PART I
CHAPTER I
FIRST NOTIONS

General Statements

The following example, illustrating the use of letters to generalise statements, is intended for oral discussion.

Example 1. Part of a post is painted black, namely the shaded portion in the figures below; the rest is white. What is the length of the white portion?

The post is 9 ft. long, and the black portion is 7 ft. long; .

.: the white portion is 9 ft. - 7 ft. long or 2 ft. long. Repeat with the following figures:

![Figures showing lengths](image)

The general statement is as follows:

If the length of the post is \( l \) feet and if the length of the part painted black is \( b \) feet, then the length of the remainder, painted white, is \( l - b \) feet.

These statements can be shortened by using brackets.

Instead of 9 ft. - 7 ft., we may write \( (9 - 7) \) ft.; and instead of \( l \) feet - \( b \) feet, \( (l - b) \) feet. The contents of a bracket are then regarded as equivalent to a single number.

Letters are used to represent numbers. Do not use them to represent quantities, i.e. numbers of things.
Do not say the length of a rod is \( l \), but say that its length is \( l \) inches or \( l \) feet or \( l \) cm., etc.

The Use of Symbols

The symbols \(+\), \(-\), \(\times\), \(\div\) have the same meanings in Algebra as in Arithmetic.

The following symbols are in common use:

\(=\) means "is equal to"; thus \(5 - 2 = 3\) and \(4 \times 5 = 20\).

\(\therefore\) means "therefore"; thus \(1 \text{ yd.} = 3 \text{ ft}\), \(\therefore 4 \text{ yd.} = 4 \times 3 \text{ ft}\).

\(>\) means "is greater than"; thus \(5 > 2\) and \(3\frac{1}{2} > 2\frac{1}{2}\).

\(<\) means "is less than"; thus \(2 < 5\) and \(2\frac{3}{4} < 3\frac{1}{2}\).

There are two other symbols it is often convenient to use.

\(\approx\) means "is approximately equal to"; thus \(3\frac{1}{4} \approx 3.14\).

\(\neq\) means "is not equal to"; thus, if \(x = 5\) and \(y = 2\), \(x \neq y\).

Do not confuse \(=\) with \(\therefore\); use the symbol \(\approx\) as a verb.

EXERCISE I. a

State in words the following:

1. \(3 \times 4 = 12\)  
2. \(7 > 4\)  
3. \(5 < 8\)  
4. \(1\frac{1}{2} = 1.25\)  
5. \(3 < 4 < 5\)  
6. \(10 > 8 > 7\)  
7. \(\frac{19}{8} = 3\frac{1}{8} > 3.3\)  
8. \(1.6 < 1\frac{3}{4}\)  
9. \(N < 3\)  
10. \(A < 8\)  
11. \(x = y = 3\)  
12. \(a \neq 2\)  
13. \(5\frac{1}{4} \approx 5.3\)  
14. \(6 > z\)  
15. \(5 < b < 7\)  
16. \(4 > l > 3\)  
17. \(x \neq 0\)  
18. \(\pi \approx 3.14\)  
19. \(a \neq b\)  
20. \(\sqrt{2} \approx 1.41\)

Write in symbols the following:

21. \(3^2\) is greater than 3.  
22. \((\frac{1}{2})^3\) is less than \(\frac{1}{4}\).

23. \(N\) is equal to 8.  
24. \(A\) is greater than 5.

25. \(z\) is not equal to nought.  
26. \(l\) is less than \(3\frac{1}{2}\).

27. \(y\) lies between 10 and 20.  
28. \(A\) and \(B\) are equal.

29. Of the two numbers \(x\) and \(y\), the greater is \(x\).

30. An approximation for \(\pi\) is \(\frac{22}{7}\).

31. The numbers \(a\) and \(b\) each equal 10.

32. The number \(N\) is less than 100 and is greater than \(C\).

33. Twice \(N\) equals fourteen, therefore \(N\) equals seven.

34. If \(N\) plus three equals eight, then \(N\) equals five.

35. If \(t\) minus four equals six, then \(t\) equals ten.
FIRST NOTIONS

EXERCISE 1 b

1. How much can the water-level rise in each of the glasses shown in Fig. 3 before the water overflows? Answer the same

question for a glass (i) 5 in. high containing water 4 in. deep, (ii) h in. high with water 5 in. deep.

2. There are two parcels of unequal weights in the scale pans of a weighing machine, the heavier on the left.

<table>
<thead>
<tr>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight in left pan -</td>
<td>6 lb.</td>
<td>10 lb.</td>
<td>8 lb.</td>
<td>W lb.</td>
</tr>
<tr>
<td>Weight in right pan -</td>
<td>4 lb.</td>
<td>3 lb.</td>
<td>w lb.</td>
<td>1 lb.</td>
</tr>
</tbody>
</table>

What weight must be placed in the right-hand scale pan to make them balance? Make a general statement.

3. Part of a rod, see Fig. 4, is painted red, another part is white and the rest is black.

<table>
<thead>
<tr>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total length of rod in cm.</td>
<td>10</td>
<td>12</td>
<td>18</td>
<td>l</td>
</tr>
<tr>
<td>Length of red part in cm. -</td>
<td>5</td>
<td>7</td>
<td>3½</td>
<td>4</td>
</tr>
<tr>
<td>Length of white part in cm.</td>
<td>3</td>
<td>4</td>
<td>4½</td>
<td>5</td>
</tr>
</tbody>
</table>

Find the length of the black part in each case. Make a general statement.

4. Find the height of a pile of equal note-books:

<table>
<thead>
<tr>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
<th>(vi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of note-books -</td>
<td>10</td>
<td>12</td>
<td>20</td>
<td>n</td>
<td>15</td>
</tr>
<tr>
<td>Thickness of each book in cm.</td>
<td>2</td>
<td>3</td>
<td>1½</td>
<td>2</td>
<td>t</td>
</tr>
</tbody>
</table>
5. Write down, without multiplying out,
   (i) the number of feet in 2 yd., 7 yd., 4½ yd., x yd.
   (ii) the number of pence in 4s., 13s., 3½s., Ps.

6. Fig. 5 gives the dimensions in yards of three rectangular fields; find the total length of fencing required for each field.

   ![Diagram of three rectangular fields]

   Fig. 5.

   Give a general statement for the perimeter, when the length is l yd. and the breadth is b yd.

7. (i) With the data of Fig. 6, state the distances of P and Q from B. What is the distance of R from B?

   ![Diagram of distances A, P, Q, R and B]

   Fig. 6

   (ii) If AB = s yd., AR = 480 yd., how far is R from B?
   (iii) If AB = s yd., AR = a yd., how far is R from B?

8. Find how much a man saves each year:

<table>
<thead>
<tr>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
<th>(iv)</th>
<th>(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income in £</td>
<td>500</td>
<td>700</td>
<td>I</td>
<td>800</td>
</tr>
<tr>
<td>Expenditure in £</td>
<td>400</td>
<td>300</td>
<td>500</td>
<td>E</td>
</tr>
</tbody>
</table>

   Make a general statement.

9. Tickets for a concert cost two shillings each. What are the receipts if (i) 100, (ii) 240, (iii) n tickets are sold?
   What are the receipts if t tickets are sold at p shillings each?

10. Write down the size of each unmarked angle in Fig. 7.

   ![Diagram of angles 90°, 115°, 132°]

   Fig. 7.

   Make a general statement.
11. A charabanc holds 40 people. How many are required for a party containing 120 people, 200 people, 480 people? Make a general statement.

12. A watch loses 10 seconds an hour; how much does it lose in 3 hours, 8 hours, 24 hours? Make a general statement.

13. Fig. 8 represents a rectangular shed EDGF in the corner of a rectangular garden; the dimensions are given in feet.

Find the lengths of AE and CG (i) if \( b = 10', \ l = 15 \), (ii) if \( b = 12', \ l = 20 \), (iii) in terms of \( b \) and \( l \).

14. How many hours is a person in bed in the following cases?

<table>
<thead>
<tr>
<th>Time of going to bed</th>
<th>10 p.m.</th>
<th>9 p.m.</th>
<th>9 p.m.</th>
<th>y p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time of getting up</td>
<td>7 a.m.</td>
<td>7 a.m.</td>
<td>x a.m.</td>
<td>6 a.m.</td>
</tr>
</tbody>
</table>

Make a general statement.

15. Fig. 9 represents the floor of a room with right-angled corners. Find the lengths of DE, EF

(i) with the data in the figure;
(ii) if \( AB = a \text{ ft.}, BC = c \text{ ft.}, AF = 14 \text{ ft.}, CD = 12 \text{ ft.} \)
(iii) if \( AB = a \text{ ft.}, BC = c \text{ ft.}, AF = x \text{ ft.}, CD = y \text{ ft.} \)

16. Fig. 10 represents a rectangular enclosure; the dimensions are given in feet. Write down the distance of B from P, measured round the enclosure, (i) through A and D, (ii) through C.

Find also these distances if \( AB = b \text{ ft.}, AD = d \text{ ft.} \)

17. When a man is 40 years old, his son is 10 years old. Copy the given table, complete it, and make a general statement.

<table>
<thead>
<tr>
<th>Age of Father</th>
<th>40</th>
<th>50</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of Son</td>
<td>10</td>
<td>6</td>
<td>( s )</td>
</tr>
<tr>
<td></td>
<td>( f )</td>
<td>( s )</td>
<td>( y )</td>
</tr>
</tbody>
</table>
18. The area of the floor of a rectangular room is 180 sq. ft. Copy the given table, complete it, and make a general statement.

