INTRODUCTION

In the first volume of Analysis of structures, statically determinate structures, i.e. the structures which could be analysed by equations of statics were dealt with. However in Engineering practice, structures which cannot be analysed by equations of statics alone are met with. These structures are called statically indeterminate structures or hyperstatic structures. These structures may be such that internal forces cannot be evaluated from statics alone, such structures are called internally indeterminate structures or the structure may be connected to foundations in such a way that all the reactions cannot be evaluated from statics alone, and such structures are called externally indeterminate structures.

The structure shown in Fig. 1 is a simply supported beam hinged at A and supported on rollers at B. Under any loading \( P \), the reactions can be worked out. At A the reaction will have vertical and horizontal components, while reaction at B will be vertical as the end is free to move horizontally. From three equations of statics \( \Sigma V = 0, \Sigma H = 0 \), and these three reaction components can be evaluated.

Consider the same beam with end A as fixed as shown in Fig. 2. Under any loading say \( P \), the reaction at A will comprise of vertical component, horizontal component and a fixing moment. At B there will be a vertical reaction. All these four unknowns cannot be evaluated from equations of statics alone. Therefore this structure called propped cantilever is indeterminate to the first degree. One extra equation is obtained from consideration of slope or deflection.

Consider the beam of Fig. 1. Suppose a support anywhere in the span as shown in Fig. 3 is introduced. Under any loading
there will be two reaction components at $A$ and vertical reactions at $B$ and $C$. Thus there will be four unknowns. One extra equation is to be obtained from consideration of deflection or slope to analyse the structure. Therefore this structure—continuous beam of two spans, is indeterminate to the first degree.

If more supports are introduced, there will be more spans and the degree of indeterminancy will increase. In case of four supports as shown in Fig. 4, there will be three spans and the reaction components will be five and, therefore, two equations are formed from considerations of deflections or slopes. Therefore this structure is indeterminate to second degree.

Consider structure of Fig. 2 with a support anywhere in span.

In this case there will be three reaction components at $A$ and vertical reactions at $B$ and $C$. Therefore two extra equations are formed from considerations of slopes and deflections. This structure is also continuous beam with one end fixed and it is indeterminate to second degree.

Consider beam $AB$ fixed at both ends $A$ and $B$ as shown in Fig. 6. Under any loading there will be three reaction components at $A$ and two reaction components at $B$. Therefore two extra equations are formed from consideration of deflections and slopes. Therefore fixed beam is indeterminate to second degree.

In general, if, $'n'$ extra equations in addition to three equations are required to analyse the structure completely then the structure is said to be indeterminate to $n$th degree.
Consider the portal frame $ABCD$ with end $A$ hinged and on rollers shown in Fig. 7. Under any loading say $P$, there will be two reaction components at $A$ and one vertical reaction at $B$. These three unknowns can be found from three equations of statics. Therefore this portal frame is determinate.

![Fig. 7](image)

![Fig. 8](image)

If the end $D$ is hinged as shown in Fig. 8, there will be two reaction components at $A$ and two reaction components at $D$. Thus one extra equation is required to solve the unknowns. Extra equation can be obtained from consideration of deflection. Therefore portal frame with ends hinged is indeterminate to first degree.

If one end say $A$ is fixed and $D$ is hinged as shown in Fig. 9, there will be in all 5 reaction components and the structure is indeterminate to the 2nd degree.

![Fig. 9](image)

![Fig. 10](image)

If both ends $A$ and $D$ are fixed as shown in Fig. 10, there will be 6 reaction components and the structure becomes indeterminate to the third degree.

Consider two-bay structure of Fig. 11. Under any loading $P$, there will be three reaction components at $A$, $C$ and $E$. Thus in all there will be nine reaction components. Thus the structure will be indeterminate to the sixth degree.
Next, consider box culvert as shown in Fig. 12. The support reactions can be found from equations of statics. Still the frame cannot be analysed as internal moments, shears and thrusts are not known. If the structure is cut anywhere, and three shear, thrust and moment are evaluated from other considerations of deflections and slopes, the B.M., shear and thrust anywhere else can be found from statics. Thus the structure is indeterminate to the third degree. In general, if there are $n$ rings and structure is determinate externally, the indeterminancy will be $3n$. Consider verendal frame of Fig. 13. There are 4 rings and externally it is determinate. The structure is indeterminate to $3 \times 4 = 12$th degree.

Consider multi-storey frame of Fig. 14. If the members $BF$, $CG$, $DH$, $FJ$, $GK$ and $HL$ are cut there will be three unknown quantities at every cut. Thus there will be in all 18 unknowns and the structure is indeterminate to the 18th degree.

In case of pin-jointed structures, if $m + r = 2j$, where $m$ is the number of members, $r$ is the number of reaction components and $j$ is the number of joints, the structure will be determinate if stable. In case the number of
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bers is more or number of reaction components is more required for determinate structure, the frame becomes determinate. Consider the pin-jointed frame of Fig. 15. There three bars meeting at O. From statics only two unknowns can be found at a joint and, therefore, the frame is indeterminate internally to first degree.

![Fig. 15](image1)

![Fig. 16](image2)

Consider truss of Fig. 16 having hinges at both supports and any loading say P, there will be two reaction components L and R. There are 11 members and 7 joints; therefore \( r = 11 + 4 = 15 \), and \( 2j = 14 \), therefore the frame is indeterminate to first degree. Consider frame of Fig. 17, the frame is determinate internally as three reaction components can be evaluated. Now \( r = 3 \), \( m = 6 \), \( j = 4 \), therefore \( m + r = 9 \), \( = 8 \) and hence structure is indeterminate internally to first degree. There is one extra member.

![Fig. 17](image3)

Consider frame supported on three supports as shown in Fig. 18. There are 4 reaction components and 23 members, therefore \( m + r = 27 \) and number of joints is 13, therefore the structure is indeterminate to first degree. One reaction component can be considered as extra and then it will be externally redundant. If any member by \( HJ, JK, KL, LM, \) or \( MN \) is considered extra, the frame will become internally redundant.