times x - 9 = 0 \]
\[ 22 - 1 \times x - 9 = 0 \]
\[ \therefore \quad x = 13 \text{ m.} \]
Taking moments about \( A \),
\[ H \times 3 - 13 \times 1 \times \frac{13}{2} - 9 \times 10 = 0 \]
\[ \therefore \quad 3H = 84.5 + 90 \]
\[ \therefore \quad H = 28.17 + 30 = 58.17 \text{ t.} \]
Maximum tension in the cable will occur at \( A \).
\[ T_{max} = \sqrt{22^2 + 58.17^2} \]
\[ = 62.13 \text{ t.} \]

Ex. 15.4. A suspension cable is supported at two points 20 m. apart. The left support is 2 m. above the right support. The cable is loaded with uniformly distributed load of 1 t/m throughout the span. The maximum dip in the cable from support level is 4 m. Find the maximum tension in the cable.

Solution. Let \( R_A \) and \( R_B \) be vertical reactions at support \( A \) and \( B \).
\[ H_A = H_B = H \quad \ldots (1) \]
\[ R_A + R_B = 1 \times 20 = 20 \text{ t} \quad \ldots (2) \]

Taking moments about \( B \),

\[ H \]

\[ R_A \]

\[ \text{1 t/m} \]

\[ 4 \text{ m} \]

\[ \text{2 m} \]

\[ R_B \]

\[ \text{1 t/m} \]

\[ \text{4 m} \]

\[ x \]

\[ H \]

\[ \text{Fig 18} \]
\[ H \times 2 - R_A \times 20 + 1 \times 20 \times 10 = 0 \]
\[ \therefore \quad H - 10 R_A + 100 = 0 \quad \cdots (3) \]

Let the lowest point of the cable \( C \) be at distance \( x \) from \( A \).
Consider free body diagram of \( AC \). Taking moments about \( A \),
\[ H \times 4 - 1 \times x \times \frac{x}{2} = 0. \]
\[ \therefore \quad H = \frac{x^2}{8} \]

For portion \( AC \),
\[ \Sigma V = 0 \text{ gives, } R_A - 1 \times x = 0 \]
\[ \therefore \quad R_A = x. \]

Substituting values of \( H \) and \( R_A \) in terms of \( x \) in (1),
\[ \frac{x^2}{8} - 10x + 100 = 0 \]
\[ x^2 - 80x + 800 = 0 \]
\[ (x - 40)^2 = 1600 - 800 = 800 \]
\[ \therefore \quad x = 40 \pm 20 \sqrt{2}. \]

Neglecting +ve sign as \( x \) cannot be greater than 20 m.
\[ x = 40 - 28.28 \]
\[ = 11.72 \text{ m}. \]
\[ R_A = x = 11.72 \text{ t.} \]
\[ H = \frac{x^2}{8} = \frac{11.72^2}{8} \]
\[ = 17.2 \text{ t.} \]

Maximum tension occurs at \( A \),
\[ T_{max} = \sqrt{R_A^2 + H^2} = \sqrt{11.72^2 + 17.2^2} \]
\[ = \sqrt{137.4 + 295.8} \]
\[ = \sqrt{433.2} \]
\[ = 20.81 \text{ t.} \]

15.4. Shape of cable under self weight.

When a cable is suspended from two supports, it takes shape of catenary under self weight. Consider a cable suspended from two supports. Let \( w \) be self weight of cable per unit length.
Let $H$ be the tension in the cable at its lowest point and $T$ be tension in cable at some section $X$. Let $\theta$ be the slope of cable at this section considering equilibrium of right portion of cable.

$$H = T \cos \theta$$

$$wS = T \sin \theta$$

where $S$ is the length of the cable between lowest point and $X$.

From (1)

$$T = \frac{H}{\cos \theta}$$

$$wS = \frac{H}{\cos \theta} \times \sin \theta$$

$$S \int_0^S ds = \int_0^x \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx$$

$$\therefore \quad w \int_0^x \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx = \frac{H \times \sin \theta}{\cos \theta} = H \tan \theta$$

$$\tan \theta = \frac{dy}{dx}.$$  

$$\therefore \quad w \int_0^x \sqrt{1 + \left( \frac{du}{dx} \right)^2} dx = H \frac{dy}{dx}$$

Differentiating both sides

$$w \sqrt{1 + (\frac{dy}{dx})^2} = H \frac{d^2y}{dx^2}$$

Let

$$\frac{dy}{dx} = p$$

$$\sqrt{1 + p^2} = \frac{H}{w} \cdot \frac{dp}{dx}$$

or

$$\sqrt{1 + p^2} = \frac{w}{H} \cdot dx$$

Integrating both sides

$$\log (p + \sqrt{1 + p^2}) = \frac{w}{H} x$$

$$p + \sqrt{1 + p^2} = e^{\frac{H}{w} x}$$

$$\sqrt{1 + p^2} = e^{\frac{H}{w} x} - p$$

or

$$1 + p^2 = (e^{\frac{H}{w} x} - p)^2$$

$$1 + p^2 = e^{\frac{H}{w} x} + p^2 - 2p e^{\frac{H}{w} x}$$

$$2p e^{\frac{H}{w} x} = e^{\frac{H}{w} x} - 1$$
\[ p = \frac{1}{2} \left[ e^{\frac{w}{H}} - e^{-\frac{w}{H}} \right] \]

\[ \frac{dy}{dx} = \frac{1}{2} \left[ e^{\frac{wx}{H}} - e^{\frac{-wx}{H}} \right] \]

\[ dy = \frac{1}{2} \left[ e^{\frac{wx}{H}} - e^{\frac{-wx}{H}} \right] dx \]

Integrating

\[ y = \frac{1}{2} \left[ e^{\frac{wx}{H}} + e^{\frac{-wx}{H}} \right] \frac{H}{w} \]

\[ \frac{H}{v} \int \frac{H}{w} \cosh \frac{wx}{H} \]

This represents equation of catenary.

Length of the cable \( S = \int ds = \int \frac{ds}{dx} \ dx \)

\[ = \int_0^x \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \ dx \]

\[ = \int_0^x \sqrt{1 + \sinh^2 \frac{wx}{H}} \ dx \]

\[ = \int_0^x \cosh \frac{wx}{H} \ dx \]

\[ = \frac{H}{w} \sinh \frac{wx}{H} + c \]

At \( x = 0, S = 0 \)

\[ S = \frac{H}{w} \sinh \frac{wx}{H} \]

15.5. Stresses in suspended wires due to self weight.

Suspended wires have very small dip at the centre. If 'w' is weight of wire per unit length, horizontal tension in the wire is given by,

\[ H = \frac{wl}{8d} \]

\[ \text{Fig. 15.10} \]

Maximum tension will be \( \sqrt{H^2 + \left( \frac{wl}{d} \right)^2} \)
For wire self weight is small. Neglecting term of self weight, maximum tension

$$H = \frac{\omega l^2}{8d}$$

Stress in the wire $$f = \frac{\omega l^2}{8dA}$$, where $$A$$ is cross-sectional area of the wire.

Thus, the stress in the wire varies inversely as the dip.


Due to change of temperature in cables and wires, there is change in the dip of wires and cables which gives rise to stresses.

Consider a cable subjected to change of temperature of $$t^\circ$$C. Change in length of the cable will be $$L \alpha t$$, where $$L$$ is the length of the cable and $$\alpha$$ is co-efficient of linear expansion of the cable.

$$\delta L = L \alpha t$$

Length of cable

$$L = l + \frac{8}{3} \frac{d^2}{l}$$

$$\therefore \delta L = \frac{16}{3} \frac{d}{l} \delta l$$

$$\delta d = \frac{L \alpha t \times 3l}{16d}$$

Putting $$l$$ approximately

$$\delta d = \frac{3}{16} \alpha t \frac{l^2}{d}$$

Due to rise in temperature, dip increases and stress decreases. Due to fall in temperature, dip decreases and stress increases.

Ex. 15.5. A steel wire 1 cm. diameter is stretched between two supports 30 m. apart the central dip being 30 cm. Find the maximum stress in the wire. What fall in temperature will increase the stress to 500 kg/cm², coefficient of linear expansion of steel = $$1.2 \times 10^{-5}^\circ$$C and weight of steel = 7850 kg/m².

Solution. Length of wire

$$= l + \frac{8}{3} \frac{d^2}{l}$$

$$= 30 + \frac{8 \times (0.3)^2}{3 \times 30}$$

$$= 30 + 0.008 = 30.008$$ m.

Weight of cable

$$= 30.008 \times \frac{\pi}{4} \times (0.01)^2 \times 7850$$

$$= 18.49$$ kg.
\[ H = \frac{WL}{8d} = \frac{18.49 \times 30}{8 \times 0.3} \]

: 231.25 kg.

Stress in the wire:
\[ \frac{\pi}{4} \times 1 \]

\[ = 294.3 \text{ kg/cm}^2. \]

Let \( d' \) be the dip of the wire corresponding to stress of 500 kg/cm².

Stress is proportional to \( 1 \)

\[ \frac{294.3}{500} = \frac{1}{1/d'} \]

\[ \therefore \quad 294.3 \times 30 = 500 \ d' \]

\[ \therefore \quad d' = \frac{294.3 \times 30}{500} \]

\[ = 17.658 \text{ cm.} \]

Change in dip required = 30 – 17.658

\[ = 12.342 \text{ cm.} \]

\[ \delta d = \frac{3xtl^2}{1vd} \]

\[ \therefore \quad 12.342 = \frac{3 \times 1.2 \times 10^{-5} \times t \times (3000)^3}{16 \times 30} \]

\[ t = \frac{12.342 \times 16 \times 30}{3 \times 1.2 \times 10^{-5} \times (3000)^2} \]

\[ = 18.28^\circ \text{C.} \]

15.7. Anchorage of suspension cables

Suspension of cables are provided with two types of supports at the ends. Either the cables pass over pulley on the pier and are back stayed or cables pass over saddles and are back stayed.

Cables over pulleys

When the cable passes over a pulley on the pier and is stayed at the back, the pier will be subjected to horizontal force as well as vertical force. The tension in the cable and back stay will be same at the pier.

Let \( \alpha \) be the angle made by the cable with the vertical and \( \beta \) be the angle made by the back stay with the vertical at the pier.

Total vertical force on the pier

\[ = T \cos \alpha + T \cos \beta \]

\[ = T(\cos \alpha + \cos \beta) \]
But \[ T \sin \alpha = H \text{ or } T = \frac{H}{\sin \alpha} , \]

where \( H \) is horizontal tension in the cable.

\[
\text{Fig. 15.11 (a)}
\]

Total vertical force on pier
\[
\frac{H}{\sin \alpha} \left[ \cos \alpha + \cos \beta \right]
\]
\[= H \left( \cot \alpha + \cot \beta \csc \alpha \right) . \]

Net horizontal force on the pier
\[= T \sin \alpha - T \sin \beta \]
\[= T (\sin \alpha - \sin \beta) \]
\[= \frac{H}{\sin \alpha} (\sin \alpha - \sin \beta) \]
\[= H(1 - \sin \beta \csc \alpha) . \]

Cable provided with saddles
When the cable passes over saddles and then it is back stayed, there will be no horizontal force acting on the pier, as saddle allows free movement and thus horizontal components of tension in cable and tension in back stay must be equal.

Let \( T \) be the tension in the cable at the pier and \( T' \) be the tension in the back stay. Let \( \alpha \) be the angle made by the cable with the vertical at the pier and \( \beta \) be the angle made by back stay with the vertical at the pier.

\[ H = T \sin \alpha = T' \sin \beta \]

\[ \therefore \quad T' = T \frac{\sin \alpha}{\sin \beta} = \frac{H}{\sin \beta} \]

Vertical force on the pier
\[= T \cos \alpha + T' \cos \beta \]
\[= \frac{H}{\sin \alpha} \cos \alpha + \frac{H}{\sin \beta} \cos \beta \]
\[= H (\cot \alpha + \cot \beta) \]
Ex. 15:6. A bridge cable slung between two piers 80 m. apart carries a load of 3000 kg/m. of span. The tops of the piers are at the same level and the cable at its lowest point sags 8 m. below this level. Calculate the maximum value of the cable tension.

Find the tension in the back-stay and the pressure on the pier if the cable passes over saddles and back-stay is inclined at 30° to horizontal. If the cable passes over pulley find the horizontal and vertical pressures on the pier, inclination of back-stay is the same.

Solution. Under uniformly distributed load cable takes shape of parabola. Equation is given by,

\[ y = \frac{4dx(l - x)}{l^2} \]

\[ = \frac{4 \times 8x(80 - x)}{80 \times 80} \]

\[ = \frac{x(80 - x)}{200} \]

**Fig. 15:12**

Horizontal thrust in the cable is given by

\[ = \frac{wl^2}{8d} = \frac{3000 \times 80 \times 80}{8 \times 8} = 300,000 \text{ kg.} \]

Vertical component of forces at A in the cable

\[ = \frac{3000 \times 80}{2} = 120,000 \text{ kg.} \]

Tension in the cable = \( \sqrt{H^2 + V^2} \)

\[ = \sqrt{(300,000)^2 + (120,000)^2} \]

\[ = 323,100 \text{ kg.} \]

\[ y = \frac{x(80 - x)}{200} \]

\[ \frac{dy}{dx} = \frac{(80 - 2x)}{200} \]

At

\[ x = 0, \]

\[ \frac{dy}{dx} = \frac{80}{200} = \frac{2}{5} \]

\[ \tan \theta = \frac{2}{5} \]
When cable passes over saddles, horizontal component of tension in cable and back-stay must balance.

\[ T' \cos 30 = T \cos \theta = H \]

\[ T' \times \frac{\sqrt{3}}{2} = 300,000 \]

\[ T' = 300,000 \times \frac{\sqrt{3}}{\sqrt{3}} = 346,420 \text{ kg.} \]

Pressure on pier = \( T' \sin 30 + T \sin \theta \)

\[ = 346,420 \times \frac{1}{2} + V_A \]

\[ = 173,210 + 120,000 \]

\[ = 293,210 \text{ kg.} \]

**Cable passing over pulley**

When cable passes over pulley tension in cable and back-stay will be same.

Horizontal force on pier = \( T \cos \theta - T \cos 30 \)

\[ = 300,000 - 323,100 \times \frac{\sqrt{3}}{2} \]

\[ = 300,000 - 279,804 \]

\[ = 20,196 \text{ kg.} \]

Vertical pressure on pier = \( T \sin \theta + T \sin 30° \)

\[ = V_A + 323,100 \times \frac{1}{2} \]

\[ = 120,000 + 161,550 \]

\[ = 231,550 \text{ kg.} \]

15-8. Stiffened Bridges.

Suspension cable bridges are suitable only for small spans and light loads. In case suspension bridges are to be used for large spans and rolling loads as for roadways, these bridges are stiffened by the use of three hinged stiffening girder or two hinged stiffening girders to make them stiff and thus decrease the sag under the rolling load.

When bridge is stiffened with three hinged stiffening girder, it is assumed that the cable retains its parabolic shape when the
loads travel on the bridge and hence the load on the cable throughout will be uniform. When the bridge is loaded with uniformly distributed load, whole of the load is directly taken by the cables and there will be no effect on the stiffening girders. In case of moving load, the cables will be assumed to carry uniform load, and, therefore, the stiffening girder will be subjected to bending moments and shear forces.

15.9. Bending moment and shear force in three hinged stiffening girders.

Consider suspension bridge with three hinged stiffening girders. Let a load $W$ act at distance $a$ from support $A$.

Taking moments about $A$,

$$R_R = \frac{W \times a}{l}$$

and

$$R_A = \frac{W(l-a)}{l}$$

These reactions are as for a simply supported beam. Cutting the bridge at central hinge and considering equilibrium of rigid portion, moment about central hinge gives,

$$\frac{Wa}{l} \cdot \frac{l}{2} - H(d+c) + H \times c = 0$$

$$H = \frac{Wa}{2d}$$

![Diagram](image)

(1)

![Diagram](image)

(2)

![Diagram](image)

(3)

To find B.M. and S.F. at any section, distance $x$ from cut the cable and the girder as shown in Fig. 15.14 (c).
The B.M. diagram for the girder can be obtained by superposing B.M. diagram as for simply supported beam on B.M. diagram due to $H \times y_a$. Value of $H$ for particular loading is constant and hence $H \times y_a$ diagram will be a parabola. The diagram is obtained by taking parabolic shape of cable with every ordinate multiplied by $H$ as shown in Fig 15-14 (d). S.F. diagram is obtained by superposing S.F. diagram as for simply supported beam on $H \tan \theta$ diagram.

\[
y = \frac{4d}{l^2} x(l-x)
\]

\[
\frac{dy}{dx} = \tan \theta = \frac{4d}{l} (l-2x)
\]

Thus $\tan \theta$ varies linearly with $x$. As $H$ is constant, $H \tan \theta$ diagram will vary linearly from $H \tan \theta_A$ at $A$ to $H \tan \theta_B$ at $B$.