<table>
<thead>
<tr>
<th>Length of room in feet</th>
<th>18</th>
<th>l</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breadth of room in feet</td>
<td>12</td>
<td>b</td>
<td>y</td>
</tr>
</tbody>
</table>

Notation in Algebra

The operation, "multiply the number N by 5," might be written $N \times 5$; to save time, it is written $5N$.

Similarly, $N \times \frac{1}{2}$ which equals $\frac{1}{2} \times N$ is written $\frac{1}{2}N$ or $\frac{N}{2}$; and $N \times \frac{3}{4}$ is written $\frac{3}{4}N$ or $\frac{3N}{4}$.

Just as $5 \times 9 = 9 \times 5$, so $a \times b = b \times a$, and the short-hand form is $ab$ or $ba$. But, just as $6 \times 1 = 1 \times 6 = 6$, so $N \times 1 = 1 \times N = N$; we therefore write $N$ instead of $1N$.

In Arithmetic, $57$ means $5 \times 10 + 7$ and $5\frac{3}{7}$ means $5 + \frac{3}{7}$; but in Algebra $5N$ always means "five times $N"$ or "$N$ times five"; $ab$ always means $a \times b$ or $b \times a$.

In other respects, notation in Algebra is the same as in Arithmetic:

- $5 \times 5 \times 5$ is written $5^3$; $N \times N \times N$ is written $N^3$.
- $2 \div 3 \text{ is written } \frac{2}{3}$; $a \div b \text{ is written } \frac{a}{b}$.
- $\sqrt{6}$ is the square root of $6$; $\sqrt{y}$ is the square root of $y$.
- $\frac{3}{7} + \frac{2}{7}$ is equal to $\frac{3+2}{7}$; $\frac{a}{x} + \frac{b}{x} \text{ is equal to } \frac{a+b}{x}$.

**Example 2.** What is the meaning of $3N - 7$? What is its value if $N$ stands for 5?

$3N$ means "multiply $N$ by 3" or "multiply 3 by $N$.

To obtain the value of $3N - 7$, multiply $N$ by 3 and then subtract 7 from the result.

If $N$ stands for 5, $3N - 7 = 3 \times 5 - 7 = 15 - 7 = 8$.

**Example 3.** If $r$ stands for 8, what is the value of $\frac{r + 2}{r}$?

$r = 8$, $\therefore \frac{r + 2}{r} = \frac{8 + 2}{8} = \frac{10}{8} = \frac{5}{4} = 1\frac{1}{4}$. 
FIRST NOTIONS

EXERCISE I.

State in words the meaning of the following, and afterwards state their values if \( N = 6, a = 2, b = 1 \).

1. \( 3N \)  
2. \( N + 3 \)  
3. \( \frac{N}{3} \)  
4. \( N - 3 \)  
5. \( N^2 \)

6. \( 2N + 2 \)  
7. \( \frac{4N}{3} \)  
8. \( \frac{6}{N} \)  
9. \( 2N \div 3 \)  
10. \( \frac{4N^2}{3} \)

11. \( aN \)  
12. \( ab \)  
13. \( a + b \)  
14. \( \frac{N}{a} \)  
15. \( b^2 \)

16. \( bN \)  
17. \( 2aN \)  
18. \( 2a + N \)  
19. \( N - 2a \)  
20. \( a - 2b \)

21. \( abN \)  
22. \( 3a^2 \)  
23. \( N - b \)  
24. \( 3 + a \)  
25. \( 5a - N \)

26. \( \frac{N}{b} \)  
27. \( 3 + aN \)  
28. \( \frac{a + b}{N} \)  
29. \( 3ab \)  
30. \( a + bN \)

If \( x = 3, y = 5, c = 8 \), find the values of the following:

31. \( 2x \)  
32. \( x^2 \)  
33. \( x + 2 \)  
34. \( x - 2 \)  
35. \( \frac{x}{2} \)

36. \( xy \)  
37. \( yx \)  
38. \( 2xy \)  
39. \( y - x \)  
40. \( 2x - y \)

41. \( c + x \)  
42. \( cx \)  
43. \( c - 2x \)  
44. \( cy - 10x \)  
45. \( 2y^3 \)

Write down the results of the following operations:

46. Add 2 to N.  
47. Multiply a by 3.

49. Subtract \( \frac{5}{c} \) from c.

50. Four times l.  
51. Add 1 to x.

52. Two-thirds of p.  
53. Double y and add 2.

54. Three less than n.  
55. Divide 12 by t.

56. Increase e by 5.  
57. Halve e and then add 2.

58. Half the sum of s and 2.  
59. Decrease k by 1.

60. Multiply N by 4 and subtract 2 from the result.

61. Divide r by 5 and subtract the result from 10.

62. Square c and multiply the result by 6.

63. Multiply together b, a, c and double the result.

64. Multiply r by s and divide the result by 5.

Like Terms

If an expression consists of various parts, some of which are connected by + or \( \cdot \) signs, and others by \( \times \) or \( \div \) signs, the parts connected by + or \( \cdot \) signs are called terms. A formal discussion of like and unlike terms is best taken later, see p. 44.
Example 4. Generalise the statement. \(9 + 9 + 9 + 9 + 9 = 9 \times 5\).

\[
\begin{array}{cccccc}
\text{Fig. 11}
\end{array}
\]

A straight fence, see Fig. 11, is made up of 5 hurdles, each 9 ft. long;
Its total length \(= (9 + 9 + 9 + 9 + 9) \text{ ft.} = 9 \text{ ft.} \times 5 = 9 \times 5 \text{ ft.}\)
Suppose each hurdle is \(x\) feet long;
The length of the fence \(= (x + x + x + x + x) \text{ ft.} = x \text{ ft.} \times 5 = 5x \text{ ft.}\)
Thus, \(x + x + x + x + x = x \times 5 = 5x\).

Example 5. Generalise the statement, \(3 \times 9 + 5 \times 9 = 8 \times 9\).
A fence is made by first setting up 3 hurdles, each 9 ft. long, and then adding 5 more hurdles, each 9 ft. long.
The lengths of the two portions are 9 ft. \(\times 3\) and 9 ft. \(\times 5\).
There are in all \((3 + 5)\) hurdles = 8 hurdles;
\(\therefore\) total length \(= (3 \times 9 + 5 \times 9) \text{ ft.} = 8 \times 9 \text{ ft.}\)
Suppose each hurdle is \(x\) feet long.
The lengths of the two portions are \(x \text{ ft.} \times 3\) and \(x \text{ ft.} \times 5\), which equal \(3x \text{ ft.}\) and \(5x \text{ ft.}\); and the total length is \(x \text{ ft.} \times 8\) or \(8x \text{ ft.}\)
\(\therefore\) total length \(= (3x + 5x) \text{ ft.} = 8x \text{ ft.}\)
Thus, \(3x + 5x = 8x\).
This is simply a short-hand statement.
\(3x = x + x + x\) and \(5x = x + x + x + x + x\).
\(\therefore\) \(3x + 5x = x + x + x + x + x + x + x + x + x + x = 8x\).

Example 6. Simplify \(7x - 2x\).
Suppose a fence is made of 7 hurdles, each \(x\) feet long; its total length is \(7x\) feet.
Now remove two of the hurdles; then the length of the part removed is \(2x\) feet.
But 5 hurdles remain; \(\therefore\) the length of the remainder is \(5x\) feet;
\(\therefore\) \(7x - 2x = 5x\).
This is a short-hand statement for the following:
\(x + x + x + x + x + x - x = x + x + x + x + x = 5x\).

Example 7. Write more shortly:
(i) \(5a \times 3\);
(ii) \(x \times 3y\);
(iii) \(2a \times 5b\);
(iv) \(2a \times 5ab\);
(v) \(6a \div 2\);
(vi) \(\frac{5a}{6} \times 4\).
FIRST NOTIONS

(i) \(5a \times 3 = 5a + 5a + 5a = 15a\).

(ii) \(x \times 3y = x \times 3 \times y = 3 \times x \times y = 3xy\).

(iii) \(2a \times 5b = 2 \times a \times 5 \times b = 2 \times 5 \times a \times b = 10ab\).

(iv) \(2a \times 5ab = 2 \times a \times 5 \times a \times b = 2 \times 5 \times a \times a \times b = 10a^2b\).

(v) \(6a \div 2 = \frac{6 \times a}{2} = 3 \times a = 3a\).

(vi) \(\frac{5a}{6} \times 4 = \frac{5 \times a \times 4}{6} = \frac{5 \times 2 \times a}{3} = \frac{10a}{3}\).

To simplify an expression containing several terms, work from the left unless, just as in Arithmetic, brackets or multiplication and division signs show that operations must be performed in a different order. Thus \(11 - 3 + 2 = 8 + 2 = 10\); similarly,
\[11x - 3x + 2x = 8x + 2x = 10x\], and \(7y + 2y - 5y = 9y - 5y = 4y\).