**Ex. 15.7.** The cables of a suspension bridge have a span of 0 m and a central dip of 10 m. Each cable is stiffened by a girder inged at the ends and at midspan to constrain the cable to retain is parabolic shape. There is uniformly distributed load of 000 kg/m of span over the whole of the span and in addition a live load of 3000 kg per horizontal metre and 20 m, long.

Determine the maximum tension in the cable when live load is situated on the left hand half of the stiffened girder with its right end over the central hinge. Sketch the S.F. and B.M. diagrams for the girder. Also find maximum B.M. and S.F.
ANALYSIS OF STRUCTURES

(a)

(b) SIMPLY SUPPORTED S.F. DIAGRAM

(c) - H TAN Q DIAGRAM

(d) S.F. DIAGRAM

(e) SIMPLY SUPPORTED B.M. DIAGRAM

(f) - H X Y DIAGRAM

(g) B.M. DIAGRAM

Fig 15.15
Solution. *Due to uniformly distributed load of 1000 kg/m.*

Horizontal thrust $H = \frac{wl^2}{8d} = \frac{1000 \times 80 \times 80}{80 \times 10} = 80,000$ kg.

Vertical component of tension in cable at support

$= 1000 \times 80$

$= 40,000$ kg.

*Due to live load*

$$R_B = \frac{3000 \times 20 \times 30}{80} = \frac{90,000}{4} = 22,500$$ kg.

$$R_A = 37,500$$ kg.

Cutting the cable at central hinge and considering the equilibrium of right hand portion

$H \times 10 = R_B \times 40$

$$= \frac{90,000}{4} \times 40$$

$\therefore$

$H = 90,000$ kg.

If $w_e$ is equivalent uniformly distributed load transferred to cable,

$$H = \frac{w_e l^2}{8d}$$

$\therefore$

$$90,000 = \frac{w_e \times 80 \times 80}{8 \times 10}$$

$w_e = \frac{9000}{8}$

Vertical component of tension in cable at supports

$$= \frac{w_e l}{2} = \frac{9000}{8} \times 40$$

$= 45,000$ kg.

Total vertical component of tension in cable at supports

$= 40,000 + 45,000$

$= 85,000$ kg.

Total horizontal thrust

$= 80,000 + 90,000 = 170,000$ kg.

Tension in cable

$$= \sqrt{85,000^2 + 170,000^2}$$

$= 86,000 \sqrt{4}$

$= 86,000 \sqrt{5}$

$= 199,000$ kg.
Due to uniformly distributed dead load of 1000 kg/m, there will be no shear force and B.M. anywhere in the stiffening girder.

Due to live load

Shear at any section = \( F_s - H \tan \theta \)

\[
y = \frac{4dx(l-x)}{l} = \frac{4 \times 10x(80-x)}{80 \times 80}
\]

\[
= \frac{x(80-x)}{160}
\]

\[
\frac{dy}{dx} = \frac{(80-2x)}{160}
\]

Considering girder as simply supported beam, S.F. diagram will be as shown in Fig. 15.15 (b).

At \( x = 0 \), \( \frac{dy}{dx} = \frac{80}{160} = \frac{1}{2} \).

\(-H \tan \theta\), at \( x = 0 = -90,000 \times \frac{1}{2} = -45,000\)

At \( x = l, H \tan \theta = +45,000\)

S.F. diagram due to \( H \tan \theta \) is shown in Fig. 15.15 (c).

Total S.F. diagram is obtained by superposing two diagrams as shown in Fig. 15.15 (d).

Maximum +ve S.F. is at \( A = 45,000 - 22,500 = 22,500 \text{ kg.} \)

Maximum -ve S.F. is at \( C = 22,500 \text{ kg.} \)

B.M. diagram

B.M. diagram for the girder as a simply supported beam is shown in Fig. 15.15 (e).

B.M. diagram due to \( H \times y \) is a parabola with vertical ordinate = 90,000 \times 10 = 900,000.

Final B.M. diagram is obtained by superposing these two diagrams.

Maximum -ve B.M. occurs in portion \( CB \).

B.M. at a distance \( x \) from \( B \) is given by

\[
M_x = M_s - H \times y
\]

\[
= 22,500 \times x - 90,000 \times \frac{x(80-x)}{160}
\]

\[
\frac{dM_x}{dx} = 22,500 - \frac{90,000}{160} \times (80-2x) = 0
\]

\[80 - 2x = \frac{22,500 \times 160}{90,000} = 40\]

\[\therefore \quad x = 20 \text{ m.}\]

\[
M_{x_0} = 22,500 \times 20 - 90,000 \times \frac{20 \times 60}{160}
\]

\[
= 450,000 - 675,000
\]

\[
= -225,000 \text{ kg. m.}\]
Maximum +ve B.M. occurs in portion CD.

B.M. at a section distance $x$ from $A$ is given by

$$M_s = M_e - Hy$$

$$M_s = 37,500x - \frac{3000(x-20)^2}{2} - \frac{90,000x(80-x)}{160}$$

$$\frac{dM_s}{dx} = 37,500 - 3000(x-20) - \frac{90,000}{160}(80-2x) = 0$$

$$600 - 48x + 960 - 720 + 18x = 0$$

$$30x = 840$$

$$x = 28 \text{ m}.$$  

$$M_{28} = 37,500 \times 28 - \frac{3600 \times 8 \times 8}{2} - \frac{97,000 \times 28 \times 52}{160}$$

$$= 135,000 \text{ kg.m.}$$

15.10. Influence lines for B.M. and S.F. in three-hinged stiffening girders.

Consider a suspension bridge with three-hinged stiffening girder as shown in Fig. 15.16 (a). Let a load $W$ act at distance $x$ from $A$.

Let $w$ per unit length be the equivalent uniformly distributed load transferred to the cable. The load acting on the stiffening girder are $W$ at distance $x$ from $A$ and uniformly distributed load $w$/unit length acting upwards as shown in Fig. 15.16.

Taking moments about $B$.

$$R_A \times l + wl \times \frac{l}{2} - W(l-x) = 0$$

$$R_A = \frac{W(l-x) - \frac{wl^2}{2}}{l}$$

$$= \frac{W(l-x)}{l} - \frac{wl}{2}$$

$$R_B = W - \frac{wl}{2} - \left[ \frac{W(l-x)}{l} - \frac{wl}{2} \right]$$

$$- \frac{wl}{2} + \frac{Wx}{l}$$
B.M. at central hinge \( C \) is zero.

\[
R_B \times \frac{l}{2} + w \frac{l}{2} \times \frac{l}{4} = 0
\]

\[
\therefore \left[ -\frac{wl}{2} + \frac{Wx}{l} \right] \times \frac{l}{2} + \frac{wl^3}{8} = 0
\]

\[
-\frac{wl^3}{4} + \frac{Wx}{2} + \frac{wl^3}{8} = 0
\]

\[
\therefore \quad \frac{Wx}{2} = \frac{wl^3}{8}
\]

\[
\therefore \quad w = \frac{4Wx}{l^3}
\]

When unit load travels the span, \( W = 1 \).

\[
\therefore \quad w = \frac{4x}{l^3}
\]

The influence line for \( w \) is a triangle with maximum ordinate at the centre equal to \( \frac{4 \times 1/2}{l^3} = \frac{2}{l} \).

Horizontal thrust at cable supports is given by

\[
H = \frac{wl^3}{8d} = \frac{4x \times l^3}{8d} = \frac{x}{2d}
\]

The influence line for \( H \) is a triangle with central ordinate at centre equal to \( \frac{l}{2 \times 2d} = \frac{l}{4d} \).

Influence line for \( R_A \).

\[
R_A = \frac{W(l-x)}{l} - \frac{wl}{2}
\]

for unit load \( W = 1 \).

\[
\therefore \quad R_A = \frac{(l-x)}{l} - \frac{wl}{2}
\]

\[
\therefore \quad \frac{(l-x)}{l} - \frac{4x}{l^3} \times \frac{l}{2}
\]

\[
\therefore \quad \frac{l-x}{l} - \frac{2x}{l}
\]

\[
\therefore \quad 1 - \frac{3x}{l} \text{ for } x \text{ between } 0 \text{ to } l/2.
\]
This represents a straight line equation

At $x = 0$, $R_A = 1$
At $x = l/3$, $R_A = 0$, At $x = l/2$, $R_A = -\frac{1}{3}$

when load is at B, $R_A = 0$.

Influence line for reaction is shown in Fig. 15-16 (d).

Influence line for $F_9$

Influence line for $H \times (-\tan \theta)$

Influence line for S.F.

(a), (b), (c), (d)

Fig. 15-17

Influence lines for B.M. and S.F. Let the section where B.M. and S.F. influence lines are required, be at distance $a$ from $A$. 
S.F. at the section is given by

\[ F_a = F - H \tan \theta_a \]

where \( F \) is shear force as for simply supported beam, \( H \) is horizontal component of tension in cable and \( \tan \theta_a \) is slope at the section. The influence line for shear force is obtained by superposing influence line at the section as for simply supported beam on influence line for \( H \) multiplied by \( \tan \theta_a \).

The influence line for \( F_a \) is shown in Fig. 15.17 (b).

The influence line for \( H \tan \theta_a \) will be a triangle with maximum ordinate at the centre \( = \frac{l}{4d} \tan \theta_a \).

\[ y = \frac{4d}{l^2} (l - x) \]

\[ \frac{dy}{dx} = \frac{4d}{l^2} (l - 2x) \]

At \( x = a \),

\[ \tan \theta_a = \frac{4d}{l^2} (l - 2a) \]

Influence line ordinate at centre \( = \frac{l}{4d} \times \frac{4d}{l^2} (l - 2a) \)

\[ = \frac{l - 2a}{l} \]

Influence line for \( H \times (-\tan \theta_a) \) is shown in Fig. 15.17 (c).

Influence line for shear force is shown in Fig. 15.17 (d).

Influence line for B.M. B.M. at a section is given by

\[ M = M_a - Hy_a \]

where \( M_a \) is the B.M. at the section as for simply supported beam, \( H \) is the horizontal component of tension in the cable and \( y_a \) is the dip of the cable at the section. Thus the influence line for B.M. at a section can be obtained by superposing influence line as for simply supported beam on influence line for \( H \) multiplied by \( y_a \).

Influence line for \( M_a \) will be a triangle with maximum ordinate \( \frac{l - a}{l} \) at the section as shown in Fig. 15.17 (e). \( Hy_a \) diagram will be influence line for \( H \) multiplied by \( y_a \). Influence
line for $H$ is a triangle with maximum ordinate at the centre equal to 

$$y_a = \frac{4da(l-a)}{l}$$

\[ \therefore \quad H \times y_a = \frac{l}{4d} \times \frac{4da(l-a)}{l^2} = \frac{a(l-a)}{l} \]

Influence line for $H \times y_a$ is shown in Fig. 15.17 (f).

Influence line for B.M. is shown in Fig. 15.17 (g).

As the two ordinates are equal, area of $+ve$ portion of diagram will be equal to area of $-ve$ portion of diagram.

**Maximum $+ve$ ordinate**

\[
= \frac{a(l-a)}{l} - \frac{a(l-a)}{l} \times \frac{a}{l/2} \\
= \frac{a(l-a)}{l} \left[ 1 - \frac{2a}{l} \right] \\
= \frac{a(l-a)}{l^2} \cdot (l-2a)
\]

**Maximum $-ve$ ordinate**

\[
= \frac{a(l-a)}{l} - \frac{a(l-a)}{l} \times \frac{l/2}{(l-a)} \\
= \frac{a(l-a)}{l} \left[ 1 - \frac{l}{2(l-a)} \right] \\
- \frac{a(l-a)}{2l(l-a)} (2l-2a \cdot l) \\
= \frac{a(l-2a)}{2l}
\]

Influence line for B.M. is shown separately in Fig. 15.17 (h).

**Absolute maximum B.M. due to concentrated load.** Maximum $+ve$ B.M. at a section occurs when the load is at the section.

In the span of the girders there will be some section where the $+ve$ B.M. will be absolute maximum. Maximum $+ve$ B.M. at a section distance $a$ from $A$ is given by

$$M_a = \frac{a(l-a)(l-2a)}{l^2}$$

For absolute maximum B.M., putting $\frac{dM_a}{da} = 0$, will give the position of section where absolute maximum $+ve$ B.M. will occur.

$$M_a = \frac{a(l-a)(l-2a)}{l^2}$$

$$= \frac{al^2 - 3a^2l + 2a^3}{l^2}$$
\[ \frac{dM_a}{da} = \frac{l^2 - 6a^2 + 6a^3}{l^2} = 0 \]

\[ a = \frac{6l \pm \sqrt{(6l)^2 - 4 \times 6l^2}}{12} = \frac{6l \pm \sqrt{36l^2 - 24l^2}}{12} = \frac{l}{2} \pm \frac{1}{12} \sqrt{12l^2} = \frac{l}{2} (1 \pm \frac{1}{\sqrt{3}}) = 0.789 \quad \text{or} \quad 0.211 \ l \]

Maximum +ve B.M. occurs at a section distance 0.211 \ l from either support.

Maximum \[ M_a = \frac{0.211 \ l (l - 0.211 \ l)(l - 0.422 \ l)}{l^2} = 0.096 \ l \]

If \( W \) is concentrated load moving on the span, Absolute maximum +ve B.M. = 0.096 \( WL \).

Maximum -ve B.M. at a section occurs when the load is at the centre of span. There will be some section in the span where the -ve B.M. will be absolute maximum.

-ve B.M. \[ M_a = \frac{a(l - 2a)}{2l} \]

Putting \[ \frac{dM_a}{da} = 0, \]

\[ \frac{l - 4a}{2l} = 0 \]

\[ a = \frac{l}{4} \]

Absolute maximum -ve B.M. occurs at quarter span.

Maximum -ve B.M. \[ = \frac{1}{4} \frac{(l - l/2)}{2l} = \frac{1}{16} \]

Due to concentrated load \( W \), absolute maximum -ve B.M. \[ = \frac{WL}{16} \]

Maximum B.M. due to uniformly distributed load. Due to uniformly distributed load, maximum +ve B.M. at the section will occur when portion \( AD \) is loaded with uniformly distributed load and maximum -ve B.M. occurs when portion \( DB \) is loaded with uniformly distributed load [Fig. 15.17 (a)].
In Fig. 15.17 (h) \( ED = \frac{a(l-a)(l-2a)}{a(l-a)(l-2a) + \frac{a}{2} l(l-2a)} \)

\[
= \frac{(l-a)}{(l-2a)} \times \frac{l}{l + \frac{1}{2}}
\]

\[
= \frac{(l-a)(l-2a)}{(3l-2a)}
\]

\( AD = a + \frac{(l-a)(l-2a)}{(3l-2a)} \).

Area of +ve portion of diagram

\[
= \frac{a(l-a)(l-2a)}{2l^2} \left[ a + \frac{(l-a)(l-2a)}{(3l-2a)} \right]
\]

\[
= \frac{a(l-a)(l-2a)}{2l^2} \left[ \frac{3al-2a^2 + l^2 - 3al + 2a^2}{(3l-2a)} \right]
\]

\[
= \frac{a(l-a)(l-2a)}{2l^2} \times \frac{l^2}{(3l-2a)}
\]

\[
= \frac{a(l-a)(l-2a)}{2(3l-2a)}
\]

Area of -ve portion of diagram will be same as area of +ve portion of diagram. If portions are loaded with uniformly distributed load of \( w/ \)unit length, maximum +ve B.M. will be

\[
\frac{w a(l-a)(l-2a)}{2(3l-2a)}
\]

when portion \( AD \) is loaded. Maximum -ve B.M. will be of same magnitude as +ve B.M. and occurs when portion \( DB \) is loaded.