EXERCISE I. d

Write down short-hand forms for the following:

1. \(A \times 3\).
2. \(B \times 4\).
3. \(C \times C\).
4. \(n + n\).
5. \(x + x + x\).
6. \(y + y + y + y\).
7. \(3x + 3z\).
8. \(3a \times 2\).
9. \(3a \times 5\).
10. \(3b + 2b\).
11. \(4c + c\).
12. \(4d \times 2\).
13. Three times N.
14. Three times \(2p\).
15. Four times \(3q\).
16. Half of \(A\).
17. One-quarter of \(4B\).
18. Two-thirds of \(C\).
19. \(12e \div 3\).
20. \(\frac{15f}{5}\).
21. \(\frac{t}{2} \times 6\).
22. \(\frac{3t}{2} \times 6\).
23. \(4 \times 7y\).
24. \(4 \times \frac{y}{2}\).
25. \(2 \times \frac{3z}{4}\).
26. \(5x \div 6\).
27. \(8x \div 6\).
28. \(\frac{3y}{4} \times 8\).
29. \(6z \times \frac{1}{2}\).
30. \(6a \times \frac{1}{4}\).
31. \(3x \div 12\).
32. \(8r \div 12\).
33. \(s \div 1\).
34. \(t \div 1\).
35. \(4y \times \frac{1}{6}\).
36. \(4z \div \frac{3}{4}\).
37. \(6a \div \frac{1}{6}\).
38. \(10b \div \frac{1}{6}\).
39. \(3 \times 7 + 2 \times 7\).
40. \(3t + 2t\).
41. \(4p + 4p\).
42. \(5 \times 8 + 8\).
43. \(5y + y\).
44. \(z + 3z\).
45. \(6 \times 9 - 2 \times 9\).
46. \(6x - 2x\).
47. \(3f - f\).
48. \(4t - 3t\).
49. \(5p - 5p\).
50. \(2q - q\).
51. \(a + a + 2a\).
52. \(3b + 2b + b\).
53. \(4c + c + 4c\).
54. \(3z + 0 + x\).
55. \(4y + y - 3y\).
56. \(2z + 5z - 4z\).
57. \(8r - 2r - 5r\).
58. \(4s - 0 - s\).
59. \(6t - 2t + 3t\).
60. \(c \times 3c\).
61. \(2d \times 3d\).
62. \(a \times 3b\).
63. \(x\times \times 4y\).
64. \(3 \times 3z\).
65. \(5 \times 4a\).
66. \(bc \times b\).
68. $3bc \times 3b$.  
69. $3bt \times 3bc$.  
70. $\frac{a}{2} \times 3xy$.  
71. $3x^2 \times 2$.

72. $\frac{3a}{2} \times 4a$.  
73. $\frac{b}{3} \times \frac{b}{3}$.  
74. $\frac{a}{5} \times 5a$.  
75. $\frac{a}{2} \times 6b$.

**Numbers and Quantities**

*Letters in Algebra are used to represent numbers, not numbers-of-things.*

A letter may stand for 2, 15, $\frac{1}{2}$, etc., but not for 2 pence, 15 days, $\frac{1}{2}$ mile, etc.

A number-of-things is called a *quantity*.

When dealing with quantities, *always state what the unit is*, as in the following examples:

A parcel weighs $W$ lb.; a book costs $C$ shillings; a room is $h$ feet high; a tank holds $n$ gallons.

**EXERCISE I**

1. I walk $s$ miles and then ride 4 miles. How far do I go? What is the answer if $s = 3$?

2. A box weighs 3 lb.; I put into it two parcels, one weighing 2 lb., the other $W$ lb. What is the total weight? What is the answer if $W = 5$?

3. A milkman sells $m$ gallons of milk out of 20 gallons; how much has he left? What is the answer if $m = 16$?

4. A bus fare is 4 pence; what is the cost of (i) 5 journeys, (ii) $n$ journeys?

5. I buy $c$ apples and then buy 2 more; how many altogether? What is the answer if $c = 10$?

6. I buy a bunch of 3$t$ bananas and cut off 3 of them; how many are left? What is the answer if $t = 5$?

7. A basket, weighing $W$ lb. when empty, contains $6n$ lb. of apples; if 2$n$ lb. of apples are sold, what is the weight of the basket and the remaining apples? What is the answer if $W = 4$ and $n = 5$?

8. Fig. 12 represents a carpet on the floor of a room; there is a margin 2 feet wide all the way round. What is the length and breadth of the carpet? What is the answer if $l = 30$, $w = 15$?

9. I can walk 4 miles an hour. How far can I walk in 3 hours, 2 hours, 3$t$ hours?
10. What is the cost of the following?
   (i) 5 lb. of butter at l pence per lb.
   (ii) 2 oz. of pepper at x pence per oz.
   (iii) m yd. of silk at 12 shillings per yd.
   (iv) x gallons of oil at p shillings per gallon.
   (v) R feet of pipe at n pence per foot.
   (vi) N dozen note-books at P pence each book.

11. How many inches are there in 4 ft., 4 ft. 9 in., l ft., l ft. m in., v yd.?

12. How many shillings have I left if I spend
   (i) 5s. out of £2; (ii) 5s. out of £N; (iii) Ps. out of £Q?

13. What is the total length in inches of the wire used to make
    the grid in Fig. 13?
    Each inch of wire weighs ½ oz.; what is the weight of the grid?
    What is the value of each answer if a = 2, b = 8?

14. It is now a quarter past ten; in how many
    minutes will it be (i) 10.25,
    (ii) t minutes past ten, (iii) n minutes to 11?

15. A man buys a horse for £40 and sells it for £P; what is his profit?

16. By selling a watch for z shillings, I gain 10 shillings; how much did it cost me?

17. (i) y apples are shared equally among 5 boys; how many does each have?
   (ii) v apples are shared equally among 5 boys and x girls; how many does each have?

18. A car uses a gallon of petrol every 24 miles. (i) How far will the car run on 2½ gall., N gall.? (ii) How much petrol is used to run 1 mile, 10 miles, s miles?

19. Fig. 14 represents a short flight of steps from A to B. Each
    step is h inches high and d inches deep. What length of carpet is needed to run from A to B?

20. A jug holds 2 pints. How many jugs are needed for k pints, n gallons?

21. A pail holds c pints. How many times can it be filled from a tank containing 100 pints, N pints, V gallons?

22. A railway porter is paid b shillings a week and receives in tips n shillings a day. How much does he get each week working 6 days?
23. A family uses 3 loaves of bread a day. How many loaves are needed for $n$ days; $t$ weeks? How long will $x$ loaves last?

24. Eggs are 3 pence each. How much change (in pence) is there out of half a crown, if you buy $x$ eggs?

25. How long (in hours) does it take to travel 20 miles at 10 m.p.h., $v$ m.p.h.? How long for $s$ miles at $u$ m.p.h.?

26. A boy is now $y$ years old and his father is now three times as old. How old will each be in 3 years' time?

27. A row of houses is $l$ yards long; each house is $w$ yards wide. How many houses are there in the row?

28. A bookshelf is $l$ feet long; how many books, each $t$ inches thick, will it hold?

29. Take the number $n$; square it and multiply the result by 6.

30. Take the number $p$; multiply it by 4 and subtract 4.

[Note. For additional examples, see Appendix, Ex. S. 1, p. 274.]

Meaning of Brackets

It was pointed out on p. 1 that the contents of a bracket may be regarded as equivalent to a single number.

Thus, $(7 + 3)$ means the number obtained by adding 3 to 7; and $(N + 5)$ means the number obtained by adding 5 to $N$.

The product of 9 and $(7 + 3)$ is written $9(7 + 3)$.

The product of 9 and $(N + 5)$ is written $9(N + 5)$.

Similarly $(p - q) \div 7$ means "subtract $q$ from $p$ and divide the result by 7." It is usually written $\frac{p - q}{7}$ or $\frac{1}{7}(p - q)$.

Also, just as $a^2$ means $a \times a$, so $(x + y)^2$ means "add $y$ to $x$ and multiply the result by itself."

Brackets show the order in which operations must be performed.

Thus $5 + 2(3 + 4)$ means "add 4 to 3, double the sum, add the result to 5."

$5 + 2(3 + 4) = 5 + 2 \times 7 = 5 + 14 = 19$.

But $(5 + 2)3 + 4$ means "add 2 to 5, multiply the sum by 3, add 4 to the result."

$(5 + 2)3 + 4 = 7 \times 3 + 4 = 21 + 4 = 25$.

And $(5 + 2)(3 + 4)$ means "add 2 to 5, add 4 to 3, multiply the first sum by the second sum."

$(5 + 2)(3 + 4) = 7 \times 7 = 49$. 
FIRST NOTIONS

Example 8. 3 oz. of cocoa are packed in a tin which weighs t oz. when empty. What is the weight of 5 tins of cocoa?

One tin when full of cocoa weighs (t + 8) oz.

∴ 5 full tins weigh 5(t + 8) oz.

Again, the 5 empty tins weigh 5t oz., and the cocoa by itself weighs 5 × 8 oz., ∴ the whole weighs (5t + 5 × 8) oz.

∴ 5(t + 8) oz. = (5t + 5 × 8) oz.
∴ 5(t + 8) = 5t + 5 × 8.

Thus, if an expression in a bracket is multiplied by a number, each term in the bracket must be multiplied by that number, when the bracket is removed.

Test this statement numerically. Suppose that the tin weighs 2 oz.; then t = 2.

5(t + 8) = 5(2 + 8) = 5 × 10 = 50.

and 5t + 5 × 8 = 5 × 2 + 5 × 8 = 10 + 40 = 50.
∴ if t = 2, each expression equals 50.

EXERCISE I.f

State in words the meaning of the following; and afterwards find their values if N = 6, a = 2.

1. 2(N + 1). 2. (N + 3)a. 3. 5(N - 5). 4. 3(a + 3).
5. a(3N + 2). 6. (2a - 1)4. 7. N(a + 1). 8. (2a - 1)N.

Find the values of the following, if x = 3, y = 5, c = 8.

9. 2(c - x). 10. c(y - x). 11. (x - 2)y. 12. \( \frac{x + y}{c} \).

Remove the brackets in the following; and afterwards test the results for the given values.

13. 3(2l + 1); l = 4.
14. 4(a - 3); a = 5.
15. 2(3p - 4); p = 3.
16. 3(5 + 2x); x = 2.
17. a(b + 3); a = 4, b = 2.
18. 2p(3a - 4); p = 5, a = 2.
19. 3x(y + z); x = 2, y = 3, z = 4.
20. 5a(b - 2c); a = 4, b = 5, c = 2.

Find the values of the following, if a = 3, b = 2, c = 6, d = 5.

21. a + b(c + d).
22. (a + b)(c + d).
23. (a + b)c + d.
24. a + (b + c)d.
25. a + bc + d.
26. (b + c) \div a.
27. b + c \div a + d.
28. (b + c) \div (a + d).
29. (b + c) \div a + d.
30. c \div (a + d).
31. b + c \div (a + d).
32. b \div a + c \div c.
33. (a + b)^2.
34. a^2 + (b + c)^2.
35. (a + b + c)^2.
Use of Brackets

Example 9. 8 oz. of cocoa are packed in a tin which weighs t oz. when empty; 30 tins of cocoa are packed in a box weighing \( W \) oz. What is the total weight?

One tin, when full of cocoa, weighs \((8 + t)\) oz.

\[ \therefore \text{30 tins of cocoa weigh } 30(8 + t) \text{ oz.} \]

\[ \therefore \text{the case of cocoa weighs } W \text{ oz.} + 30(8 + t) \text{ oz.} \]

It is simpler to write this answer in the form \([W + 30(8 + t)]\) oz.

**Note.** If one set of brackets is enclosed, as here, inside another set, different shapes of brackets should be used, because this makes it easier to see what the expression means.

**EXERCISE I. g**

*Use brackets when answering the following questions; do not remove the brackets.*

1. 10 lb. of jam fills a jar which weighs \( w \) lb. when empty. What is the weight of 10 full jars? The jars are packed in a box weighing \( P \) lb. What is the total weight?

2. I have 10 coins; \( n \) of them are sixpennies and the rest are shillings. What is their value in pence?

3. A cellar contains \( k \) bottles; six dozen of them hold a pint each and the rest a quart each. How many pints are there altogether?

4. A man buys glasses at the rate of 6d. each for the first dozen and 5d. each for the rest. What is the cost in pence (i) of 20 glasses, (ii) of \( x \) glasses, where \( x > 12 \)?

5. In Fig. 15, \( AB \) is 10 inches, \( AP \) is \( l \) inches long. \( PB \) is divided into three equal parts. What is the length of each part?

![Fig. 15](image)

6. In Fig. 15, \( AB \) is \( a \) inches, \( AP \) is 2 inches; and \( PB \) is divided at \( Q, R \) into three equal parts. What are the lengths of (i) \( PQ \); (ii) \( AQ \); (iii) \( BR \); (iv) \( AR \)?

7. A man at a hotel is charged £1 a day for the first four days, and 16s. a day afterwards. What is the amount of his bill in shillings for (i) 7 days, (ii) \( n \) days, if \( n > 4 \)?

8. A journey by car takes \( 3\frac{1}{2} \) hours; for the first \( t \) hours, the speed is 20 miles per hour, and for the remainder it is 15 miles per hour. What is the distance travelled (\( t < 3\frac{1}{2} \))?
9. A man's rate of pay is as follows: ordinary time, 10d. an hour; overtime, 15d. an hour. The regular working day is 7 hours. What does he receive for a day on which he works (i) 5 hours; (ii) 10 hours; (iii) \( t \) hours if \( t < 7 \); (iv) \( T \) hours if \( T > 7 \)? Answer in pence.

10. A workman is paid \( xs. \ yd. \) per day. What does he receive (in shillings) for \( t \) days' work?

Write down expressions for the following, Nos. 11-24:

11. The result of multiplying \( a - b \) by \( 2\frac{1}{2} \).
12. The result of subtracting \( P + 2Q \) from \( R \).
13. The result of subtracting \( p + q \) from \( r - s \).
14. Five times the sum of \( x \) and \( y \).
15. Subtract \( c \) from \( d \) and divide the result by \( 3 \).
16. Three-quarters of the sum of \( A \) and \( B \).
17. The average of \( p, q, r \).
18. The product of \( 2a \) and \( (y - z) \).
19. The number by which \( 10 \) exceeds the sum of \( x \) and \( y \).
20. The square of the sum of \( a \) and \( b \).
21. The number of pence in \( (p + q) \) shillings.
22. The number of feet in \( (a + b) \) yards \( l \) inches
23. Subtract from \( a \) half the sum of \( b \) and \( c \).
24. The product of three consecutive whole numbers of which (i) \( l \) is the least; (ii) \( g \) is the greatest; (iii) \( m \) is the middle number.

25. A box full of sugar weighs \( W \) lb. and when empty weighs \( w \) lb. What is the weight of the sugar in \( n \) boxes?

26. When a boy is \( x \) years old his father is \( y \) years old; how much younger is the boy than his father? How old is the boy when his father is \( z \) years old?

27. A cask when empty weighs \( P \) lb. and when full weighs \( Q \) lb. What is the weight of the contents when the cask is full? What is the weight of the cask and contents when the cask is half-full?

28. A man was earning \( (a + 3b) \) shillings a week. What are his new wages if the old wages are increased by one-tenth?

29. The letter rate for inland postage is as follows: 1½d. for the first 2 oz. and 1⅛d. for each additional 2 oz. or part thereof. What is the cost in pence of sending a letter weighing \( W \) oz., if \( W \) is an even whole number?

30. Use the rule in No. 29 to find the cost in pence of postage on a letter weighing \( (2W + 1) \) oz. where \( W \) is a whole number.
CHAPTER II
FORMULAE

Construction of Formulae

The following illustrative examples are intended for oral discussion.

Example 1. The Area of a Rectangle.

If you have a rectangle 4 in. long, 3 in. broad, you can divide it as in Fig. 16, so that there are 3 rows and each row contains 4 one-inch squares. Therefore there are $4 \times 3$ one-inch squares altogether. But the area of a one-inch square is called 1 sq. inch.

:. the area of the rectangle is $4 \times 3$ sq. in.

It would be a waste of time to use this process whenever the area of a rectangle is required. We therefore look for the general method.

Suppose a rectangle is $l$ inches long and $b$ inches broad. Show, by repeating the argument used above, that, if $l$ and $b$ are whole numbers, its area is $l \times b$ sq. inches. Call the area $A$ sq. inches:

Then $A = l \times b$.

This relation is called a formula. Although only proved here for integral values of $l$ and $b$, it can be shown to be true for all values; we therefore use it whenever we wish to find the area of a rectangle.

Example 2. The Measurement of Speed.

A policeman times a car over a measured distance of 270 feet.

If the car takes 6 seconds, its average speed is $\frac{270}{6} = 45$ ft. per sec.

If the car takes $4 \frac{1}{2}$ seconds, its average speed is $\frac{270}{4\frac{1}{2}} = 60$ ft. per sec.
FORMULAE

The policeman wishes to know the speed in miles per hour.

\[ \text{45 ft. per sec.} = \frac{45 \times 60 \times 60}{3 \times 1760} \text{ miles per hour} \]

\[ = \frac{45 \times 15}{22} \text{ m.p.h.} = 30 \frac{1}{2} \text{ m.p.h.} \]

Similarly, 60 ft. per sec. may be expressed in miles per hour; but time is saved by using the general formula.

If the car takes \( t \) sec. to travel \( s \) ft., its average speed is \( \frac{s}{t} \) ft. per sec.

\[ = \frac{s}{t} = \frac{60 \times 60}{3 \times 1760} \text{ miles per hour} \]

\[ = \frac{s}{t} \times \frac{15}{22} \text{ m.p.h.} = \frac{15s}{22t} \text{ m.p.h.} \]

Therefore the average speed of a car, which takes \( t \) sec. to travel \( s \) ft., is \( V \) miles per hour, where

\[ V = \frac{15s}{22t} \]

This is the policeman's formula; he does not waste time by working through the argument by which it is proved, whenever he uses it.

EXERCISE II. a

[If brackets occur in the Answer, do not remove them.]

1. What is the cost in pence of an inland telegram with (i) 10 words, (ii) 17 words, (iii) \( n \) words? [The charge is: 12 words or less, 1s.; each word, more, 1d.]

2. Find a formula for the third angle of a triangle, given the other two angles, \( A^\circ \), \( B^\circ \). [Invent an example with special numbers.]

3. Find a formula for the time a train takes to go a given distance, \( s \) miles, at \( v \) miles an hour. [Invent an example with special numbers.]