Corresponding to some section +ve B.M. and -ve B.M. will be absolute maximum.

\[
\text{Max. B.M.} = \frac{wa(l-a)(l-2a)}{2(3l-2a)}
\]

For absolute maximum B.M., put

\[
\frac{dM_a}{da} = 0.
\]

\[
\frac{w}{2} \left[ (l^2 - 6al + 6a^2) - \frac{(al^2 - 3a^2l + 2a^3)}{(3l-2a)^2} \right] = 0
\]

\[
(3l-2a)(l^2 - 6al + 6a^2) - (al^2 - 3a^2l + 2a^3)(-2) = 0
\]

\[
3l^3 - 20al^2 + 30a^2l - 12a^3 + 2al^2 - 6a^3l + 4a^4 = 0
\]

\[
3l^3 - 18al^2 + 24a^2l - 8a^3 = 0
\]

Solving for 'a' by trial and error,

\[
a = 0.234l \text{ approximately}
\]
Maximum area of the diagram

\[
A_{\text{max}} = \frac{0.234 l (l - 0.234 l) (l - 0.468 l)}{2 (3l - 0.468 l)} = \frac{0.234 \times 0.766 \times 0.532 l}{2 \times 2.532 l} = 0.01883 l
\]

Due to uniformly distributed load of intensity \( w \), absolute maximum B.M. = \( 0.01883 \, w l^2 \).

The influence line for S.F. and B.M. at a section obtained above can also be obtained from influence line for \( w \) the load carried by cables and influence line for \( H \) obtained in Fig. 15.16 (b) and (d), as shown below.

**I.L. for S.F.**

When the unit load is to the right of section, S.F. at the section is

\[
F_a = R_A + wa
\]

\[
\therefore \text{Influence line for } F_a \text{ will be obtained by superposing influence line for } R_A \text{ on influence line for } w \text{ multiplied by } a.
\]

When the unit load is to the left of the section,

\[
F_a = R_A + wa - 1
\]

Thus influence line for S.F. at the section when load is to left of the section is obtained by deducting unity from the superposed diagrams obtained above when the load is to the right of the section as shown in Fig. 15.18 (d) and (e).

**I.L. for B.M.**

When the load is to the right of the section, B.M. at the section is

\[
M_a = R_A \times a + \frac{wa^2}{2}
\]
Thus influence line for $M_a$ is obtained by superposing influence line for $R_a$ multiplied by $a$ on influence line for $w$ multiplied by $a^2/2$ as shown in Fig. 15.18 (h). When the load is to the left of section,

$$M_a = R_a \times a + \frac{wa^2}{2}$$

$$-l(a-x)$$

Thus the influence line for $M_a$ is obtained by subtracting $l(a-x)$ from the superposed diagrams as obtained above as shown in Fig. 15.18 (h).

**Fig. 15.18**

Maximum $-ve$ B.M. = $\frac{a}{2} - \frac{a^2}{l}$

= $\frac{w}{2l} (l-2a)$

Maximum $+ve$ B.M. = $\frac{a \times (l/3-a)}{l/3} + l \times \frac{-a(l-3a)}{l/2}$

= $\frac{a(l-3a)}{l} + \frac{2a^2}{l}$

= $\frac{1}{\ell} (l^2-3al+2a^3)$

= $\frac{w}{l^2} (l-a)(l-2a)$

*The values are same as obtained previously.*

**Ex. 15.8.** A suspension bridge with three hinged stiffening girder has span of 100 m and central dip of 10 m. The self weight of the bridge carried by one set of cables is 1500 kg/m. The bridge is to be designed to carry a live load of 3000 kg/m. to be equally divided between two sets of cables. The working stress is 1500 kg/cm.² for cable and 1200 kg/cm.² for girder. Find (a) cross-sectional area of one set of suspension cable and (b) necessary section modulus of the stiffening girder.

**Sol.** Live load per set of cables = $\frac{3000}{2}$

= 1500 kg/m.

Maximum tension in the cable occurs when the entire span is loaded with live load.
Horizontal component of tension in cable
\[ H = w \times \frac{l^2}{8d} = \frac{(1500+1500) \times 100 \times 00}{8 \times 10} = \frac{3000 \times 100 \times 100}{800} = 375,000 \text{ kg.} \]

Vertical component of tension in cable at support
\[ = (1500+1500) \times \frac{100}{2} = 150,000 \text{ kg.} \]

Maximum tension in the cable
\[ = \sqrt{H^2 + V^2} = \sqrt{375,000^2 + 150,000^2} = 403,875 \text{ kg.} \]

Area of one set of cable
\[ = \frac{403,875}{1500} = 269.25 \text{ cm}^2 \]

Due to uniformly distributed dead load of 1500 kg/m, there will be no bending moment in the girder.

Due to live load maximum +ve B.M. and -ve B.M. are equal to 0.01883 \( w \times l^2 \).

\[ \therefore \text{ Maximum B.M.} = 0.01883 \times 1500 \times 100 \times 100 = 282,450 \text{ kg.m.} \]

\[ Z \text{ required} = \frac{M}{f} = \frac{282,450}{1200} \times 100 \text{ cm} = 23,538 \text{ cm}^3. \]

**Ex. 159.** The towers of 150 m span suspension bridges are of unequal heights. One is 20 m. and other is 5 m. above the lowest point of the cable which is immediately above the inner pin of a three hinged stiffening girder hinged at the towers. Find the maximum tension in the cable due to a point load 10,000 kg. crossing the bridge.

**Sol.** Let \( x \) be the distance of the lowest point of the cable from end \( A \). Let \( w/\text{unit length} \) be load transferred to the cable. Let concentrated load act at distance \( a \) from \( A \).

Considering equilibrium of left hand portion of cable, taking moments about \( A \),

\[ u = \frac{w \times x^2}{2 \times 20} \]

Considering the equilibrium of right hand portion of cable, taking moments about \( B \),

\[ = \frac{w(150-x)^2}{2 \times 5} \]
Equating two values of $H$,
\[
\frac{wx^2}{2 \times 20} = \frac{w(150-x)^2}{2 \times 5}
\]

\[
\begin{align*}
\text{Fig. 15.19} \\
x^2 & \quad (150-x)^2 \\
\therefore \quad \frac{x}{2} &= 150 - x \\
\therefore \quad \frac{3}{2} x &= 150 \\
\therefore \quad x &= 100 \text{ m.}
\end{align*}
\]

Taking moments about $A'$,
\[
B' \times 1 - W \times a - H \times 15 = 0
\]
\[ R'_B = \frac{10,000 \times a}{150} + \frac{H \times 15}{150} \]

Cut the cable and stiffening girder at \( C \) and \( C' \).

Moment at \( C' \) is zero.

\[ H \times 5 - R'_B \times 50 = 0 \]

\[ H \times 5 - \left( \frac{10,000 \times a}{150} + \frac{H \times 15}{150} \right) 50 = 0 \]

\[ H - \frac{10,000 \times a}{15} + H = 0 \]

\[ H = \frac{10,000 \times a}{30} \]

The value of \( H \) increases as distance \( a \) increases. Maximum value of \( a \) is 100 m, i.e., when concentrated load is at central hinge.

\[ H = \frac{10,000}{30} \times 100 \]

\[ = \frac{100,000}{3} \]

\[ H = \frac{wx}{2 \times 20} \]

\[ \frac{100,000}{3} = \frac{w \times 100 \times 100}{2 \times 20} \]

\[ w = \frac{400}{3} \text{ kg/m.} \]

Vertical reaction at \( A = 100 \times \frac{400}{3} = \frac{40,000}{3} \text{ kg.} \)

Vertical reaction at \( B = 50 \times \frac{400}{3} = \frac{20,000}{3} \text{ kg.} \)

Maximum tension in cable occurs at \( A \).

Maximum tension in cable = \( \sqrt{H^2 + V_A^2} \)

\[ \sqrt{\left( \frac{100,000}{3} \right)^2 + \left( \frac{40,000}{3} \right)^2} \]

\[ - \frac{20,000}{3} \sqrt{5^2 + 2^2} \]

\[ = \frac{20,000}{3} \sqrt{29} \]

\[ = 35,925 \text{ kg.} \]
15.11. Suspension Bridges with two-hinged stiffening girders.

As already discussed a two-hinged stiffening girder makes suspension bridge more stiff. In the analysis of suspension bridges with two-hinged stiffening girders it is assumed that cable retains its parabola shape under concentrated loads, thus the load transferred to cable will be uniformly distributed. It is further assumed that the reactions at two supports of stiffening girder are equal and opposite. Therefore the load on the cable due to concentrated load $W$ will be $W/l$, where $l$ is the span, whatever be the position of the load.

Let the concentrated load $W$ act at distance $x$ from left support. Let $w$/unit length be uniformly distributed load transferred to cable, $w = W/l$.

Taking moments about $B$,

$$R_A - W(l-x) = 0$$

$$R_A = \frac{Wl}{l} - Wx$$

$$R_A = \frac{\frac{Wl}{2} - Wx}{l}$$

$$R_A = \frac{W}{2} - \frac{Wx}{l}$$

Influence line for $R_A$ is straight line with value $+ \frac{W}{2}$ when load is at $A$ and $- \frac{W}{2}$ when load is at $B$.

$$H = \frac{wl^2}{8d}$$

$$= \frac{W}{l} \times \frac{l^2}{8d}$$

$$= \frac{Wl}{8d}.$$
Thus the value of $H$ is constant whatever be the position of the load.

Influence line for S.F. and B.M. Let the section be at a distance $a$ from $A$.

I.L. for S.F.

S.F. at a section is given by $F_s = F_s - H \tan \theta_a$ where $F_s$ is shear force at the section, considering girder as simply supported, $H$ is horizontal component of tension in cable and $\theta_a$ is the slope of cable at the section.

At $x = a$, $\tan \theta_a = \frac{4d(l-2a)}{v_a}$
Thus influence line for shear force is obtained by subtracting from influence line for \( F_o \), considering girder as simply supported beam, quantity \( \frac{(l-2a)}{2l} \) as shown in Fig. 15.21 (d).

When the load is at the section,

\[
\begin{align*}
-\text{ve S.F.} & = \frac{a}{l} + \frac{l-2a}{2l} - \frac{1}{2} \\
+\text{ve S.F.} & = 1 - \frac{1}{2} = \frac{1}{2}
\end{align*}
\]

Thus maximum +ve S.F. and -ve S.F. due to rolling load is half of the rolling load, irrespective of the position of the section on the girder.

Distance in which shear force is +ve,

\[
\frac{l}{2} + \frac{(l-2a)}{2l} \times (l-a)
\]

Due to uniformly distributed load, maximum +ve S.F.

\[
= w \times \frac{1}{2} \times \frac{l}{2} \times \frac{1}{2} = \frac{wl}{8}
\]

-ve S.F. due to uniformly distributed load

\[
i = \text{intensity of loading} \times \text{area of negative portion of diagram}
\]

\[
= -w \left[ \frac{(l-2a)}{2l} \times a + \frac{a}{l} \times \frac{a}{2} + \frac{(l-2a)}{2l} \times \left( \frac{l}{2} - a \right) \times \frac{1}{2} \right]
\]

\[
= -w \left[ \frac{(l-2a)a}{2l} + \frac{a^2}{2l} + \frac{(l-2a)^2}{8l} \right]
\]

\[
= -w \left[ \frac{(l-2a)}{2l} \left( a + \frac{l-2a}{4} \right) + \frac{a^2}{2l} \right]
\]

\[
= -w \left[ \frac{l^2-4a^2}{8l} + \frac{a^2}{2l} \right]
\]

\[
= -w \left[ \frac{l^2-4a^2 + 4a^2}{8l} \right]
\]

\[
= -w \frac{wl}{8}
\]

Thus maximum +ve S.F. and -ve S.F. due to uniformly distributed load are equal. This value is independent of position of section.
I.L. for B.M.

B.M. at a section is given by

\[ M_a = M_s - Hy_a \]

where \( M_s \) is the B.M. at the section, considering girder as simply supported, \( H \) is horizontal component of tension in the cable and \( y_a \) is the dip of the cable at the section.

\[ \therefore M_a = M_s - \frac{l}{8d} \times \frac{4da(l-a)}{l^2} \]

\[ = M_s - \frac{a(l-a)}{2l} \]

Thus influence line for B.M. is obtained by subtracting \( \frac{a(l-a)}{2l} \) from influence line considering girder as simply supported beam. I.L. for B.M. is shown in Fig. 15.21 (e).

The areas of influence line diagram due to \( M_s \) and due to \( \frac{a(l-a)}{2l} \) are same, hence in the superposed diagram, areas of positive portion of diagram and negative portion of diagram will be same.

Due to single concentrated load maximum +ve B.M. at the section will be \( \frac{a(l-a)}{2l} \), when the load is at the section. Maximum −ve B.M. at the section will also be \( \frac{a(l-a)}{2l} \) when the load is at either supports.

For some section the B.M. will be absolute maximum.

\[ M_a = \frac{a(l-a)}{2l} \]

\[ \frac{dM_a}{da} = \frac{l-2a}{2l} = 0 \]

\[ a = l/2 \]

\[ \therefore \text{At } a = l/2 \text{, B.M. will be absolute maximum.} \]

\[ \text{Max. B.M. } = \frac{l}{2l} \left( l - \frac{l}{2} \right) = \frac{l}{8} \]

Due to load \( W \), maximum +ve B.M. and −ve B.M. will be each equal to \( \frac{WL}{8} \).

Absolute maximum B.M. due to live load

For maximum +ve B.M., length in +ve portion of diagram

i.e. \( \frac{l}{2} \) should be loaded.
\[ M_a = \frac{a(l-a)}{2l} \times \frac{l}{2} \times \frac{1}{2} = \frac{a(l-a)}{8} \]

For some section this quantity will be absolute maximum

\[ \frac{dM_a}{da} = \frac{l-2a}{8} = 0 \]

\[ a = l/2 \]

Absolute maximum +ve B.M.

\[ \frac{l}{2} \left( l-l/2 \right) = \frac{l^2}{8} \]

Due to uniformly distributed load of intensity \( w \), maximum B.M.

\[ \frac{w}{32} \]

15.12. Temperature stresses in stiffening girder

Consider a suspension bridge with stiffening girders and subjected to temperature change of \( t^\circ \).

Length of the cable is given by

\[ L = l + \frac{8}{3} \frac{d^2}{l} \]

where \( d \) is the dip of the cable at centre and \( l \) is the horizontal span of the cable.

\[ \delta L = \frac{16}{3} \frac{d}{l} \times \delta d \]

Due to temperature change

\[ \delta L = L \times \Delta t \]

\[ L \times \Delta t = \frac{16}{3} \frac{d}{l} \times \delta d \]

\[ L = l \text{ approximately} \]

\[ \delta d = \frac{3}{16} \Delta t \frac{l^2}{d} \]

...(1)

Due to change in dip of cable by \( \delta d \), the girder will be deflected by same amount \( \delta d \).

Assuming that due to temperature change the load transfer between cable and girder is uniform throughout the length and equal to \( w \)/unit length.

Central deflection of girder due to uniformly distributed load is given by

\[ \delta d = \frac{5}{384} \frac{wl^4}{EI} \]

\[ = \frac{wl^4}{8l} \times \frac{5l^2}{48E} \]

\[ = \frac{M}{l} \times \frac{5l^3}{48E} \]
where \( M \) is B.M. at centre due to uniformly distributed load

\[
M = f \frac{5l^3}{48E} \left[ \frac{f}{\gamma_{mas}} = \frac{M}{I} \right]
\]

where \( f \) is maximum stress in extreme fibres and \( D \) is depth of girder.

\[
\delta d = \frac{f}{D} \frac{5l^3}{48E}
\]

Substituting \( \delta d \) from (1)

\[
\frac{3 \alpha t l^3}{16d} = \frac{f}{D} \frac{5l^3}{24E}
\]

\[
f = \frac{24 \times 3 \alpha t ED}{5 \times 16}
\]

\[
= \frac{9 \alpha t ED}{10 \frac{d}{d}}
\]

\[
= 0.9 \alpha t ED
\]

**Ex. 15.10.** A suspension bridge has span of 30 m. The central dip of cable is 3 m. and stiffening girder has depth of 1 m. Find the change in the stress due to the change in temperature of 25°C.

\[ E = 2 \times 10^8 \text{ kg/cm}^2, \quad \alpha = 1.2 \times 10^{-5}/\text{°C} \]

**Solution.**

\[
f = \frac{0.9 \alpha t ED}{d}
\]

\[
= \frac{0.9 \times 1.2 \times 10^{-5} \times 25 \times 2 \times 10^8 \times 1}{3}
\]

\[
= 0.36 \times 500
\]

\[
= 180 \text{ kg/cm}^2.
\]

**Ex. 15.11.** An unstiffened suspension cable carries a uniformly distributed load of 6.6 t/m over a span of 30 m. as shown in Fig. 15.22. The suspension cable is supported on frictionless rollers fixed to the piers. The anchor cables are inclined at 30° to the horizontal. One pier is 4.5 m. below the other and the maximum dip at the lowest point is 3 m. below the lower pier. Calculate (a) the maximum and minimum tension in the cable, and (b) the horizontal and vertical forces at each pier. (A.M.I.E. Nov. 1967)

**Solution.**

Let the lowest point \( C \), be \( x \) metres from \( A \).