4. When making tea, put in one spoonful for each person and one for the pot. How much tea is required for \( n \) people (i) if one teapot is used, (ii) if \( k \) teapots are used?

5. Find a formula for the cost of \( n \) collars, sold as follows:

<table>
<thead>
<tr>
<th>Number of collars</th>
<th>1</th>
<th>4</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost</td>
<td>1s. 3d.</td>
<td>5s.</td>
<td>15s.</td>
</tr>
</tbody>
</table>

6. A member of a club receives 3 free tickets and is charged 5s. for each additional ticket. How much does he pay for (i) 7 tickets, (ii) \( p \) tickets? Give each answer firstly in shillings, secondly in £.

---

p.s.a.
7. String is sold by weight, as follows:

<table>
<thead>
<tr>
<th>Weight</th>
<th>8 oz.</th>
<th>12 oz.</th>
<th>1 lb.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost</td>
<td>1s. 4d.</td>
<td>2s.</td>
<td>2s. 8d.</td>
</tr>
</tbody>
</table>

Find a formula for the cost of W oz.

8. Use the data of No. 7 to find a formula for the weight of a ball of string which costs (i) p pence, (ii) z shillings, (iii) x shillings y pence.

9. A book is t in. thick, each cover is c in. thick, and there are n sheets. What is the thickness of each sheet? [First invent a numerical example.]

10. A piece of cotton is wound n times round a cylinder. When unwound, it measures l inches. What is the girth of the cylinder (i) in inches, (ii) in feet?

11. The wheel of a car makes r revolutions when the car travels s yards. What is the circumference of the wheel (i) in yd. (ii) in ft.?

12. What is (i) the perimeter, (ii) the area of the rectangle in Fig. 18?

13. Fig. 19 represents a cross with four equal arms. What is (i) the perimeter, (ii) the area of the cross?

14. How many times can a jug holding p pints be filled from a cask holding g gallons? [1 gallon = 8 pints.]

15. In Fig. 20, AB is longer than BC by the same amount that BC is longer than CA. What is the distance of A from B?

16. How many posts are required for a straight fence AB, if a post is needed every 5 yards, see Fig. 21, and if the length of the fence is 35 yards, 100 yards, 5l yards, l being an integer?

17. How many posts are required for the fence of a field ABCD, see Fig. 21, 5l yards long, 5b yards broad, if a post is needed every 5 yards; l and b being integers?
19. A closed box, see Fig. 22, is \( x \) in. long, \( x \) in. wide, \( y \) in. deep, outside measurements. Find (i) the total length of all its edges, (ii) the total area of its outside surface, (iii) its volume. What do these results become in terms of \( x \), if \( y = 2x \) ?

![Fig. 22 and Fig. 23](image)

19. Fig. 23 represents two cubical tins, one of side \( 2l \) in., the other of side \( l \) in., inside measurements; the larger is full of water, the smaller is empty. How much water is left in the larger tin, when the smaller has been filled from it? How many more tins, each equal to the smaller tin, can be filled from the larger?

20. Neither tin in Fig. 23 has a lid. With the data of No. 19, find the area of the inside surface of each tin.

21. Find the area of Fig. 24 by taking the sum of two rectangles.

22. Find the area of Fig. 25 by taking the difference of two rectangles.

23. Find the area of Fig. 26 by taking the difference of two rectangles.

24. Find the perimeter of the areas bounded by the continuous lines in (i) Fig. 24, (ii) Fig. 25, (iii) Fig. 26.

25. In Fig. 27, \( AB = b \) in., \( AC = c \) in., and \( P \) is the mid-point of \( BC \). What is the length of (i) \( BC \), (ii) \( PC \), (iii) \( AP \)?
26. Fig. 28 represents a rectangular brick wall pierced by three equal windows, each \( \frac{a}{b} \) ft. high, \( w \) ft. wide. The units for the dimensions shown in the figure are feet. What is (i) the length and height of the wall, (ii) the area of the brickwork?

27. Fig. 29 represents a sheet of paper from which equal squares, side \( h \) in., have been cut away at each corner. The paper is folded to form a box, without a lid; the dotted lines show the creases. (i) Find the length, breadth and height of the box. (ii) What is the volume of the box? (iii) Write down, without any working, the total area of its outside surface.

(Note. For additional examples, see Appendix, Ex. S. 2, p. 275.)

Substitution

The process of finding the numerical value of an expression, when the letters it contains stand for given numbers, is called substitution. It is required for making use of formulae.

Example 3. If \( x = 3 \), \( y = 2 \), find the values of:

(i) \( 2x^2 \); (ii) \( 5xy \); (iii) \( xy^2 \).

(i) \( 2x^2 = 2 \times 3^2 = 2 \times 9 = 18 \).

(ii) If \( x = 3 \), \( y = 2 \), \( 5xy = 5 \times 3 \times 2 = 30 \).

(iii) If \( x = 3 \), \( y = 2 \), \( xy^2 = 3 \times 2^2 = 3 \times 4 = 12 \).

EXERCISE II. b

If \( x = 4 \), \( y = 3 \), find the values of the following:

1. \( 2x + y \). 2. \( 2xy \). 3. \( y^2 \). 4. \( 2x^3 \).
5. \( xy - x \). 6. \( \frac{2y}{x} \). 7. \( x^2 - y^2 \). 8. \( (x - y)^2 \).
FORMULAE

9. $\frac{1}{2}xy^2$. 10. $3x - 4y$. 11. $x - \frac{6}{y}$. 12. $6 + \frac{x}{2}$.

If $p = 3$, $q = 0$, find the values of the following:
13. $r + 2$. 14. $2q$. 15. $2pq$. 16. $3p - 2q$.
17. $p^2q$. 18. $p^2 + q^2$. 19. $p(p - q)$. 20. $\frac{q}{p}$.

If $r = 3$, $s = 5$, $t = 4$, find the values of the following:
21. $rst$. 22. $2rst$. 23. $r + s + t$. 24. $t(r + s)$.
25. $r + s - 2t$. 26. $ts^2$. 27. $\frac{rt}{s}$. 28. $s^2 - 2rt$.

If $a = 1$, $b = \frac{1}{2}$, find the values of the following:
29. $a^2$. 30. $b^2$. 31. $ab$. 32. $a + b$.
33. $6ab$. 34. $1 - b$. 35. $\frac{1}{b}$. 36. $a^2b^2$.

If $c = 3$, find the values of the following:
37. $c^2 - 1$. 38. $2c^3$. 39. $\frac{c}{12}$. 40. $\frac{24}{c}$.
41. $c^2 - 3c$. 42. $(c - 1)^2$. 43. $(2c)^2$. 44. $(c - 2)$.

If $e = 6$, $f = 2$, $g = 0$, find the values of the following:
45. $ef + fg$. 46. $3f^2 + 2eg$. 47. $\frac{e + f}{e - f}$.
48. $\frac{1}{2}efg$. 49. $e^2 - 3f^2$. 50. $(e - f)^2 + (f - g)^2$.

If $l = \frac{1}{2}$, $m = \frac{1}{2}$, find the values of the following:
51. $l - m$. 52. $lm$. 53. $\frac{l}{m}$. 54. $1 - l$.
55. $\frac{1}{l}$. 56. $6lm^2$. 57. $2l^2 + 3m^2$. 58. $\frac{3}{l} - \frac{2}{m}$.

59. If $a = 3b$ and $b = 4$, what is $a$?
60. If $y = x^2$ and $x = 5$, what is $y$?
61. If $R = \frac{1}{2}r$ and $r = 16$, what is $R$?
62. If $y = 2x + 3$ and $x = 4$, what is $y$?
63. If $c + d = 8$ and $d = 3$, what is $c$?
64. If $py = 48$ and $y = 8$, what is $p$?
65. If $s = 3t$ and $t = 2$, what is $st$?
66. If $N - n = 3$ and $n = 7$, what is (i) $N$; (ii) $N + n$; (iii) $Nn$?
67. If $y - z = 2$ and $y = 9$, what is (i) $z$; (ii) $y + z$?
68. If \( \frac{a}{b} = 6 \) and \( b = 2 \), what is (i) \( a \); (ii) \( gb \)?

69. If \( bl = 3 \) and \( l = 2 \), what is (i) \( b \); (ii) \( 2(b + l) \)?

70. If \( y = \frac{1}{2}x - 1 \) and \( x = 6 \), what is \( y \)?

71. If \( y = x^2 - 2x \) and \( x = 5 \), what is \( y \)?

[Note: For additional drill-examples, see Exercise E.P. 1, p. 133.]

Use of Formulae

Example 4. From a masthead, \( h \) feet above the surface of the sea, it is possible to see a distance of \( \sqrt{\frac{3h}{2}} \) miles. How far can an observer see, if he is (i) at the top of a mast 54 feet above the sea, (ii) at the top of a cliff 150 feet high?

(i) Put \( h = 54 \). Then the distance of the horizon is

\[
\sqrt{\frac{3 \times 54}{2}} \text{ mi.} = \sqrt{81} \text{ mi.} = 9 \text{ miles.}
\]

(ii) Put \( h = 150 \). Then the distance of the horizon is

\[
\sqrt{\frac{3 \times 150}{2}} \text{ mi.} = \sqrt{225} \text{ mi.} = 15 \text{ miles.}
\]

The process of obtaining special results from a general formula is called substituting in the formula.

EXERCISE II. c

1. M miles is nearly the same distance as \( \frac{8M}{5} \) kilometres. Find the number of kilometres in 20 miles.

2. P pints of water weigh about \( 1\frac{1}{4}P \) lb. What is the weight of (i) 2 pints of water, (ii) 1 gallon of water? Find in ounces the weight of half a pint of water.

3. \( v \) miles an hour is the same speed as \( \frac{22v}{15} \) feet per second. Express in ft. per sec. (i) 60 m.p.h.; (ii) 45 m.p.h.; (iii) 5 m.p.h.

4. \( s \) yards a minute is the same speed as \( \frac{3s}{88} \) miles an hour. Express in m.p.h. (i) 880 yd. \( \frac{1}{8} \) hr., (ii) 88 yd. per min.

5. When making tea for \( N \) people, if \( t \) teapots are required, you must use \( (N + t) \) teaspoonfuls of tea. How much is used for (i) 5 people (1 teapot); (ii) 10 people (2 teapots); (iii) 30 people (3 teapots)?

6. With summer time, the middle of the day may be taken as 1 o'clock; and so, if the Sun rises at \( t \) o'clock a.m., it sets at \( (14 - t) \) o'clock p.m. On May 20, in London, the Sun rises at 5 a.m., when does it set? What is the length of the day?
7. Fig. 30 represents a polygon with 5 sides (i.e. a pentagon). If a polygon has $n$ sides, the sum of its interior angles is $(2n - 4)$ right angles. What is the sum of the angles of (i) a quadrilateral, (ii) a pentagon, (iii) a decagon (10 sides)? Does the formula hold for a triangle?

8. $n$ postcards cost $(n + 1)$ pence if $n < 12$ and cost $(n + 2)$ pence if $11 < n < 23$. What is the cost of (i) 8 postcards, (ii) 20 postcards?

9. For corrugated iron roofing, see Fig. 31, the relation between the pitch, $P$ in., and the depth, $d$ in., is $d = \frac{1}{4}P$.

The pitch should not be less than 3 inches; what can you say about the depth?

The pitch should not be more than 5 inches; what can you say about the depth?

10. The "rise" $AB$, $R$ inches, and the "tread" $BC$, $T$ inches, of a staircase, see Fig. 32, are often connected by the rule $R = \frac{1}{4}(24 - T)$. The tread should not be less than 9 inches; what can you say about the rise? The tread should not be more than 12 inches; what can you say about the rise?

11. Repeat No. 10, using the rule, $R = \frac{66}{T}$, which is sometimes employed.

12. The $n$th day of March is the same day of the week as the $(n + 5)$th day of September. If the 1st of March is a Monday, what is the date of the first Monday in September? What day of the week is Sept. 1?

13. Using the rule in No. 12, if the 4th of March is a Monday, what is the date of the first Monday in September?

If the 27th of March is a Wednesday, what is the date of the last Wednesday in September?

14. $F^\circ$ Fahrenheit is the same temperature as $C^\circ$ Centigrade, if $F - 32 = \frac{9C}{5}$, see Fig. 33; this is equivalent to $F = 32 + \frac{9C}{5}$. 
Express in degrees Fahrenheit: (i) 100° C., (ii) 0° C., (iii) 15° C., (iv) 35° C.

15. The formula connecting $F$ and $C$ in No. 14 may also be stated in either of the following ways:

(i) $F = \frac{9}{5}(C + 40) - 40$; (ii) $C = \frac{5}{9}(F + 40) - 40$.

From (i), find $F$ for $15°$ Centigrade; then use this value of $F$ to find $C$ from (ii).

Show in a similar way that the formulae agree for $50°$ Fahrenheit.

16. Use the rules in No. 15 to state in words how to convert degrees Centigrade to degrees Fahrenheit, and vice versa.

17. With the data of Fig. 34, the triangle is right-angled. What results are obtained by taking (i) $x = 2$, (ii) $x = 3$, (iii) $x = 4$?

![Fig. 34.](image)

18. A parcel whose weight does not exceed $(3n - 1)$ lb. may be sent by post for $(3n + 3)$ pence, where $n$ is an integer. But parcels for which $n > 4$ are not accepted by the post office. State in words what this rule means by taking $n = 1, n = 2, n = 3, n = 4$.

19. An oak beam, $l$ feet long, $b$ inches wide, $d$ inches thick, is built into a wall at one end and carries a load of $W$ cwt. at the other end, see Fig. 35. It will break if $W > \frac{5bd^2}{4l}$. Will such a beam, 5 ft. long, 8 in. wide, 3 in. thick, break under a load of 1 ton?

If the thickness is 6 inches, instead of 3 inches, will it break under a load of 3 $\frac{1}{2}$ tons?

20. The marks obtained in an examination run from 20 to 70. They are converted so that an original mark $n$ becomes $t$, where $t = 2(n - 20)$. What is the new mark if the original mark is (i) 31, (ii) 58, (iii) 40?

What is the new top mark and the new bottom mark?

21. A wind blowing at $v$ miles an hour exerts a direct pressure of $P$ lb. per sq. foot of surface it strikes, where $P = \frac{v^2}{200}$. What pressure must a hoarding 10 ft. high, 20 ft. wide, be able to withstand against (i) a breeze of 10 m.p.h., (ii) a gale of 25 m.p.h., (iii) a storm of 50 m.p.h.?

[Note. For additional examples, see Appendix, Ex. S. 3. p. 280.]
FORMULAE

Generalised Statements about Numbers

Numbers represented by letters need not be whole numbers.

Example 5. Give a general statement which includes the following special facts:

\[ 4 \times 7 = 7 \times 4 ; \quad 8 \times 6 = 6 \times 8 ; \quad 3\frac{1}{2} \times 2\frac{1}{2} = 2\frac{1}{2} \times 3\frac{1}{2}. \]

In words, if we take any two numbers, the result of multiplying the first by the second is equal to that obtained by multiplying the second by the first.

Using letters, if \( a \) and \( b \) are any two numbers,

\[ a \times b = b \times a. \]

Example 6. Give a general statement to include the following; and test one of them.

\[ 7^2 - 5^2 = 4 \times 6 ; \quad 10^2 - 8^2 = 4 \times 9 ; \quad 14^2 - 12^2 = 4 \times 13. \]

Test the last statement; \( 14^2 = 196, \quad 12^2 = 144 ; \]

\[ \therefore \quad 14^2 - 12^2 = 196 - 144 = 52 = 4 \times 13. \]

In words, if we take three consecutive integers, the square of the largest minus the square of the smallest equals four times the middle number.

Using letters, if \( l, \ l + 1, \ l + 2 \) are three consecutive integers,

\[ (l + 2)^2 - l^2 = 4(l + 1). \]

The similarity of 3 or 4 special facts suggests that a general statement, which includes all of them, is true; but you cannot be certain that this is so, unless you prove it. Try to prove the statement in Example 6 by using Fig. 65, p. 49, and stating the area of the whole figure and the area of each compartment.

EXERCISE II. d.

Give general statements which include the following:

1. \[ 5 + 9 = 9 + 5 ; \quad 2 + 8 = 8 + 2 ; \quad 11 + 3 = 3 + 11 ; \quad A + B = \ldots. \]
2. \[ 5 + 5 = \text{twice } 5 ; \quad 8 + 8 = \text{twice } 8 ; \quad 2\frac{1}{2} + 2\frac{1}{2} = \text{twice } 2\frac{1}{2}; \quad N + N = \ldots. \]
3. \[ \frac{1}{3} \times 3 = 1 ; \quad \frac{1}{7} \times 7 = 1 ; \quad \frac{10}{7} \times 10 = 1 ; \quad \frac{1}{c} \times c = \ldots. \]
4. \[ 5 \times \frac{3}{2} = 4 ; \quad 7 \times \frac{3}{2} = 4 ; \quad 11 \times \frac{3}{2} = 4. \]
5. \[ 6 \times 1 = 6 ; \quad 7 \times 1 = 7 ; \quad 12 \times 1 = 12. \]
6. \[ 3 \times 0 = 0 ; \quad 8 \times 0 = 0 ; \quad 12 \times 0 = 0. \]
7. 8 exceeds \(5\) by \(8 - 5\); \(13\) exceeds \(7\) by \(13 - 7\); \(A\) exceeds \(B\) by \(\ldots\).

8. What must you add to \(5\) to make \(8\)? \(\text{Answer, } 8 - 5\).
What must you add to \(9\) to make \(15\)? \(\text{Answer, } 15 - 9\).
What must you add to \(4\) to make \(N\)?
What must you add to \(A\) to make \(12\)?
What must you add to \(P\) to make \(Q\)?

9. You obtain \(20\) if you multiply \(5\) by \(\frac{4}{5}\).
You obtain \(36\) if you multiply \(2\) by \(\ldots\).
You obtain \(24\) if you multiply \(N\) by \(\ldots\).
You obtain \(c\) if you multiply \(10\) by \(\ldots\).
What is the general statement?

10. \(\frac{4 \times 7}{5} = \frac{3 \times 11}{8} = \frac{3 \times 9}{7} = \frac{6 \times 9}{6}\).

11. \(2 \times 5 = 10\) and this is an even number.
\(2 \times 13 = 26\) and this is an even number.
If \(N\) is any whole number, \(2 \times N\) is \(\ldots\).