Cutting the cable at supports and at \( C \),

Moment about \( A \)

\[
H \times 3 = \omega x \times \frac{x}{2}
\]
\[ H = \frac{wx^2}{6} \]

**Moment about B.**

\[ H \times 7'5 - \frac{w(30-x)^2}{2} \]

\[ \therefore \ H = w(30-x)^2 \]

\[ \therefore \ \frac{wx^2}{6} = \frac{w(30-x)^2}{15} \]

\[ 2'5x^2 = 900 - 60x + x^2. \]

\[ 1'5x^2 + 60x - 900 = 0. \]

\[ x^2 + 40x - 600 = 0. \]

\[ x = -40 \pm \sqrt{1600 + 2400} \]

\[ -40 + \sqrt{4000} = 11'62 \text{ m.} \]

Minimum tension will be at the lowest point

\[ H = \frac{6 \times 6 \times 11'62^3}{6} = 148'6 \text{ t.} \]

Maximum tension will be at B

\[ = \sqrt{H^2 + [w(30-x)]^2} \]

\[ = \sqrt{(148'6)^2 + [6'6(18'38)]^2} \]

\[ = 191'8 \text{ t.} \]

Tension at A = \[ \sqrt{(148'6)^2 + (6'6x)^2} \]

\[ = \sqrt{(148'6)^2 + (11'62 \times 6'6)^2} \]

\[ = 167'1 \text{ t.} \]

**Forces at pier A**

- **Vertical** = \[ 11'62 \times 6'6 + 167'1 \times \sin 30^\circ \]
  = \[ 11'62 \times 6'6 + 167'1 \times \frac{1}{2} \]
  = \[ 76'7 + 83'55 \]
  = \[ 160'25 \text{ t.} \]

- **Horizontal** = \[ 148'6 - 167'1 \cos 30^\circ \]
  = \[ 148'6 - 144'7 \text{ t.} \]
  = \[ 3'9 \text{ t.} \]

**Forces at pier B.**

- **Vertical** = \[ (30 - 11'62) \times 6'6 + 191'8 \sin 30^\circ \]
  = \[ 18'38 \times 6'6 + 191'8 \times \frac{1}{2} \]
  = \[ 121'3 + 95'9 \]
  = \[ 217'2 \text{ t.} \]

- **Horizontal** = \[ 148'6 - 191'8 \cos 30^\circ \]
  = \[ 148'6 - 166'1 \]
  = \[ -17'5 \text{ t.} \]
PROBLEMS

1. A light suspension bridge is constructed to carry a pathway 3 m. broad over a channel 24 m. wide. There are 7 equidistant suspension rods. The central dip of the cable is 2·0 m. and the platform load is 1 t/m². Find the maximum tension in the cable.

2. A suspension cable having supports at same level has span of 20 m. and maximum dip of 2 m. It is loaded with uniformly distributed load of 1 t/m throughout its length and with concentrated loads of 2 t and 6 t at right hand and left hand middle third points respectively. Find the maximum tension in the cable.

3. Show that the permissible span for a steel cable suspended between supports at same level and having central dip of \( \frac{1}{10} \) of span is given approximately by \( 138.25 \cdot \rho \), where \( f \) is the permissible stress in kg/cm² and \( \rho \) is the density of the material of cable in kg/cm³. Assume the cable to hang in parabolic form.

4. A bridge cable slung between two piers 100 m. apart carries a load of 2000 kg/m of span. The tops of the piers are at the same level and the cable at its lowest point sags 10 m. below this level. Calculate the maximum value of cable tension.

If the cable passes over saddles on frictionless rollers and backstay is inclined at 30° to the horizontal, find the tension in the backstay and pressure on the pier. If the cable passes over a pulley find the horizontal and vertical pressures on the pier.

[Ans. \( T_{max} = 269,250 \text{ kg} ; 288,683 \text{ kg}, 24,44,342 \text{ kg} ; 17,262 \text{ kg} ; 234,875 \text{ kg} \)]

5. A bridge cable of total weight 320,000 kg is supported across a span of 80 m by two sets of suspension cables hanging in parabolic form with central dip of 10 m. and strengthened on each side by a stiffening girder hinged at the centre and ends.

Find for a live load of 2000 kg/m longer than the span, the necessary section modulus for the stiffening girders and the required cross-sectional area of one set of cables. Permissible stresses in girders and cables are 1200 kg/cm² and 1500 kg/cm² respectively.

[Ans. \( A = 179 \text{ cm}^2, Z = 10,443 \text{ cm}^3 \)]

6. A suspension cable of span \( l \), hanging in the shape of a symmetrical parabola is strengthened by a stiffening girder pinned at the abutments and at the centre.

Show that for the passage across the bridge of a uniformly distributed load longer than the span, the maximum positive and negative shear forces occur at the abutments. Find the magnitude of these forces and of the maximum positive and negative shearing force at the centre of the span and state the corresponding loading conditions in all cases.
7. Fig. 15.24 shows suspension bridge stiffened with three hinged stiffening girder. Find maximum force in member PQ due to single rolling load of W. Assume that this load is taken by one set of cables. [Ans. 0.695 W]

8. A suspension bridge with 3 hinged stiffening girder has a span of 100 m, and a central dip of 10 m, and carries dead load of 300 tonnes. It is to be designed to carry a single rolling load of 10 tonnes. The load may be assumed to be equally divided between the two suspension cables. Determine the sectional area of one cable if working stress is 1500 kg/cm². Also find maximum B.M. in the stiffening girder.

9. A suspension chain of 130 m span and 15 m dip, stiffened with a three-hinged girder, carries a dead load of 1 t/m of span. If a uniformly distributed load of 3 t/m run, 50 m long, travels slowly across the span determine.
   
   (a) the maximum tension in the cable.
   
   (b) the maximum positive and negative shearing forces in the stiffening girder at a section 30 m from left end, and
   
   (c) maximum bending moment at the above section.

10. A suspension cable 100 m span and 15 m dip is stiffened with a three-hinged girder and carries a dead load of 1 t/m of span. If a uniform moving load of 6 t/m, longer than the span travels slowly across the span, determine the maximum tension in the cable, and the greatest B.M. and S.F. in the stiffening girder. State the loaded length in each case.

11. A suspension bridge of 60 m span is supported by two sets of cables hanging in a parabolic form and has a central dip of 6 m. It is strengthened on each side by a stiffening girder of two hinged type. The total weight of the bridge is 80 tonnes. The bridge is likely to be travelled by a live load of 2 t/m run longer than the span. Determine the required modulus of section for the stiffening girder and also cross-sectional area of one set of cables if the permissible stress in the girder and the cables are 1200 kg/cm² and 1500 kg/cm², respectively.
12. A cable is used to support six equal and equidistant loads over a span of 14.7 m. The central dip of the cable is 1.5 m and the loads are 2 tonnes each. Find the length of the cable required and its sectional area, if the safe tensile stress is 1.5 tonnes/sq. cm. The distance between the loads is 2.1 m.  
(A.M.I.E. May 1969)

13. A suspension bridge with two hinged stiffening girders has a span of 167 m and the cables have a central dip of 13.3 m. If the stiffening girders are 5 m deep, calculate the flange stress due to a fall of 14°C and also the increase in tension in the cable. The moment of inertia of the girder is $104 \times 10^5$ cm$^4$, $E = 2000$ t/cm$^2$ and $a = 10.8 \times 10^{-5}$ per °C.  
(A.M.I.E. May 1970)

14. A cable of a suspension bridge has a span of 400 m over supports which are at the same level, and a sag of 40 m measured vertically from the line of supports to the lowest point on the cable at mid span. It is stiffened by a three-hinged girder with hinged supports at the two ends and the third hinged at its mid point. The girder carries 3 loads of 30 t, 50 t, 40 t, acting at 70 m, 140 m and 300 m respectively from the left hand end.

(i) Draw the B.M.D. and S.F.D. for the girder giving values at salient points.

(ii) Calculate the maximum tension in the cable for a U.D. live load.  
(A.M.I.E. Nov. 1970)
16·1. Masonry Dams. Dams are used to retain water to required height. Forces acting on dam section are weight of masonry, pressure of water and uplift pressure. As the masonry is not capable of taking any tension, the resultant of the forces on the dam section should lie within the middle third of the section. As the pressure of the water increases with the depth, width of dam section is made broader with the increase in depth. At the top of the dam minimum width is provided from considerations of traffic and operation of any machinery. Same width is provided for some height of the dam so that no tension is induced in the section. After this height, the width of dam section is increased on the downstream side up to some height so that no tension is developed at the section and also the compressive stress is within permissible limits as shown in Fig. 16·1 (a). With the increase in the height of the dam, it becomes necessary to increase the width of the section on upstream side in addition to downstream side so that no tension is induced at the base when tank is empty as shown in Fig. 16·1 (b).

16·2. Causes of failures of Dams. A dam section may fail due to any of the following causes:

(i) Tension in the section.
(ii) Crushing at the toe.
(iii) Sliding.
Consider one metre length of the dam. Let $W$ be the weight of dam above the section considered, $B$ be the width of the section and $P$ be total pressure of water. Let the resultant of $W$ and $P$ strike the section at an eccentricity of $e$.

$$p_{\text{max}} = \frac{W}{B} + \frac{W \times e}{B^2/6}$$
$$= \frac{W}{B} \left[ 1 + \frac{6e}{B} \right]$$

$$p_{\text{min}} = \frac{W}{B} - \frac{W \times e}{B^2/6}$$
$$= \frac{W}{B} \left[ 1 - \frac{6e}{B} \right]$$

(i) If no tension is to occur $1 - \frac{6e}{B} \geq 0$

In the limiting case $1 - \frac{6e}{B} = 0$

$$\therefore e = \frac{B}{6} \text{ i.e. the resultant should lie within the middle third.}$$

(ii) Maximum stress at the toe is $\frac{W}{B} \left( 1 + \frac{6e}{B} \right)$. To avoid crushing of toe, the maximum stress should not exceed allowable stress for masonry.

(iii) If $\mu$ is the co-efficient of friction, the maximum frictional resistance will be $\mu W$. The force tending to cause sliding is water pressure $P$. The factor of safety against sliding will be $\frac{\mu W}{P}$. This should not be less than 1.5.

16.3. Rectangular trapezoidal dam sections.

Rectangular sections. Consider rectangular dam section of height $H$ and breadth 'b'. Let $w$ be the density of water and $\rho$ be the density of masonry. Consider one metre length of dam.

Weight of the dam $= W = bH\rho$

Pressure of water $= P = \frac{wH^2}{\pi}$

The reactions from the soil will be $W$ upwards and $P$ horizontal passing through the point where resultant of weight of masonry and water pressure will strike the base. Let the resultant strike the base at $x$ distance from toe.
Taking moments about the toe,
\[
\frac{Wb}{2} - \frac{PH}{3} - Wx = 0
\]
\[
x = \frac{\frac{Wb}{2} - \frac{PH}{3}}{W} = \frac{b}{2} - \frac{PH}{3W}
\]
\[
e = \frac{b}{2} - x = \frac{PH}{3}
\]
\[
\frac{\frac{wH^2}{2}}{3bH^2} = \frac{wH^3}{6b^2}
\]

(i) For no tension to develop at base
\[
e = \frac{b}{6} = \frac{wH^2}{6b^2}
\]
\[
b^2 = \frac{wH^3}{6b^2}
\]
\[
H = b\sqrt{\frac{\rho}{w}}
\]
\[
\therefore \text{limiting height of dam for no tension to develop is}
\]
\[
b\sqrt{\frac{\rho}{w}}.
\]

(ii) For factor of safety of 1.5 against sliding.
\[
\mu W = 1.5 P \text{ where } \mu \text{ is coefficient of friction}
\]
\[
\therefore \mu \times \frac{bH^2}{2} = 1.5 \times \frac{wH^3}{2}
\]
\[
\mu b^2 = \frac{3}{4} wH
\]
\[
\therefore H = \frac{4}{3} \mu b^2.
\]

(iii) If the compressive stress is to be within permissible limit of \(\rho\)
\[
\rho = \frac{W}{b} \left( 1 + \frac{6e}{b} \right)
\]
In the limiting case of no tension to develop
\[
e = b/6
\]
\[ p = \frac{W}{b} (1+1) = 2 \frac{W}{b} \]
\[ p = 2 \frac{bH \rho}{b} = 2H \rho \]
\[ H = \frac{p}{2 \rho} \]

Limiting height of dam \( \frac{p}{2 \rho} \)

**Trapezoidal section.** Consider trapezoidal dam section of height \( H \), top width \( b \) and bottom width \( B \), with upstream face vertical.

Let \( W_1 \) be the weight of rectangular portion of section and \( W_2 \) be the weight of triangular portion of the section.

Pressure of water \( P \)

\[ P = 2 \text{ acting at } H/3 \text{ from base.} \]

\[ W_1 = bH \rho \text{ acting at } \left( B - \frac{b}{2} \right) \text{ from toe.} \]

\[ W_2 = \frac{(B-b)}{2} H \rho \text{ acting at } \frac{2}{3} (B-b) \text{ from toe.} \]

Total weight \( W \)

\[ W_1 + W_2 = \frac{(b+B)}{2} H \rho. \]

Let resultant of \( W_1, W_2 \) and \( P \) strike the base at distance \( \bar{x} \) from toe. The reaction from soil will be \( W = W_1 + W_2 \) upwards and \( P \) as shown in Fig. 16.4.

Taking moments about toe,

\[ W_1 (B - \frac{b}{2}) + W_2 \times \frac{2}{3} (B-b) - \frac{PH}{3} - W \bar{x} = 0 \]

\[ \therefore \bar{x} = \frac{W_1 (B - \frac{b}{2}) + \frac{W_2}{W} \times \frac{2}{3} (B-b) - \frac{PH}{3}}{W} \]

\[ \bar{x} = \frac{b \left( B - \frac{b}{2} \right) + 2}{(b+B)} - \frac{2}{3} \left( B - \frac{b}{2} \right) \left( B - \frac{b}{2} \right) \frac{2}{3} \left( B - \frac{b}{2} \right) \frac{wH^3}{3(b+B)^2} \]

\[ \frac{b(2B-b)}{(b+B)} + \frac{\frac{b}{3}(B-b)^2}{(b+B)} - \frac{wH^3}{3(b+B)^2} \rho \]
\[ e = \frac{B}{2} - \varepsilon \]

Maximum stress \[ = \frac{W}{B} \left( 1 + \frac{6e}{B} \right) \]

Minimum stress \[ = \frac{W}{B} \left( 1 - \frac{6e}{B} \right) \]

(iii) Section having batter on upstream and downstream faces. Consider dam section shown in Fig. 16.5.

![Fig. 16.5.](image)

A vertical is drawn at the edge of base on upstream side. The weight of masonry will be

\[ W_1 + W_2 + W_3 + W_4 + W_6 \]

The weight of water on upstream side above the dam will be \( W_6 + W_7 \). The water pressure is considered to act on the vertical drawn at the edge of base on upstream side, as shown in Fig. 16.5.

Let the resultant of weights \( W_1, W_2, W_3, W_4, W_5, W_6 \) and \( W_7 \) and \( P \) cut the base at eccentricity \( e \) from the middle of the base.

Maximum stress
\[
= \frac{W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7}{B} \left( 1 + \frac{6e}{B} \right)
\]

Minimum stress
\[
= \frac{W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7}{B} \left( 1 - \frac{6e}{B} \right)
\]

Factor of safety against sliding
\[
= \frac{\mu(W_1 + W_2 + W_3 + W_4 + W_5 + W_7)}{P}
\]

When reservoir is empty, only forces acting are weight \( W_1 + W_2 + W_3 + W_4 + W_5 \). The resultant of these weights should act in the middle third of the base.
Ex. 16.1. A rectangular masonry dam is 2 m. at the base. What is the maximum height (a) when there is no tension to occur, (b) when the factor of safety against sliding is 1.5.

Density of masonry is 2½ times the density of water, \( \mu = 0.5 \)

Solution. Let \( H \) be the height of dam. Consider one metre length of dam. Let \( w \) be density of water.

Weight of dam

\[
W = 2H \times 2.5w = 5Hw
\]

Water pressure

\[
P = \frac{wH^2}{2}
\]

Let the resultant cut the base at distance \( \bar{x} \) from toe. Taking moments about toe.

\[
W \times 1 - P \times \frac{H}{3} - W \times \bar{x} = 0
\]

\[
\bar{x} = 1 - \frac{PH}{3w} = 1 - \frac{wH^2}{3 \times 5Hw}
\]

\[
= 1 - \frac{H^3}{30}
\]

Eccentricity \( = 1 - \left(1 - \frac{H^3}{30}\right) = \frac{H^3}{30} \).

(i) For no tension to develop

\[
e = \frac{b}{6} = \frac{2}{6}
\]

\[
\frac{2}{6} \frac{H^3}{30}
\]

\[
\therefore \quad H = \sqrt{10} = 3.162 \text{ m.}
\]

(ii) For factor of safety of 1.5 against sliding,

\[
\frac{wW}{P} = 1.5
\]

\[
\frac{0.5 \times 5Hw}{wH^2} = 1.5
\]

\[
H = \frac{10}{3} \text{ m.} = 3.33 \text{ m.}
\]

Ex. 16.2. A reservoir 4 m. high is one metre wide at top and 3 m. wide at base and, has vertical water face. Calculate the maximum and minimum stresses at the base for tank full and tank empty conditions. Density of masonry = 2000 kg/m².
Solution. Consider one metre length of the wall.
Weight of rectangular portion of masonry
\[ W_1 = 1 \times 4 \times 2000 = 8000 \text{ kg. acting at } 2.5 \text{ m. from toe}, \]

(a)

Weight of triangular portion of masonry
\[ W_3 = \frac{2}{2} \times 4 \times 2000 = 8000 \text{ kg. acting at } 4/3 \text{ m. from toe.} \]

Total weight = \( W_1 + W_3 = 16,000 \text{ kg.} \)

Water pressure = \( P = \frac{wH^2}{2} = \frac{1000 \times 4^2}{2} = 8000 \text{ kg. acting at } 4/3 \text{ m. from base.} \)

Let resultant cut the base at \( \bar{x} \) metres from toe.
Taking moments about toe,
\[ 2.5 W_1 + \frac{4}{3} W_2 - P \times \frac{\bar{x}}{3} - W \times \bar{x} = 0 \]
\[ 2.5 \times 8000 + \frac{4}{3} \times 8000 - 8000 \times \frac{4}{3} - 16,000 \bar{x} = 0 \]
\[ \bar{x} = 2.5 + \frac{4}{3} - \frac{4}{3} = 1.25 \text{ m.} \]

Eccentricity \( e = 1.5 - 1.25 = 0.25 \text{ m.} \)
Maximum and minimum stresses are given by
\[ p = \frac{W}{B} \left[ 1 \pm \frac{6e}{B} \right] \]
\[ -\frac{16,000}{3} \left[ 1 \pm \frac{6 \times 0.25}{3} \right] \]
\[ + \frac{16,000}{3} \left[ 1 \pm 0.5 \right] \]
Reservoir Empty

Taking moments about toe,

\[ W_1 \times 2.5 + W_2 \times \frac{4}{3} - W \times x = 0 \]

\[ 8000 \times 2.5 + 8000 \times \frac{3}{4} - 1600 \times x = 0 \]

\[ \frac{2.5}{2} + 11.5 \]

Eccentricity \( e = \frac{11.5}{6} - 1.5 = \frac{25}{6} \) m

Maximum and minimum stresses are given by

\[ W \left[ 1 \pm \frac{6e}{B} \right] \]

\[ \frac{16,000}{3} \left[ 1 \pm \frac{6 \times 2.5}{3} \right] \]

\[ \frac{16,000}{3} \left[ 1 \pm \frac{25}{3} \right] \]

\[ P_{\text{max}} = \frac{16,000}{3} \left[ 1 + \frac{25}{3} \right] = 8000 \frac{\text{kg}}{\text{m}^2} \]

\[ P_{\text{min}} = \frac{16,000}{3} \left[ 1 - \frac{25}{3} \right] = 8000 \frac{\text{kg}}{\text{m}^2} \]

Ex. 16.3. A dam 4 m. high and 1 m. top width has vertical water face. Find the bottom width of the dam if no tension is to develop at base. Density of masonry is 2000 kg/m².

If the co-efficient of friction is 0.6 and maximum allowable stress is 20 t/m², investigate the stability of the dam.

Solution

Consider one metre length of the dam. Let \( B \) be the breadth of the dam section at the base.

Weight of rectangular portion of dam = \( W_1 \)

\[ = 4 \times 1 \times 2000 = 8000 \text{ kg} \]

acting at \((B - 0.5)\) from toe.

Weight of rectangular portion of dam section

\[ = W_2 = \frac{(B - 1)}{2} \times 4 \times 2000 \]

\[ = 4000 \frac{(B - 1)}{3} \text{ acting at } \frac{3}{4}(B - 1) \text{ from toe.} \]
Total weight

\[ W_1 + W_2 = 8000 + 4000(B - 1) = 4000(B + 1) \]

Water pressure = \( P \)

\[ 1000 \times 4^2 = 8000 \text{ kg. acting at } 4 \text{ m. from base.} \]

Let \( \varepsilon \) be distance from toe where resultant of forces \( W \) and \( P \) strikes the base.

In the limiting case, for no tension to develop at base eccentricity \( \varepsilon \) should be equal to \( B/6 \).

\[ \frac{B}{2} - \varepsilon = \frac{B}{6} \]

\[ \varepsilon = B/3 \]

Taking moments about toe,

\[ W_1(B - 0.5) + W_2 \times \frac{2}{3} (B - 1) - P \times \frac{4}{3} - W \times \varepsilon = 0 \]

\[ 8000(B - 0.5) + 4000(B - 1) \times \frac{2}{3}(B - 1) - 8000 \times \frac{2}{3} - \left(\frac{4000(B+1)}{3}\right) \times \frac{B}{3} = 0 \]

\[ 8000B - 4000 + \frac{8000}{3}(B^2 - 2B + 1) - \frac{32000}{3} - \frac{8000B}{3} \]

\[ 4000 \frac{B^2}{3} - 4000B - 12000 = 0 \]

\[ B^2 + B - 9 = 0 \]

\[ B = \frac{-1 \pm \sqrt{1 + 36}}{2} \]

\[ B = \frac{-1 + \sqrt{37}}{2} = \frac{-1 + 6.1}{2} \]

\[ B = 2.55 \text{ m.} \]

Check for crushing

\[ W = 4000(B + 1) \]

\[ = 4000 \times 3.55 \]

\[ = 14200 \text{ kg.} \]

Maximum stress = \( \frac{2W}{B} = \frac{2 \times 14200}{2.55} \)

\[ = 11140 \text{ kg/m}^2 < 20 \text{ t/m}^2. \text{ Safe.} \]

Check for sliding

\[ \frac{\mu W}{P} = \frac{0.6 \times 14200}{8000} = 1.065 \]

\[ \therefore \text{ Factor of safety against sliding is } 1.065. \]
Ex. 16.4. A masonry dam 12 m. high and 3 m. top width is of trapezoidal section. The water face has slope of 1 : 6 to vertical and downstream face has slope of 1 : 4 to vertical. Investigate the stability of the dam if allowable compressive stress is 40 t/m² and coefficient of friction is 0.6. Water is retained in the dam up to height one metre below top. Density of masonry is 2000 kg/m³.

Solution. Consider one metre length of the dam. Stability will be checked both for reservoir full and reservoir empty conditions.

Reservoir full

Weight of upstream triangular portion of dam section

\[ W_1 = \frac{2 \times 12}{2} \times 2000 \]

\[ = 24,000 \text{ kg.} \]
acting at 6\(\frac{1}{2}\) m. from the toe.

Weight of rectangular portion of dam section.

\[ W_2 = 3 \times 12 \times 2000 \]

\[ = 72,000 \text{ kg.} \]
acting at 4.5 m. from toe.

Weight of downstream triangular portion of dam

\[ W_3 = \frac{1}{2} \times 12 \times 2000 \]

\[ = 36,000 \text{ kg.} \]
acting at 2 m. from the toe.

Weight of water in upstream triangular portion

\[ W_4 = \frac{1}{2} \times \frac{11}{6} \times 11 \times 1000 \]

\[ = 30,250 \text{ kg.} \]
acting at 7\(\frac{1}{18}\) m. from toe.

Pressure of water \( P = \frac{1000 \times 11^3}{2} \)

60,500 kg. acting at \(\frac{11}{3}\) m. from base.
Taking moments about toe,

\[ W_1 \times \frac{20}{3} + W_2 \times 4.5 + W_3 \times 2 + W_4 \times 7 - \frac{P \times 11}{18} - W \times \varepsilon = 0 \]

\[ 24,000 \times \frac{20}{3} + 72,000 \times 4.5 + 36,000 \times 2 + \frac{30,250}{3} \times \frac{133}{18} \]

\[ -60,500 \times \frac{11}{3} - \left( 24,000 + 72,000 + 36,000 + \frac{30,250}{3} \right) \varepsilon = 0 \]

\[ 160,000 + 324,000 + 72,000 + 74,500 - 221,833 - 142,083 \varepsilon = 0 \]

\[ \therefore \ 
\varepsilon = \frac{408,667}{142,083} 
= 2.877 \text{ m.} \]

Eccentricity \( e = 4 - \varepsilon \)

\[ = 4 - 2.877 \]

\[ 1.123 < \frac{B}{\delta} \]

**Check for crushing**

Maximum stress

\[ \frac{W}{B} \left( 1 + \frac{6\varepsilon}{B} \right) \]

\[ = \frac{142,083}{8} \left( 1 + \frac{6 \times 1.123}{8} \right) \]

\[ = \frac{142,083}{8} \times 1.842 \]

\[ = 32,710 \text{ kg/m}^2. < 40 \text{ t/m}^2. \text{ Safe.} \]

**Check for sliding**

\[ \mu W = 0.6 \times 142,083 = 85,250 \]

Factor of safety against sliding

\[ = \frac{\mu W}{P} \]

\[ = \frac{85,250}{60,500} \]

\[ = 1.409 \]

**Reservoir empty**

When reservoir is empty, weight \( W_4 \) and water pressure \( P \) will not act.

Taking moments about toe,

\[ W_1 \times \frac{20}{3} + W_2 \times 4.5 + W_3 \times 2 - (W_1 + W_2 + W_3) \varepsilon = 0 \]

\[ \therefore 24,000 \times \frac{20}{3} + 72,000 \times 4.5 + 36,000 \times 2 - 132,000 \varepsilon = 0 \]
\[
\tau = \frac{160,000 + 324,000 + 72,000}{132,000} = \frac{556,000}{132,000} = 4.288 \text{ m.}
\]

Eccentricity \( e = 4.288 - 4 = 0.288 < \frac{8}{8} \)

Maximum stress \[
= \frac{W}{B} \left( 1 + \frac{6e}{B} \right)
= \frac{132,000}{8} \left( 1 + \frac{6 \times 0.288}{8} \right)
= 16,500 \times 1.216
= 20,064 \text{ kg/m}^2. \text{ Safe.}
\]

**Ex. 16.5.** A gravity dam of trapezoidal section has a vertical water face 25 m. high and is 3 m and 12 m wide at the top and bottom respectively. It is intended to raise water by light shuttering erected on top of the dam in line with the vertical face. Determine to what extent above the crust of dam water can be stored if the resultant pressure on the dam is to cut the base at point not further than one-sixth of the bottom width from the centre of the bottom width? Density of masonry is 2240 kg/m³.

**Sol.** Let \( ABCD \) be the profile of the dam as shown in Fig. 16.10.

Let \( AE \) equal to \( a \) metres be the height of shuttering

![Diagram](image_url)

Consider one metre length of the dam.
If no tension is to develop, resultant of weight of dam and water pressure should cut the base at middle third i.e. 4 m from the toe.

**Weight of rectangular portion of dam section**

\[ W_1 = 15 \times 3 \times 1 \times 22 \times 2240 \]

\[ = 100,800 \text{ kg. acting at } 10.5 \text{ m from the toe.} \]

**Weight of triangular portion of dam**

\[ W_2 = 15 \times \frac{9}{2} \times 2240 \]

\[ = 151,200 \text{ kg. acting at } 6 \text{ m from the toe.} \]

**Total weight** = \( W_1 + W_2 = 252,000 \text{ kg.} \)

**Water pressure** \( P = \frac{100}{2} \times (15+a) \)

\[ = 5000 \times (15+a)^3 \]

Taking moments of all forces about toe,

\[ W_1 \times 10.5 + W_2 \times 6 - P \times \frac{15+a}{3} - W \times 4 = 0 \]

\[ 100,800 \times 10.5 + 151,200 \times 6 - 500 \times (15+a)^3 \times \frac{15+a}{3} \]

\[ - 252,000 \times 4 = 0 \]

\[ 100,800 \left[ 10 \frac{5}{2} \times 6 - \frac{5}{2} \times 4 \right] - \frac{500}{3} (15+a)^3 = 0 \]

\[ 100,800 \times 9.5 - \frac{500}{3} (15+a)^3 = 0 \]

\[ (15+a)^3 = 100,800 \times \frac{9.5 \times 3}{500} \]

\[ 15+a = \sqrt[3]{1008 \times 5.7} \]

\[ 15+a = 17.91 \]

\[ \therefore \quad a = 2.91 \text{ m.} \]

\[ \therefore \quad \text{Height of shuttering should not exceed } 2.91 \text{ m.} \]

**16.4. Retaining Walls**

A wall designed to maintain unequal level of ground on its two faces is called a retaining wall. The earth on the side of the wall where ground level is higher is called backfill and the retaining wall is to retain this earth. A retaining wall which supports the load of a bridge and also retains earth is called abutment. The walls placed at an inclination to the normal to
the direction of fill and used to retain the ends of an approach fill for a bridge are known as wing walls. (Fig. 16.11).

Various portions of the retaining wall with their names are illustrated in Fig. 16.12.
16·5. Types of Retaining Walls
Following types of retaining walls are generally used.

1. Gravity Walls.
This is a simplest type of retaining wall [Fig. 16·12 (a)]. These walls consist of mass of concrete or masonry. Main function of mass of concrete or masonry is to provide dead weight to give stability against thrust of retained earth. The tensile stresses in concrete or masonry are very low and these walls are so proportioned that there are no tensile stresses developed. Because of massive construction these retaining walls are more resistant to destructive agencies than other walls.

2. Semi gravity Walls.
Gravity retaining walls require greater toe to give sufficient base width so that no tension is developed at the base. Also in gravity retaining walls fairly heavy section of stem is required. By providing some reinforcement in toe and stem as shown in Fig. 16·13 the section of retaining wall can be reduced.

3. Cantilever Walls.
This type of retaining wall consists of three cantilever beams—vertical stem, toe projection and heel projection. In some cases toe projection may be missing Fig. 16·14 (b). Common types of cantilever walls are illustrated in Fig. 16·14. The types shown in Fig. 16·14 (a) and (b) are suitable for small heights. Fillets are sometimes provided at the junction of stem with heel and toe slabs for higher walls [Fig. 16·14 (c)].
The resistance to sliding of a cantilever type of retaining wall is sometimes increased by providing vertical projection known as key at the base [Fig. 16.14 (d)]. The key may be provided near toe, near heel or at the middle of base.

4. Counterfort retaining walls

The cantilever type of retaining walls are economical only upto height of about 6 m. to 8 m. However for higher walls the design of cantilever slabs, stem and heel will prove uneconomical. Greater economy can be achieved by providing counterforts as shown in Fig. 16.15. The greater economy is achieved in this case as the stem and heel are not designed as cantilevers but as continuous slabs over counterforts and thus bending moments are reduced. However in this type of retaining wall additional concrete, reinforcement and formwork is required for the counterforts.

Sometimes retaining walls with counterforts on the side of toe, in addition to counterforts on the side of the heel, are constructed but these types of walls are not commonly used.

5. Buttressed walls.

A buttressed wall is similar to a counterfort type of wall but in this case counterforts are placed on the front of the walls as shown in Fig. 16.16 and are known as buttresses rather than counterforts. In this type of wall heel projection is small and hence backfill contributes less to the stability of wall. Also buttresses reduce clearance in front of the wall. These types of walls are rarely used.