12. \(2 \times 6 + 1\) is odd; \(2 \times 11 + 1\) is odd.
If \(N\) is any whole number, \(\ldots\).

13. \(2 \times 6 - 1, 2 \times 9 - 1, 2 \times 20 - 1\) are all odd numbers.

14. Since \(12 + 5 = 17\), \(\therefore 12 = 17 - 5\).
Since \(9 + 11 = 20\), \(\therefore 9 = 20 - 11\).
If \(a + b = c\), then \(\ldots\).

15. Since \(4 \times 5 = 20\), \(\therefore 4 = \frac{20}{5}\); since \(9 \times 6 = 54\), \(\therefore 9 = \frac{54}{6}\).
If \(N \times 3 = 21\), then \(N\) \(\ldots\). If \(a \times b = c\), then \(a\) \(\ldots\).

16. \(5 - 1, 5, 5 + 1\) are consecutive whole numbers.
\(14 - 1, 14, 14 + 1\) are consecutive whole numbers.

17. \(5 - 2, 5, 5 + 2\) are consecutive odd numbers.
\(13 - 2, 13, 13 + 2\) are consecutive odd numbers.
If \(N\) is \(\ldots\).

18. \(6 - 2, 6, 6 + 2\) are consecutive even numbers.
\(14 - 2, 14, 14 + 2\) are consecutive even numbers.

19. What is the next even number (i) above \(18\), (ii) above \(N\) if \(N\) is even, (iii) above \(N\) if \(N\) is odd?

20. What is the next odd number, (i) above \(22\), (ii) above \(N\) if \(N\) is even, (iii) above \(N\) if \(N\) is odd?

21. What is the odd number (i) just below \(21\), (ii) just below \(t\), if \(t\) is even?

22. The greatest of four consecutive odd numbers is \(N\). What is the least?

23. How many whole numbers are there between (i) \(6\) and \(10\), (ii) \(3\) and \(11\), (iii) the whole numbers \(x\) and \(y\), if \(y > x\)?
24. Write down 5 consecutive whole numbers, such that the middle number is (i) 6, (ii) 10, (iii) the whole number \( n \).

25. Up to and including 8, there are 4 even numbers.
   Up to and including 20, there are 10 even numbers.
   What is the general statement?

26. Up to and including 9, there are 5 odd numbers.
   Up to and including 15, there are 7 odd numbers.
   What is the general statement?

27. The sum of 4, 5, 6 is three times 5.
    The sum of 11, 12, 13 is three times 12.
    The sum of \( N, N + 1, N + 2 \) is ...
    The sum of \( a - 1, a, a + 1 \) is ...

28. The sum of 5 and 7 is twice 6.
    The sum of 16 and 18 is twice 17.
    What is the general statement?

29. Can you say at a glance the number of crosses in each of the groups (i), (ii), (iii) in Fig. 36? How many are there in the various compartments of each group?

![Fig. 36.](image)

What is the value of (i) \( 1 + 3 \), (ii) \( 1 + 3 + 5 \), (iii) \( 1 + 3 + 5 + 7 \)?

Draw a group containing 5 rows of crosses with 5 crosses in each row and divide it up in the same way. What is the value of \( 1 + 3 + 5 + 7 + 9 \)?

What is the sum of (i) the first 6 odd numbers, (ii) the first 20 odd numbers, (iii) the first \( n \) odd numbers?

30. Take a number of groups of crosses as in Fig. 36 and add them up along diagonals. Prove in this way from group (iii) that \( 1 + 2 + 3 + 4 + 3 + 2 + 1 = 4^2 \). Can you generalise this result?

[Note. For additional drill examples, see Exercise E.P. 2, p. 134. For a revision exercise on Ch. I-1, see Appendix, Ex. R. 1, p. 257.]
CHAPTER III

EASY PROBLEMS AND EQUATIONS

General Instructions

(i) Numbers may be represented by letters. Do not use letters to represent quantities, i.e. numbers-of-things. (See p. 10.)

(ii) In any problem which deals with quantities, state your units clearly.

(iii) Whenever you use a letter to represent an unknown number, first write down a sentence, stating exactly what this letter represents.

Example 1. Express by an equation the following statement:
I think of a number; I then double it and add 7 to it; the result is 25.

We might say
(Twice the number thought of) + 7 = 25.
But it is simpler to say: denote the number thought of by \( n \). Then
\[ 2n + 7 = 25. \]

The form in which this statement is now written is called an equation, and \( n \) is called "the unknown." The process of discovering the unknown number is called solving the equation; and the value of the unknown number is called the root of the equation.

The reader may be able to see what this value is; methods for finding it will be given later.

Example 2. The weight of a box and its contents is 10 lb.; the contents weigh 3 lb. more than the box.

Express these facts by an equation.

Suppose that the box by itself weighs \( W \) lb.

Then the contents of the box weigh \((W + 3)\) lb.

\[ W \text{ lb.} + (W + 3) \text{ lb.} = 10 \text{ lb.} \]

This is a relation between quantities; we can obtain from it a relation between numbers, \( W + W + 3 = 10 \).

\[ 2W + 3 = 10. \]
EXERCISE III. a

Criticise and correct, where necessary, the following statements:

1. Let the weight of the man be \( w \).
2. Suppose the length of the room is \( l \).
3. Let \( x \) be the cost of the house.
4. The distance of A from B is \( s \) miles.
5. Let the speed of the train be \( v \).
6. Eggs are sold at N for a shilling.
7. Let \( \angle ABC = x \).
8. Let \( t \) be the time it takes to walk a mile.
9. Let the required even number be \( N \).
10. If the radius of a circle is \( r \), its diameter is \( 2r \).
11. \( x + y = 180^\circ \).
12. If a room is \( l \) ft. long, \( b \) ft. broad, \( h \) ft. high, its volume is \( l \times b \times h \).
13. His age is \( x \).
14. Let the price of butter be \( y \).
15. If \( n \) is an odd number, \( n + 3 \) is an even number.

EXERCISE III. b

What expressions represent the results in Nos. 1\textsuperscript{16}?

Think of a number \( n \),

1. Double it.
2. Add 2 to it.
3. Halve it.
4. Subtract 2 from it.
5. Divide it by 10.
6. Diminish it by 4.
7. Multiply it by 3.
8. Increase it by 3.
9. Subtract it from 12.
10. Square it.
11. Take two-thirds of it.
12. Divide 24 by it.
13. Multiply it by 3 and subtract 3 from the result.
14. Add to it one half of itself.
15. Increase it by 5 and halve the result.
16. Diminish it by 3 and multiply the result by 4.

Write the statements in Nos. 17-25 as equations. If you can say at sight what the root of the equation is, do so.

17. I think of a number, then subtract 9; the result is 13.
18. I think of a number, then double it; the result is 48.
19. I think of a number, divide it by 4, add 7; the result is 12.
20. I think of a number, then add 25; the result is the same as multiplying the original number by 4.

21. From the cube of a number, I subtract the square of the same number; the result is 180.

22. The sum of two consecutive numbers is 37.

23. The difference between a number and its square is 72.

24. I think of a number, add 5, double the result, then subtract the original number. This gives 17.

25. A number exceeds 5 by half the amount that it falls short of 17.

Write the statements in Nos. 26-37 as equations. State clearly in each case what the unknown represents. If you can say at sight or with very little work what the root of the equation is, do so.

26. A scuttle of coal weighs 28 lb.; the coal weighs three times as much as the scuttle.

27. The rods AB and CD in Fig. 37 are cut down by equal amounts, so that one is just twice the length of the other.

28. A is now 53 years old and B is 21; in a certain number of years' time, A will be just twice as old as B.

29. In Fig. 38, BP exceeds PC by 1\(\frac{1}{2}\) inches and BC is 5 inches.

30. In Fig. 38,
\[ \angle BAC = 4 \angle ABC = 4 \angle ACB. \]

31. In Fig. 38, AB = AC = \(\frac{1}{2}\)BC, and the perimeter of the triangle ABC is 10\(\frac{1}{2}\) inches.

32. A hall is twice as broad and three times as long as it is high; it contains 6000 cu. ft. of air.

33. One tap pours water into a bath twice as fast as another tap; it takes 108 gallons to fill the bath; this is done, by both taps together, in 6 minutes.

34. The sum of the angles of a certain polygon is 14 right-angles. [If a polygon has \(n\) sides, the sum of its angles is \((2n - 4)\) right-angles.]

35. A man walks at 4 miles an hour from his house to a station, see Fig. 39, and returns home at 3 miles an hour; the two journeys together take 3\(\frac{1}{2}\) hours.
36. The boundary of the shaded area in Fig. 40, formed by cutting a quadrant away from a square is \(5\) inches. [Take the length of the circumference of a circle of radius \(r\) inches as \(\frac{44r}{7}\) inches.]

37. The shaded area in Fig. 40 (see No. 36) is \(10\frac{1}{2}\) sq. inches. [Take the area of a circle of radius \(r\) inches as \(\frac{22r^2}{7}\) sq. inches.]

Give statements about numbers which correspond to the equations in Nos. 38-45.

38. \(n + 6 = 33\). 39. \(n - 7 = 19\).
40. \(n + 5n = 42\). 41. \(n + \frac{n}{4} = 20\).
42. \(20 - n = 9\). 43. \(n + 18 = 3n\).
44. \(n + (n + 1) + (n + 2) = 27\). 45. \(2n^3 - 3n = 1000\).