The magnitude of pressure exerted by earth depends on the state of equilibrium in the soil. Two states of equilibrium exist in soil; one is state of elastic equilibrium and other is state of plastic equilibrium. When shearing stresses in a soil mass are less than the shearing strength of soil, the soil is said to be in state of elastic equilibrium. When shearing stresses in a soil mass are equal to shearing strength of soil, the soil is said to be in a state of plastic equilibrium. In this state every soil particle is on the verge of failure.

The earth pressure exerted is divided in three categories i.e. earth pressure at rest, active earth pressure and passive earth pressure, depending on the lateral deformation of the soil.

Consider a semi-infinite soil mass which has been deposited artificially and which has remained undisturbed. This soil mass

![Diagram](image)

**Fig. 16-17**

is said to be in state of elastic equilibrium. On any plane \(X-X\), the pressure exerted at a depth \(h\) is \(p_0\) and is known as earth pressure at rest, \(p_0 = k_0wh\) where \(w\) is the density of soil and \(k_0\) is the coefficient of earth pressure at rest. The value of \(k_0\) generally lies between 0.4 to 0.5.

![Diagram](image)

**Fig. 16-18**

If the soil mass discussed above is allowed to stretch
laterally by sufficient amount, shearing stresses are developed along several planes and these shearing stresses are equal to shearing strength of soil. The shear planes are inclined at an angle of $45 + \phi/2$ as shown in Fig. 16.18. Under these conditions, the soil mass is said to be under state of plastic equilibrium and the soil is said to be in active state. The earth pressure exerted is known as active pressure. This state of pressure was investigated by Rankine and is known as Rankine’s active pressure.

Earth pressure $p_A$ at depth $h$ is given by

$$p_A = k_A wh$$

where $k_A$ is coefficient of active earth pressure.

If the soil mass discussed above is allowed to compress instead of stretching, shearing stresses are developed along several planes and these shearing stresses are equal to shearing strength of the soil. The shearing planes are inclined at an angle $45 - \phi/2$ to the horizontal as shown in Fig. 16.19. The whole soil mass is in state of plastic equilibrium. In this case, soil is said to be in passive state and the earth pressure exerted is known as passive earth pressure.

![Diagram](image)

Fig. 16.19

Earth pressure $p_P$ at a depth $h$ is given by

$$p_P = k_P wh$$

where $k_P$ is coefficient of passive earth pressure.

16.7. Theories of Earth Pressure

Two theories of earth pressure, Rankine’s theory of earth pressure, and Coulomb’s theory of earth pressure are used for computation of earth pressures.

Rankine’s Theory of earth pressure.

In computation of pressure by Rankine’s theory it is assumed that retaining wall yields by sufficient amount so that state of plastic equilibrium exists in soil mass in the immediate vicinity of the wall. The remaining soil is in the state of elastic equilibrium. In Rankine’s theory it is further assumed that there is no friction between soil and wall.
Consider semi-infinite soil mass. Consider a small element of soil at depth \( h \).

The forces acting on the soil mass are vertical pressure of soil \( p_1 = wh \), where \( w \) is the density of soil, and horizontal pressure \( p_2 \). Since there are no frictional forces, \( p_1 \) and \( p_2 \) are principal stresses.

Shearing resistance of non-cohesive soil is given by \( s = \rho \tan \phi \) where \( \rho \) is the normal stress on the surface of sliding and \( \phi \) is the angle of internal friction.

This is represented graphically by lines \( OP \) and \( OQ \) in Fig. 16:20 (b) and (c).

When Mohr’s circle for principal stresses \( p_1 \) and \( p_2 \) is drawn, if the Mohr’s circle does not cut the lines \( OP \) and \( OQ \), there is no plane in the element considered where frictional resistance of the soil is exceeded and hence failure cannot take place. On the other hand, if the Mohr’s circle cuts line \( OP \) and \( OQ \), this represents a case when shearing resistance on the planes defined by points above lines \( OP \) and \( OQ \) has exceeded the shearing resistance of soil, which is not possible. Hence in the limiting case Mohr’s circle should just touch lines \( OP \) and \( OQ \).

Two cases arise, when \( p_1 = wh \) is major principal stress and \( p_2 \) is minor principal stress, this represents Rankine’s active state. In the second case, when \( p_2 \) is major principal stress and \( p_1 = wh \) is minor principal stress, this represents Rankine’s passive state.

**Rankine’s Active State**

From Fig. 16:20 (b),

\[
\sin \phi = \frac{\rho C}{OC} = \frac{p_1 - \rho}{p_1 + \rho} = \frac{p_1 - \rho}{\rho + \rho} = \frac{p_1 - \rho}{\rho p_2 + \rho}.
\]
\[ \frac{1 - \sin \varphi}{1 + \sin \varphi} \cdot \frac{P_1 + P_2 - (P_3 - P_3)}{P_1 + P_2 + P_3 - P_1} = \frac{P_3}{P_1} \]

\[ P_3 = P_1 \left( \frac{1 - \sin \varphi}{1 + \sin \varphi} \right) \]

\[ = \left( \frac{1 - \sin \varphi}{1 + \sin \varphi} \right) wh \]

\[ = k_A wh \]

where \( k_A \) is coefficient of active earth pressure.

**Fig. 16.21**

Total earth pressure at depth \( h \) is given by

\[ P_A = \frac{1}{2} h \times wh \left( \frac{1 - \sin \varphi}{1 + \sin \varphi} \right) \]

\[ = \frac{wh}{2} \left( \frac{1 - \sin \varphi}{1 + \sin \varphi} \right) \]

and will act at \( h/3 \) from base.

**Rankine Passive State**

From Fig. 16.20 (c),

\[ \sin \varphi = \frac{a'C'}{OC} \]

\[ P_3 - P_1 = \frac{2}{P_1 + P_2} \cdot \frac{P_3 - P_3}{P_1 + P_2} \]

\[ \frac{1 + \sin \varphi}{1 - \sin \varphi} = \frac{P_1 + P_2 + P_3 - P_1}{(P_1 + P_2 - (P_3 - P_1))} \]

\[ = \frac{P_3}{P_1} \]

\[ \therefore \]

\[ P_3 = P_1 \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right) \]

\[ = \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right) wh \]

\[ = k_P wh \]
$k_p = \text{coefficient of passive earth pressure}$

$$= \frac{1 + \sin \phi}{1 - \sin \phi}$$

**Fig. 16.22**

Total earth pressure up to depth $h$ is given by

$$P_p = \frac{1}{2} \times wh \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right) \times h$$

$$= \frac{wh^2}{\phi} \left( \frac{1 + \sin \phi}{1 - \sin \phi} \right)$$

and will act at $h/3$ from base.

**Surcharged Walls**

When the earth at the back of the wall does not have horizontal surface but sloping, the wall is said to be surcharged.

**Fig. 16.23 (a)**

The maximum inclination that sloping surface can have is the angle of repose $\phi$.

Consider retaining wall with earth surcharged at an angle $\alpha$. Consider the equilibrium of a small sphereloped $UVWX$ at depth $h$. The pressures $p_r$ on the faces $UV$ and $XW$ are vertical weight of earth above it equal to $w h$ and this is parallel to faces
$UX$ and $VW$. Therefore the resultant stresses $p_h$ on the faces $UX$ and $VW$ must be parallel to faces $UV$ and $XW$. The stresses $p_v$ and $p_h$ are conjugate. These stresses are not principal stresses. Let $p_1$ and $p_2$ be the corresponding principal stresses.

Suppose values of $p_h$ and $p_v$ are known. Set out $OP$ and $OQ$ equal to $p_h$ and $p_v$ respectively at an angle $\alpha$ to the horizontal $OX'$. $PQ$ is bisected at $R$ and $RC$ is drawn perpendicular to $PQ$, cutting horizontal $OX'$ at $C$. A circle is drawn with $C$ as centre and passing through $P$ and $Q$. $Ob$ and $Od$ represent principal stresses $p_h$ and $p_1$ respectively.

\[
OC = \frac{p_1 + p_2}{2}, \quad Ca = CP = CQ = \frac{p_1 - p_2}{2}
\]

\[
OC = \frac{p_1 + p_2}{2} = p_h + p_v - \frac{1}{2} \quad \text{(1)}
\]

\[
CQ^2 = CR^2 + RQ^2 \quad OC^2 - OR^2 + RQ^2
\]

\[
CQ^2 = \left(\frac{p_h + p_v}{2 \cos \alpha}\right)^2 - \left[\left(\frac{p_h + p_v}{2}\right)^2 - \left(\frac{p_h + p_v}{2}\right)^2\right]
\]

\[
\left(\frac{p_h + p_v}{2 \cos \alpha}\right)^2 - p_h p_v
\]

But

\[
CQ = \frac{p_1 - p_2}{2}
\]

\[
\therefore \quad \frac{p_1 - p_2}{2} = \sqrt{\left(\frac{p_h - p_v}{2 \cos \alpha}\right)^2 - p_h p_v} \quad \text{(2)}
\]

Squaring (2) and (1) and dividing

\[
\frac{\left(\frac{p_h + p_v}{2 \cos \alpha}\right)^2 - p_h p_v}{\left(\frac{p_h + p_v}{2 \cos \alpha}\right)^2} = \left(\frac{p_1 - p_2}{p_1 + p_2}\right)^2
\]

\[
\therefore \quad 1 - \frac{4p_h p_v \cos^2 \alpha}{(p_h + p_v)^2} = \left(\frac{p_1 - p_2}{p_1 + p_2}\right)^2
\]
\[ \frac{p_1 - p_2}{p_1 + p_2} = \frac{aG}{\gamma'G} = \sin \phi \]

\[ \frac{4p_a p_v \cos^2 \alpha}{(p_a + p_v)^2} = \sin^2 \alpha \]

\[ \frac{4p_a p_v \cos^3 \alpha}{(p_a + p_v)^2} = 1 - \sin^2 \phi = \cos^2 \phi \]

\[ (p_a + p_v)^2 = \frac{4p_a p_v \cos^2 \alpha}{\cos^2 \phi} \]  \hspace{1em} (1)

Subtract 4p_a p_v from both sides.

\[ (p_a - p_v)^2 = 4p_a p_v \left( \frac{\cos^2 \alpha}{\cos^2 \phi} - 1 \right) \]

\[ = 4p_a p_v \left( \frac{\cos^2 \alpha - \cos^2 \phi}{\cos^2 \phi} \right) \]  \hspace{1em} (4)

Eq. 4 \[ \frac{p_a - p_v}{p_a + p_v} \] gives

\[ \left( \frac{p_a - p_v}{p_a + p_v} \right)^3 = \frac{\cos^2 \alpha - \cos^2 \phi}{\cos^2 \alpha} \]

\[ p_a - p_v = \pm \frac{\sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha} \]

\[ p_a = \frac{\cos \alpha \pm \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \]

\[ p_v = \frac{\cos \alpha \mp \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \]

\[ \eta = \frac{\cos \alpha \pm \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \]

In the Rankine active state, \( p_a \) is minor stress and \( p_v \) is major stress.

\[ \eta = p_a \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}} \]

In the Rankine passive state, \( p_a \) is major stress and \( p_v \) is minor stress.

\[ \eta = p_v \frac{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}} \]

\[ p_v = wh \cos \alpha, \text{ as the parallelepiped considered is of unit width, horizontal component of width is } 1 \times \cos \phi. \]

\[ : \]

\[ p_a = \frac{wh \cos \alpha}{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}, \text{ for } \cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi} \]

Active Rankine state

\[ p_a = \frac{wh \cos \alpha}{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}, \text{ for } \cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi} \]

Passive Rankine state.

It is assumed in the Rankine's theory of earth pressure that the back of retaining wall is smooth and thus there is no friction between wall and earth. This assumption is however seldom justified as the back of the retaining wall is never smooth and thus the earth pressures computed on the basis of Rankine's theory lead to erroneous results. Coulomb's theory can be applied to any boundary conditions. In the Coloumb's theory it is assumed that the retaining wall yields by sufficient amount and causes state of plastic equilibrium in soil. This is also the assumption made in Rankine's theory. In the Coloumb's theory it is further assumed that surface of sliding is plane though in reality the surface of sliding is slightly curved:

However the error due to assuming that the surface of sliding is plane is negligible for lower values of $\varphi$.

Active Earth Pressure

Notations, $\varphi =$ Angle of internal friction of soil.
$\delta =$ Angle of friction between retaining wall and earth.
$\alpha =$ Angle of surcharge.
$\theta =$ Angle made by back of wall with vertical.

Intensity of active earth pressure at a depth $h$ is given by

$$p_A = wh \cos \theta \times k_A$$

where $k_A = \frac{\cos^2 (\varphi - \theta)}{\cos^2 \theta \cos (\delta + \theta) \left[ 1 + \sqrt{\frac{\sin(\delta + \varphi)\sin (\varphi - \alpha)}{\cos(\delta + \theta)\cos (\theta - \alpha)}} \right]^2}$

Total pressure upto depth $h$ is given by

$$P_A = \frac{1}{2} wh^2 k_A - (\text{Inclined length} = \frac{h}{\cos \theta})$$

The resultant earth pressure acts at lower middle third point $L = \frac{h}{2}$ at an angle $\delta$ to the face of the wall as shown in
Passive Earth Pressure.

Intensity of passive earth pressure at a depth 'h' is given by

\[ p_F = wh \cos \theta \, k_F \]

and total pressure upto h is given by

\[ P_F = \frac{1}{2} wh^2 \, k_F \left( \text{Inclined length} = \frac{h}{\cos \theta} \right) \]

where \( k_F = \frac{\cos^2 (\phi + \delta)}{\cos^2 \theta \cos(\theta - \delta) \left[ 1 - \sqrt{\frac{\sin (\phi + \delta) \sin (\phi + \alpha)}{\cos (\theta - \delta) \cos (\theta - \alpha)}} \right]^{\frac{1}{2}}} \)

The resultant pressure acts at lower middle third point and is inclined at angle \( \delta \) to the face of the wall as shown in Fig. 16.25.

![Diagram of passive earth pressure](image)

Fig. 16.25


![Graphical representation](image)

Fig. 16.26
By Rehmann's construction, total active earth pressure against a retaining wall can be computed based on Coulomb's theory. The construction is illustrated in Fig. 16·26.

Let $AB$ be the back of the wall, $\phi$ the angle of internal friction of soil, $\alpha$ the angle of surcharge and $\delta$ the angle of friction between soil and retaining wall.

Through $A$ draw a horizontal line $AD$. Draw a line $AC$ at an angle $\phi$ to the horizontal meeting ground surface at $C$. Describe a semicircle with $AC$ as diameter. Through $B$ draw a line $BE$ making angle $\delta + \phi$ with $AB$ and meeting $AC$ in $E$. At $E$ draw a perpendicular to $AC$ cutting semicircle at $F$. With $A$ as centre and $AF$ as radius draw an arc cutting $AC$ at $R$. Draw $RP$ parallel to $BE$ cutting $BC$ at $P$. With $R$ as centre and $RP$ as radius draw an arc cutting $AC$ in $Q$. Join $PQ$ and $AP$.

Area of triangle $PQR$ multiplied by unit weight of soil represents total earth pressure against the wall. $AP$ represents plane of sliding.

Rehmann's construction for particular case when $\alpha = \phi$ is illustrated in Fig. 16·27.

In this case the line $AC'$ drawn through $A$ at an angle $\phi$ will be parallel to ground level as $\alpha = \phi$.

![Fig. 16·27](image)

Through $B$ draw a line at an angle $\delta + \phi$ 'meeting $AC'$ in $E$. Select any point $P$ on the ground surface and through this point draw a line $PR$ parallel to $BE$ meeting $AC'$ in $R$. With $R$ as centre and $RP$ as radius draw an arc cutting $AC'$ in $Q$.

Join $PQ$. Area of triangle $PQR$ multiplied by unit weight of soil represents total earth pressure against the wall.

16·10. Earth Pressure due to Submerged Soil.

When the pores of permeable soil are filled with water, the water exerts full hydrostatic pressure. The pressure of water
causes buoyant effect on the soil with the result that effective weight of soil which causes earth pressure is reduced by an amount equal to weight of water displaced by soil. The reduced weight is called submerged weight.

Let the submerged weight of soil be \( w_1 \) and unit weight of water be \( w_w \). Pressure at depth \( h \) exerted by submerged soil will be \( P_s = w_w h k \) where \( k \) is coefficient of earth pressure. Pressure exerted by water at depth \( h \) is \( P_w = w_w h \).

Pressure against the base of a retaining wall for various cases of submerged soil is shown in Fig. 16.28.

\[ P_w = \frac{1}{2} w_w h^2 \]
\[ P_s = \frac{1}{4} w_1 h^3 k. \]

(a) Water level above ground surface

(b) Ground surface above water level

16.11. Minimum Depth of Foundation.

By Rankine theory of earth pressure, it is possible to get formula for minimum depth of foundation.

Consider a small cube under the foundation. The vertical downward pressure of the footing is considered as major principal stress and lateral pressure \( P_2 \) will be Rankine earth pressure.

\[ P_2 = P_1 \frac{1 - \sin \varphi}{1 + \sin \varphi} \quad \ldots (1) \]

Next consider another small cube, adjacent to the cube considered previously but outside the foundation area. The lateral intensity of pressure transmitted to this cube is \( P_2 \). For this cube this pressure is considered major principal stress and hence the vertical weight of soil on this cube will be minor principal stress.

\[ \therefore w_h = P_2 \frac{1 - \sin \varphi}{1 + \sin \varphi} \]
Substituting \( p \) from (1)

\[
wh = p_1 \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2
\]

\[
:\therefore \quad h = \frac{p_1}{w} \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2
\]

If \( W \) is weight of structure and \( A \) is area of foundation,

\[
p_1 = \frac{W}{A}
\]

\[
:\therefore \quad h = \frac{W}{Aw} \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right)^2
\]

This is the minimum depth of foundation required.

**Ex. 16.6.** A masonry retaining wall, 6 m. high, has vertical face on the earth side and retains earth which has level surface. The wall has top width of 1 m and bottom width 3 m. The weight of earth is 1600 kg/m\(^3\) and angle of repose 30\(^\circ\). The weight of masonry is 2200 kg/m\(^3\). Investigate the stability of the wall. Co-efficient of friction = 0.5.

**Solution.** Co-efficient of active earth pressure

\[
k_A = \frac{1 - \sin \phi}{1 + \sin \phi}
\]

\[
= \frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{3}
\]

Weight of rectangular portion

\[
W_1 = 1 \times 6 \times 2200
\]

\[
= 13,200 \text{ kg. acting at } 2.5 \text{ m. from toe.}
\]

Weight of triangular portion

\[
W_2 = \frac{2 \times 6}{2} \times 2200
\]

\[
= 13,200 \text{ kg. acting at } \frac{4}{3} \text{ m. from toe.}
\]

Total weight \( W = W_1 + W_2 = 13,200 + 13,200 = 26,400 \text{ kg.}
\]

Total earth pressure

\[
= \frac{wh^2}{2} \times \frac{1 - \sin \phi}{1 + \sin \phi}
\]

\[
= \frac{1600 \times 6 \times 6}{2} \times \frac{1}{3}
\]

\[
= 9600 \text{ kg. acting at 2 m. from base.}
\]

Taking moments about toe,

\[
W_1 \times 2.5 + W_2 \times \frac{4}{3} - P \times \frac{H}{3} - W \times 2 = 0
\]
\[
2.5W_1 + \frac{4}{3}W_2 - \frac{PH}{3} = \frac{26,400}{W}
\]
\[
2.5 \times 13,200 + \frac{4}{3} \times 13,200 - 9600 \times 2
\]
\[
33,000 + 17,600 - 19,200 = \frac{26,400}{31,400} = 1.19 \text{ m.}
\]

Eccentricity \( e \)
\[
e = 1.5 - 1.19 = 0.31 \text{ m.}
\]

Pressure at base
\[
\frac{W}{B} \left( 1 \pm \frac{6e}{B} \right)
\]
\[
\frac{26,400}{3} \left( 1 \pm \frac{6 \times 0.31}{3} \right)
\]
\[
P_{\text{max}} = 8800(1 + 0.62) = 14,260 \text{ kg/m}^2.
\]
\[
P_{\text{min}} = 8800(1 - 0.62) = 8800 \times 0.38 = 3344 \text{ kg/m}^2.
\]

Factor of safety against sliding
\[
\frac{\mu W}{P}
\]
\[
= 0.5 \times \frac{26,400}{9600}
\]
\[
= \frac{13,200}{9,600}
\]
\[
= 1.375.
\]

Ex. 16:7. A retaining wall 8 m. high, 1 m. wide at top and 3.4 m. wide at base has vertical face retaining earth at a surcharge of 20° to the horizontal. If the density of earth is 1800 kg/m³, with an angle of repose of 30°, calculate the earth pressure per metre length of wall by Rankine formula. If the density of masonry is 2400 kg/m³, calculate the normal stress distribution at base. Also calculate factor of safety against sliding, if co-efficient of friction is 0.6.

Solution. Co-efficient of active earth pressure
\[
\cos \alpha \times \frac{\cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \phi}}{\cos \alpha + \sqrt{\cos^2 \alpha - \cos^2 \phi}}
\]
\[
\cos 20 \times \frac{\cos 20 - \sqrt{\cos^2 20 - \cos^2 30}}{\cos 20 + \sqrt{\cos^2 20 - \cos^2 30}}
\]
Consider 1 m, length of wall.
Total earth pressure against the wall

\[
\frac{wH^2}{2} \times k_A \\
1800 \times 8 \times 8 \times 0.4145
\]

\[= 23,880, \text{ acting at } \frac{3}{8} \text{ m. from base.}\]

Horizontal component of earth pressure

\[P_H = P \cos \alpha\]

\[= 23,880 \times 0.9397\]

\[= 22,440 \text{ kg.}\]

Vertical component of earth pressure

\[P_V = P \sin \alpha = 23,880 \times 0.342\]

\[= 8,166 \text{ kg.}\]

Weight of rectangular portion of wall section

\[= W_1 = 8 \times 1 \times 1 \times 2400 = 19,200 \text{ kg., acting at 2.9 m. from toe.}\]

Weight of triangular portion of wall section

\[= W_2 = \frac{1}{2} \times 2.4 \times 8 \times 2400 = 23,040 \text{ kg., acting at 1.6 m from toe.}\]

Taking moments about toe,

\[W_1 \times 2.9 + W_2 \times 1.6 + P_V \times 3.4 = P_H \times \frac{8}{3} - W \times \bar{z} = 0\]

\[\therefore \quad \bar{z} = \frac{19,200 \times 2.9 + 23,040 \times 1.6 + 8166 \times 3.4 - 22,440 \times \frac{8}{3}}{19,200 + 23,040 + 8166}\]

\[= \frac{55,680 + 36,864 + 27,764 - 59,840}{50,406}\]

\[= \frac{60,408}{50,406}\]

\[= 1.20 \text{ m.}\]

Eccentricity \(\epsilon = \frac{B}{2} - \bar{z} = \frac{3.4}{2} - 1.2\)

\[= 0.5 \text{ m.}\]

Maximum and minimum pressures at base are given by

\[p = \frac{W}{B} \left( 1 \pm \frac{\epsilon}{B} \right)\]
\[ P = \frac{50.406}{3.4} \left(1 \pm 0.5 \times 1.8822\right) \]

\[ P_{\text{max}} = \frac{50.406}{3.4} \times 1.8822 = 27,900 \text{ kg/m}^2. \]

\[ P_{\text{min}} = \frac{50.406}{3.4} \times 0.1178 = 1746 \text{ kg/m}^2. \]

Factor of safety against sliding

\[ = \frac{\mu W}{P} \]

\[ = 0.6 \times \frac{50.406}{22.440} \]

\[ = 1.348. \]

**Ex. 16.8.** Fig. 16.32 (a) shows the section of a gravity retaining wall, retaining earth having density of 1600 kg/m\(^3\) and angle of repose 30°. The weight of masonry is 2400 kg/m\(^3\). Find pressure distribution at base and factor of safety against sliding if coefficient of friction is 0.3.

**Solution.** Consider one metre length of retaining wall.

Co-efficient of active earth pressure

\[ = \frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{3}. \]

Pressure is considered to act on the vertical through last step. The retaining wall will be considered to be consisting of masonry and earth column above the stepped portions.
\[ W_1 = 2 \times 6 \times 2400 = 28,800 \text{ kg, acting at 1-0 m. from toe;} \]
\[ W_2 = 1 \times 4 \times 2400 + 1 \times 2 \times 1600 = 12,800 \text{ kg, acting at 2-5 m. from toe} \]
\[ W_3 = 1 \times 2 \times 2400 + 1 \times 4 \times 1600 = 11,200 \text{ kg, acting at 3-5 m. from toe} \]
\[ W = W_1 + W_2 + W_3 = 52,800 \text{ kg.} \]

Total earth pressure \( P = k_A \frac{wH^2}{3} \times 1600 \times \frac{6}{2} = 9600 \text{ kg, acting at 2 m. from base.} \)

Let the resultant force cut the base at \( z \) distance from toe.

Taking moments about toe,
\[ W_1 \times 1 + W_2 \times 2.5 + W_3 \times 3.5 - P \times 2 - W \times z = 0 \]
\[ 28,800 \times 1 + 12,800 \times 2.5 + 11,200 \times 3.5 - 9600 \times 2 - 52,800z = 0 \]
\[ z = \frac{28,800 + 32,000 + 39,200 - 19,200}{52,800} = \frac{80,000}{52,800} = 1.531 \text{ m.} \]

Eccentricity \( e = \frac{4}{5} - 1.531 = 0.469 \text{ m.} \)

Maximum and minimum pressures at base are given by
\[ \frac{W}{B} \left( 1 \pm \frac{6e}{B} \right) \]
\[ = \frac{52,800}{4} \left( 1 \pm \frac{6 \times 0.469}{4} \right) \]
\[ = 13,200 (1 \pm 0.7035) \]
\[ P_{\text{max.}} = 13,200 \times 1.7035 = 22,490 \text{ kg/m}^2. \]
\[ P_{\text{min.}} = 13,200 \times 0.2965 = 3913 \text{ kg/m}^2. \]

Factor of safety against sliding
\[ = \frac{\mu W}{P} \]
\[ = \frac{0.3 \times 52,800}{9600} \]
\[ = 1.584 \]
\[ = 1.65. \]

Ex. 16-9. Fig. 16-33 shows an R.C.C. retaining wall. The earth retained has density of 1800 kg/m\(^3\) and angle of repose of 30\(^\circ\). If the earth fill is level with the top of the wall and carries a super-imposed load of 900 kg/m\(^3\), find the dimension \( B \) if no tension is to develop at the base. Density of R.C.C. = 2400 kg/m\(^3\).

Solution. Coefficient of active earth pressure
\[ k_A = \frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{2} \]
DAMS, RETAINING WALLS AND CHIMNEYS

Effect of surcharge of 900 kg/m² is to give horizontal pressure on wall $= 900 \times \text{coefficient of active earth pressure} = 900 \times \frac{1}{2} = 300 \text{ kg/m}^2$.

![Diagram of earth pressure on a retaining wall]

Fig. 16·33

The pressure is constant throughout.

Weight of different portions, pressures and moments about $P$ are given in Table 16·1.

**TABLE 16·1**

<table>
<thead>
<tr>
<th>No.</th>
<th>Item</th>
<th>Force Vertical</th>
<th>Force Horizontal</th>
<th>Lever arm</th>
<th>Moment about $P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3\cdot5 \times (B-0.5) \times 1800$</td>
<td>6300 $(B-0.5)$</td>
<td>$(B-0.5)^2/2$</td>
<td>$3150 \ (B-0.5)^2$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>900 $(B-0.5)$</td>
<td>900 $(B-0.5)$</td>
<td>$(B-0.5)^2/2$</td>
<td>$450 \ (B-0.5)^2$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$0.5 \times (B-0.5) \times 2400$</td>
<td>1200 $(B-0.5)$</td>
<td>$(B+0.5)^2/2$</td>
<td>$800 \ (B-0.5)^2$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$4 \times 0.5 \times 2400$</td>
<td>4800</td>
<td>$(B-0.25)$</td>
<td>$4800 \ (B-0.25)$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>300 $\times 4$</td>
<td>1200</td>
<td>2</td>
<td>$2400$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$\frac{1}{2} \times 2400 \times 4$</td>
<td>4800</td>
<td>$\frac{1}{2}$</td>
<td>$1200$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Sigma$</td>
<td>$8400(B-0.5)$</td>
<td></td>
<td>$4200 \ (B-0.5)^2 + 4800(B-0.25) + 800$</td>
<td></td>
</tr>
</tbody>
</table>

$\Sigma \frac{4200(B-0.5)^2 + 4800(B-0.25) + 800}{8400(B-0.5) + 4800}$
\[ W_1 = 2 \times 6 \times 2400 = 28,800 \text{ kg, acting at 1.0 m from toe}; \]
\[ W_2 = 1 \times 4 \times 2400 + 1 \times 2 \times 1600 = 12,800 \text{ kg, acting at 2.5 m from toe}; \]
\[ W_3 = 1 \times 2 \times 2400 + 1 \times 4 \times 1600 = 11,200 \text{ kg, acting at 3.5 m from toe}; \]
\[ W = W_1 + W_2 + W_3 = 52,800 \text{ kg}. \]

Total earth pressure \[ P = k_a \frac{1}{3} \times 1600 \times \frac{6^3}{2} \approx 9600 \text{ kg, acting at 2 m from base.} \]

Let the resultant force cut the base at \( x \) distance from toe.

Taking moments about toe,
\[ W_1 \times 1 + W_2 \times 2.5 + W_3 \times 3.5 - P \times x - W_3 \times 2 = 0 \]
\[ 28,800 \times 1 + 12,800 \times 2.5 + 11,200 \times 3.5 - 9600 \times 2 - 52,800x = 0 \]
\[ x = \frac{28,800 + 32,000 + 39,200 - 19,200}{52,800} \]
\[ x = \frac{80,800}{52,800} = 1.531 \text{ m.} \]

Eccentricity \( e = \frac{4}{2} - 1.531 \]
\[ e = 2 - 1.531 = 0.469 \text{ m.} \]

Maximum and minimum pressures at base are given by
\[ \frac{W}{B} \left( 1 \pm \frac{6e}{B} \right) \]
\[ = \frac{52,800}{4} \left( 1 \pm \frac{6 \times 0.469}{4} \right) \]
\[ = 13,200 \left( 1 \pm 0.7035 \right) \]
\[ P_{\text{max}} = 13,200 \times 1.7035 = 22,490 \text{ kg/m}^2. \]
\[ P_{\text{min}} = 13,200 \times 0.2965 = 3913 \text{ kg/m}^2. \]

Factor of safety against sliding
\[ = \frac{\mu W}{P} \]
\[ = \frac{0.3 \times 52,800}{9600} \]
\[ = 15.840 \]
\[ = \frac{9600}{9600} \]
\[ = 1.65. \]

Ex. 16.9. Fig. 16.33 shows an R.C.C. retaining wall. The earth retained has density of 1800 kg/m\(^3\) and angle of repose of 30\(^\circ\). If the earth fill is level with the top of the wall and carries a superimposed load of 900 kg/m\(^3\), find the dimension \( B \) if no tension is to develop at the base. Density of R.C.C. = 2400 kg/m\(^3\).

Solution. Coefficient of active earth pressure
\[ k_A = \frac{1 - \sin 30}{1 + \sin 30} = 0.5 \]
Effect of surcharge of 900 kg/m² is to give horizontal pressure on wall

\[ = 900 \times \text{coefficient of active earth pressure} \]

\[ = 900 \times \frac{1}{3} = 300 \text{ kg/m}^2. \]

---

Fig. 16.33

The pressure is constant throughout.

Weight of different portions, pressures and moments about \( P \) are given in Table 16.1.

**TABLE 16.1**

<table>
<thead>
<tr>
<th>No.</th>
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<tr>
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<td>( 3.5 \times (B-0.5) \times 1800 )</td>
<td>6300 ( (B-0.5) )</td>
<td>( \frac{B-0.5}{2} )</td>
<td>( 3150 \ (B-0.5)^3 )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>900 ( (B-0.5) )</td>
<td>900 ( (B-0.5) )</td>
<td>( B-0.5 )</td>
<td>( 450 \ (B-0.5)^3 )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.5 ( (B-0.5) \times 2400 )</td>
<td>1200( (B-0.5) )</td>
<td>( \frac{B+0.5}{2} )</td>
<td>( 500 \ (B-0.5)^3 )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( 4 \times 0.5 \times 2400 )</td>
<td>4800</td>
<td>( (B-0.25) )</td>
<td>( 4800 \ (B-0.25) )</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>300 \times 4</td>
<td>1200</td>
<td>2</td>
<td>2400</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>( \frac{1}{2} \times 2400 \times 4 )</td>
<td>4800</td>
<td>( \frac{1}{6} )</td>
<td>6400</td>
<td></td>
</tr>
</tbody>
</table>

\[ \Sigma \]

\[ = 3400 (B-0.5) + 4800 \]

\[ = 4200 (B-0.5)^2 + 4800(B-0.25) + 8800 \]

\[ \varepsilon = \frac{4200 (B-0.5)^2 + 4800(B-0.25) + 8800}{3400(B-0.5) + 4800} \]
Eccentricity \[ e = \frac{R}{2} \]

For no tension to develop, \[ e = \frac{B}{6} \]

\[ \therefore \quad x = e + \frac{B}{2} = \frac{2B}{3} \]

\[ \therefore \quad \frac{4200(B-0.5)^2 + 4800 (B-0.25) + 8800}{8400 (B-0.5) + 4800} = \frac{2}{3} B \]

\[ 21(B-0.5)^2 + 24 (B-0.25) + 44 = \frac{1}{3} B [42 (B-0.5) + 24] \]

\[ 63 (B^2 + \frac{1}{3} B + \frac{1}{3}) + 72 (B-0.25) + 132 = 84 B (B-0.5) + 48B \]

\[ 63B^2 + 63B + 15.75 + 72B-18+132 = 84B^2 - 42B + 48B \]

\[ 21B^2 - 3B = 129.75 \]

\[ 7B^2 - B = 43.25 \]

\[ B^2 - 0.143B = 6.18 \]

\[ (B-0.0715)^2 = 6.18 + 0.0715^2 \]

\[ = 6.23 \]

\[ \therefore \quad B = 2.496 + 0.0715 \]

\[ = 2.5675 \text{ m.} \]

**Ex. 16.10.** Fig. 16.34 (a) shows a section of gravity retaining wall. In the top 4 m soil is dry and below this level soil is water logged. Calculate the maximum pressure on the base of the wall. Density of dry soil is 1600 kg/m³ and density of submerged soil is 1100 kg/m³ and angle of repose of soil is 30°. Density of masonry is 2400 kg/m³.

**Solution.** Consider one metre length of the wall.

In the top 4 m of the wall, pressure on wall will be due to earth. It will vary from zero at top to \( \frac{1}{3} \times 1600 \times 4 \) at depth of
In the bottom 4 m. of the wall pressure will be due to submerged weight of soil, due to water and due to earth in top 4 m. of the soil. Pressure diagram is shown in Fig. 16.34.

\[ W_1 = 1 \times 8 \times 2400 = 19,200 \text{ kg. acting at } 4.5 \text{ m from toe} \]

\[ W_2 = \frac{4}{3} \times 8 \times 2400 = 38,400 \text{ kg. acting at } \frac{4}{3} \text{ m from toe} \]

\[ W = W_1 + W_2 = 57,600 \text{ kg.} \]

\[ P_1 = \frac{1}{3} \times 600 \times 4 \times \frac{4}{3} = \frac{12,800}{3} \text{ kg. acting at } \frac{1}{3} \text{ m from base} \]

\[ P_3 = \frac{1}{3} \times 1600 \times 4 \times \frac{4}{3} = \frac{25,600}{3} \text{ kg. acting at } 2 \text{ m from base.} \]

\[ P_3 = \frac{1}{3} \times 1100 \times 4 \times \frac{4}{3} = \frac{8,800}{3} \text{ kg. acting at } \frac{4}{3} \text{ m from base.} \]

\[ P_4 = 1000 \times 4 \times \frac{4}{3} = 8000 \text{ kg. acting at } \frac{4}{3} \text{ m from base.} \]

Let the resultant of horizontal pressure and weight strike the base at distance \( x \) from the toe.

Taking moments about toe.

\[ \frac{1}{5} W_1 + \frac{8}{3} W_2 - P_1 \times \frac{16}{3} - P_3 \times 2 - P_4 \times \frac{4}{3} - W_2 \times x = 0 \]

\[ 4.5 \times 19,200 + \frac{8}{3} \times 38,400 - \frac{12,800}{3} \times \frac{16}{3} - \frac{25,600}{3} \times 2 \]

\[ \frac{8,800}{3} \times \frac{4}{3} - 8000 \times \frac{4}{3} - 57,600 \times x = 0 \]

\[ 86,000 + 102,400 - \frac{204,800}{9} - \frac{51,200}{3} - \frac{35,200}{9} - \frac{32,000}{3} \]

\[ = \frac{134,133}{57,600} \]

\[ = 2.329 \text{ m.} \]

Eccentricity \( e = \frac{0}{2} - 2.329 = 0.171 \text{ m.} \)

Maximum pressure \( = \frac{W}{B} \left( 1 + \frac{6e}{B} \right) \)

\[ = \frac{57,600}{5} \left( 1 + \frac{6 \times 0.171}{5} \right) \]

\[ = 11,520 \times 1.2052 \]

\[ = 13,880 \text{ kg/m}^2. \]
Ex. 16.11. Fig. 16.35 shows the section of gravity retaining wall, retaining cohesionless soil having angle of repose 30°. In the top 3 m. the density of soil is 1500 kg/m³, below this level the density is 1800 kg/m³. The wall weighs 2400 kg/m³.

(a) Find the magnitude and point of application of the resultant earth pressure on the wall per linear metre, neglecting the wall friction.

(b) Calculate the maximum bearing pressure at the base of the wall, assuming linear pressure distribution.

(c) Investigate stability of the wall against sliding if the coefficient of friction is 0.4.

Take into account passive resistance of soil in front of the wall.

![Diagram showing the section of gravity retaining wall with calculations for earth pressure and stability analysis.]

**Solution.** Coefficient of active earth pressure

\[ k_a = \frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{3} \]

Coefficient of passive earth pressure

\[ k_p = \frac{1 + \sin 30}{1 - \sin 30} = 3. \]

Pressure diagram due to soil on front and back of wall shown in Fig. 16.35.

Net pressure on wall

\[ = \frac{1}{2} \times 1500 \times 3 + 1500 \times 5 + 3000 \times \frac{1}{2} : \\
- 5400 \times \frac{1}{2} \times 1 \]

\[ = 2250 + 7500 + 7500 - 2700 \]

\[ = 14,550 \text{ kg.} \]
Let \( y \) be distance of line of action of resultant pressure from base. Taking moments about base.

\[
y = \frac{\frac{1}{2} \times 1500 \times 3 \times 6 + 1500 \times 5 \times \frac{1}{2} + 3000 \times 1 \times 5 \times \frac{1}{2} - 5400 \times 1 \times \frac{1}{2}}{14,550}
\]

\[
y = \frac{13,500 + 18,750 + 12,500 - 900}{14,550}
\]

\[
y = \frac{43,850}{14,550} = 3.014 \text{ m.}
\]

Weight of retaining wall is

\[
W_1 = 2 \times 8 \times 2400
\]

\[
= 38,400 \text{ kg.}, \text{ acting at 3 m. from toe.}
\]

\[
W_2 = \frac{1}{2} \times 7 \times 1 \times 2400
\]

\[
= 8400 \text{ kg.}, \text{ acting at } \frac{1}{2} \text{ m. from toe.}
\]

\[
W_3 = 1 \times 2 \times 2 \times 2400
\]

\[
= 4800 \text{ kg.}, \text{ acting at 1 m. from toe.}
\]

Let \( z \) be the distance from toe where resultant of weight of retaining wall and earth pressure strike the base. Taking moments about toe

\[
W_1 \times 3 + W_2 \times \frac{1}{2} + W_3 \times 1 - P \times 3.014 - (W_1 + W_2 + W_3) \times z = 0
\]

\[
\therefore z = \frac{38,400 \times 3 + 8400 \times \frac{1}{2} + 4800 \times 1 - 14,500 \times 3.014}{38,400 + 8400 + 4800}
\]

\[
= \frac{115,200 + 14,000 + 4,800 - 43,850}{51,600}
\]

\[
= \frac{90,150}{51,600} = 1.741 \text{ m.}
\]

Eccentricity \( e = \frac{B}{2} - 1.741
\]

\[
= 2 - 1.741
\]

\[
= 0.259 \text{ m.}
\]

Maximum pressure at base

\[
= \frac{W}{B} \left(1 + \frac{6e}{B}\right)
\]

\[
= \frac{51,600}{4} \left(1 + \frac{6 \times 0.259}{4}\right)
\]

\[
= 12,900(1 + 0.3885)
\]

\[
= 17,920 \text{ kg/m}^2.
\]

Factor of safety against sliding

\[
= \frac{\mu W}{P} = \frac{0.4 \times 51,600}{14,550}
\]

\[
= 1.419.
\]
Ex. 16.12. A trench 2 m. deep is excavated in earth weighing 1600 kg./m³ and having angle of repose 30°. The trench is supported by vertical boardings with horizontal longitudinal walings at top and bottom only. Horizontal struts are fixed at 2 m. intervals to support the vertical boarding. Calculate

(a) Maximum B.M. per metre width in boarding.

(b) Thickness of boarding if maximum stress is not to exceed 200 kg/cm².

(c) Pressure on top and bottom struts.

Solution.

Coefficient of active earth pressure

\[
\frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{3}
\]

Intensity of pressure at base

\[
\frac{1}{3} \times 1600 \times 2 = 3200 \text{ kg/m}^2.
\]

The boarding is supported at top and bottom and is subjected to load varying from zero at top to 3200 kg./m², at bottom.

Consider 1 m. length of trench. Pressure diagram on boarding is shown in Fig. 16.36 (b).

At depth \( y \), B.M. in boarding is

\[
M = \frac{3200}{9} y - \frac{3200}{3} \times \frac{1}{2} \times y \times \frac{y}{2} \times \frac{y}{3}
\]

\[
= \frac{3200}{9} y - \frac{3200}{36}
\]

For maximum B.M., put

\[
\frac{dM}{dy} = 0
\]
\[
\therefore \frac{3200}{9} \times 3y^2 = 0
\]
\[
\therefore \frac{3200}{9} = \frac{3200}{36} \times 3y^2
\]
\[
y^2 = \frac{4}{3}
\]
\[
= \frac{2}{\sqrt{3}}
\]
\[
\text{Maximum DM} = \frac{3200}{9} \times \frac{2}{\sqrt{3}} < \frac{3 \sqrt{3}}{3}
\]
\[
= \frac{12,800}{27 \sqrt{3}} \text{ kg. m.}
\]
\[
= 273.6 \text{ kg. m.}
\]
\[
= 27,360 \text{ kg. cm.}
\]
\[
M = fZ
\]
\[
\therefore 27,360 = 200 \times \frac{1}{3} \times bd^2
\]
\[
27,360 = 200 \times \frac{1}{3} \times 100 \times d^1
\]
\[
d^2 = \frac{27,360 \times 3}{1,600}
\]
\[
d = \sqrt{8.208}
\]
\[
= 2.865 \text{ cm}
\]

Provide 3 cm. thick boarding.

Pressure on ton strut,
\[
= \frac{3200}{9} \times s \text{ where } s \text{ is spacing of struts}
\]
\[
= \frac{3,200}{9} \times \sqrt{4}
\]
\[
= \frac{3,200}{9} \times \frac{2}{2}
\]
\[
= \frac{3,200}{9}
\]
\[
= 711.11 \text{ kg.}
\]

Pressure on bottom strut
\[
= \frac{6400}{9} \times 2
\]
\[
= 1422.22 \text{ kg.}
\]

**Ex. 16.13. Using Rebbann's construction, find earth pressure at the back of retaining wall section shown in Fig. 16.37 (a).**

Check the results by Coulomb's formula. Angle of friction between wall and earth is 20°. Angle of repose of soil is 30°. Density of earth = 1800 kg./m³.

**Solution.**

Earth pressure = Area of triangle \(PQR\times w\)
\[
= 4.6 \times 1800
\]
\[
= 8280 \text{ kg.}
\]
\[
\varphi = 30^\circ \\
\theta = 20^\circ \\
\delta = 20^\circ \\
\alpha = 10^\circ
\]

Fig. 16:37 (b)

Coefficient of active earth pressure, by coulomb's formula

\[
\frac{\cos^2 (\varphi - \theta)}{\cos^2 \theta \cos (\delta + \theta) \left[ 1 + \sqrt{\frac{\sin (\delta + \varphi) \sin (\varphi - \alpha)}{\cos (\delta + \theta) \cos (\theta - \alpha)}} \right]^2}
\]

\[
= \frac{\cos^2 10}{\cos^2 20 \cos 40 \left[ 1 + \sqrt{\frac{\sin 50 \sin 20}{\cos 40 \cos 10}} \right]}
\]

\[
= \frac{(0.9848)^3}{(0.9397)^3 (0.766) \left[ 1 + \sqrt{\frac{0.766 \times 0.3420}{0.766 \times 0.9848}} \right]^3}
\]
\[
\text{Total pressure} = \frac{1}{2} \cdot w \cdot h^2 \cdot k_A = \frac{1}{2} \times 1800 \times 4 \times 4 \times 0.5678
\]

\[
= 8,176.32 \text{ kg.}
\]

**Ex. 16.14.** An R.C.C. retaining wall is supported on piles as shown in Fig. 16.38. Find the maximum pressure in pile. Density of earth is 1600 kg/m³ and density of concrete is 2400 kg/m³. Angle of repose of soil is 30°.
Solution. Consider one metre length of the retaining wall.

\[ W_1 = \frac{25}{100} \times 8.5 \times 2400 \]
\[ = 5100 \text{ kg. acting at } 1.625 \text{ m. from toe} \]

\[ W_2 = \frac{25}{100} \times \frac{1}{2} \times 8.5 \times 2400 \]
\[ = 2550 \text{ kg. acting at } \frac{2}{3} \text{ m. from toe} \]

\[ W_3 = 4.5 \times 0.5 \times 2400 \]
\[ = 5400 \text{ kg. acting at } 2.25 \text{ m. from toe} \]

\[ W_4 = 0.737 \times 2.75 \times 1600 \]
\[ = 1621.4 \text{ kg. acting at } \frac{10.75}{3} \text{ m. from toe} \]

\[ W_5 = 8.75 \times 2.75 \times 1600 \]
\[ = 37,400 \text{ kg. acting at } 3.125 \text{ m. from toe} \]

\[ W_6 = 1.25 \times 1.0 \times 1600 \]
\[ = 2000 \text{ kg. acting at } 0.625 \text{ m. from toe} \]

\[ W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 = 54,071.4 \text{ kg.} \]

\[ kA = wH^2 \]

\[ \cos \alpha = \left( \cos \alpha - \sqrt{\cos^2 \alpha - \cos^2 \varphi} \right) \times 1600 \times (9 + 0.737)^2 \]
\[ = \cos 15 \left( \frac{\cos 15 - \sqrt{\cos^2 15 - \cos^2 30}}{\cos 15 + \sqrt{\cos^2 15 - \cos^2 30}} \right) \times 800 \times 9 \times 737^2 \]
\[ = 0.37 \times 800 \times 9 \times 737^2 \]
\[ = 28,060 \text{ kg.} \]

\[ P_H = P \cos \alpha = 28,060 \times \cos 15^\circ \]
\[ = 27,100 \text{ kg. acting at } 3.246 \text{ m. from base} \]

\[ P_V = P \sin \alpha = 28,060 \times \sin 15^\circ \]
\[ = 7263 \text{ kg. acting at } 4.5 \text{ m. from toe} \]

Let the resultant of \( W_1, W_2, W_3, W_4, W_5, W_6, P_H \) and \( P_V \) cut the base at \( x \) distance from toe.

Taking moments about toe,

\[ 1.625W_1 + \frac{4}{3} W_3 + 2.25W_4 + \frac{10.75}{3} W_6 = 3.125W_5 \]
\[ + 0.625W_6 - P_H \times 3.246 + P_V \times 4.5 - (W + P_V)x = 0 \]

\[ \therefore 1.625 \times 5100 + \frac{4}{3} \times 2550 + 2.25 \times 5400 + \frac{10.75}{3} \times 1621.4 \]
\[ + 3.125 \times 37,400 + 0.625 \times 2000 - 27,100 \times 3.246 \]
\[ + 7,263 \times 4.5 - (54,071.4 + 7263)x = 0 \]