Solving Equations

Example 3. What is \(n\), if \(n + 6 = 19\) ?
I think of a number; then add 6; the result is 19.
By adding 6, I obtain 19; \(\therefore\) the number = 19 - 6 = 13.
\(\therefore\) \(n = 13\).

Check: \(n + 6 = 13 + 6 = 19\).

Example 4. What is \(x\), if \(7x = 63\) ?
I think of a number; then multiply it by 7; the result is 63.
By multiplying by 7, I obtain 63; \(\therefore\) the number = \(\frac{63}{7}\) = 9.

Check: \(7x = 7 \times 9 = 63\).

EXERCISE III. c

Express each equation as a "think of a number" problem; then find the answer and check it.

1. \(n + 8 = 17\). 2. \(n - 5 = 11\). 3. \(3y = 21\).
4. \(\frac{z}{2} = 9\). 5. \(x + 1\frac{1}{2} = 2\). 6. \(n - 2\frac{1}{2} = 5\).
7. \(N - \frac{n}{7} = 0\). 8. \(8x = 0\). 9. \(\frac{y}{5} = 7\).
10. \(z + 2z = 18\). 11. \(3n - 2 = 10\). 12. \(\frac{x}{3} + 4 = 9\).
13. \(4y - y = 21\). 14. \(\frac{z + 2}{4} = 5\). 15. \(\frac{z}{4} + 2 = 8\).
General Methods for solving Simple Equations

Example 5. Solve \( n - 17 = 46 \).
Since the numbers \( n - 17 \) and 46 are equal, if we add 17 to each of them, the results will be equal.
\[
\therefore \ n - 17 + 17 = 46 + 17 ;
\]
\[
\therefore \ n = 63.
\]

Example 6. Solve \( x + 17 = 46 \).
Since the numbers \( x + 17 \) and 46 are equal, if we subtract 17 from each of them, the results will be equal.
\[
\therefore \ x + 17 - 17 = 46 - 17 ;
\]
\[
\therefore \ x = 29.
\]

Example 7. Solve \( 7y = 91 \).
Since the numbers \( 7y \) and 91 are equal, if we divide each number by 7, the results will be equal.
\[
\therefore \ 7y \div 7 = 91 \div 7.
\]
\[
\therefore \ y = 13.
\]

Example 8. Solve \( \frac{z}{3} = 14 \).
Since the numbers \( \frac{z}{3} \) and 14 are equal, if we multiply each number by 3, the results will be equal.
\[
\therefore \ \frac{z}{3} \times 3 = 14 \times 3 ;
\]
\[
\therefore \ z = 42.
\]

The solution of every simple equation is performed by applying one or more of these four arguments.

Example 9. Solve \( 3x - 12 = \frac{2x}{3} + 16 \).
Add 12 to each side, \( \therefore \ 3x - 12 + 12 = \frac{2x}{3} + 16 + 12 \);
\[
\therefore \ 3x = \frac{2x}{3} + 28.
\]
Subtract \( \frac{2x}{3} \) from each side, \( \therefore \ 3x - \frac{2x}{3} = \frac{2x}{3} + 28 - \frac{2x}{3} \);
\[
\therefore \ 3x - \frac{2x}{3} = 28.
\]
EASY PROBLEMS AND EQUATIONS

Multiply each side by 3, \( \therefore 3x \times 3 - \frac{2x}{3} \times 3 = 28 \times 3; \)
\( \therefore 9x - 2x = 84; \)
\( \therefore 7x = 84. \)
Divide each side by 7, \( \therefore x = 12. \)
Check: Left side = \( 3x - 12 = 3 \times 12 - 12 = 36 - 12 = 24. \)
Right side = \( \frac{2x}{3} + 16 = \frac{2 \times 12}{3} + 16 = 8 + 16 = 24. \)
\( \therefore \) when \( x = 12, \) left side = right side.

The four arguments used in solving simple equations may be summarised as follows:

(i) Equal numbers may be added to each side.
(ii) Equal numbers may be subtracted from each side.
(iii) Each side may be multiplied by equal numbers.
(iv) Each side may be divided by equal numbers.

From (i), if \( x - a = b, \) then \( x = b + a. \)

From (ii), if \( x + a = b, \) then \( x = b - a. \)

This shows that any term may be moved from either side of an equation to the other side if the sign in front of it is changed.

It is sometimes convenient to reverse the order of an equation.
Thus, if \( 3 = x, \) we can at once say \( x = 3, \) without using any of the arguments (i)-(iv) above.

Example 10. Solve \( \frac{5}{x} = \frac{2}{3}. \)
Multiply each side by 3x, \( \therefore \frac{5}{x} \times 3x = \frac{2}{3} \times 2x; \)
\( \therefore 15 = 2x; \therefore 2x = 15. \)
Divide each side by 2, \( \therefore x = \frac{15}{2} = 7 \frac{1}{2}. \)

When solving an equation

Start by writing down the equation exactly as it stands in the book: do not try to simplify it in your head.

Be careful not to confuse the symbols = and \( \therefore \).
= means "is equal to"; \( \therefore \) means "therefore."

Thus we say: \( 3x = 15; \therefore x = 5. \)

D.S.A.
When checking an equation
Substitute for "the unknown" in each side separately, as in Example 9.
Substitute in the equation as it is given, not in any simplified form of it. The object of checking is to make sure that your answer is right. You cannot be certain that it is, unless you substitute in the actual equation given you.

EXERCISE III. d

Solve the equations in Nos. 1-10, explaining each step in the argument as in Examples 9, 10 above: and check each answer.

1. (i) \(3n = 21\);
   (ii) \(8x = 64\);
   (iii) \(1\frac{1}{2}R = 12\);
   (iv) \(0.3m = 1.2\).

2. (i) \(a - 4 = 7\);
   (ii) \(p - 7 = 0\);
   (iii) \(z - 2\frac{1}{2} = 6\frac{1}{2}\);
   (iv) \(w - 3.2 = 1.9\).

3. (i) \(l + 5 = 12\);
   (ii) \(x + 2\frac{1}{2} = 7\);
   (iii) \(t + \frac{3}{5} = 1\);
   (iv) \(t + 7.4 = 9\).

4. (i) \(\frac{1}{3}n = 3\);
   (ii) \(\frac{y}{4} = 7\);
   (iii) \(\frac{n}{3} = \frac{2}{7}\);
   (iv) \(\frac{q}{5} = 4.2\).

5. (i) \(\frac{4R}{5} = 2\);
   (ii) \(\frac{3p}{8} = 12\);
   (iii) \(\frac{2m}{3} = 10\);
   (iv) \(\frac{3x}{5} = 1\).

6. (i) \(3 \geq n - 2\);
   (ii) \(7 = p + 5\);
   (iii) \(4\frac{1}{2} = t + 1\frac{1}{2}\);
   (iv) \(6.2 = 3.8 + k\).

7. (i) \(3y - 4 = 8\);
   (ii) \(4R + 3 = 13\);
   (iii) \(5N + 2 = 17\);
   (iv) \(7z - 3 = 25\).

8. (i) \(6y - 15 = 0\);
   (ii) \(5y + \frac{1}{2} = 7\frac{1}{2}\);
   (iii) \(7l - 3 = 9\);
   (iv) \(9t + 8 = 20\).

9. (i) \(0.3x = 6\);
   (ii) \(2.5t = 11\);
   (iii) \(1.6z = 12\);
   (iv) \(0.5k = 0\).

10. (i) \(\frac{3y}{2} = 1\);
    (ii) \(\frac{4a}{7} = 2\frac{2}{7}\);
    (iii) \(\frac{3t}{8} = 0\);
    (iv) \(\frac{5t}{8} = \frac{1}{5}\).
Solve the following equations and check each answer.

11. \(2p - 8 = p - 3\).
12. \(2l + 4 = 19 - l\).
13. \(t + 7 = 17 - 4t\).
14. \(3(n - 7) = 12\).
15. \((2k + 1) = 20\).
16. \(7(3y - 1) = 28\).
17. \(4(t - 5) = 0\).
18. \(l - \frac{1}{4}l = 6\).
19. \(m + \frac{m}{5} = 24\).
20. \(5 = 3R\).
21. \(0 = 2t - 7\).
22. \(10y = y\).
23. \(x - \frac{2x}{7} = 10\).
24. \(\frac{p}{2} - \frac{p}{3} = 1\).
25. \(l = 1 + \frac{l}{4}\).
26. \(\frac{R}{5} - \frac{2}{7}R = 0\).
27. \(\frac{1}{4}(3x - 1) = 7\).
28. \(3 = \frac{1}{4}(2x + 1)\).
29. \(3t = 5 - 7\).
30. \(\frac{1}{4}k = 1 - 7\).
31. \(2 - 3y = 6\).
32. \(\frac{3}{x} = 2\).
33. \(\frac{3}{4} = \frac{2}{x}\).
34. \(\frac{5}{2p} = 6\).
35. \(\frac{2R - R}{3} - \frac{R}{2} = 1\).
36. \(\frac{t}{3} + \frac{t}{5} = 0\).
37. \(\frac{2y - 3}{5} = 3\).
38. \(\frac{6x}{7} + 2 = 11\).
39. \(5 = \frac{3k - 1}{4}\).
40. \(2 - 3 = \frac{3}{4}\).
41. \(\frac{t + 1}{5} = \frac{t + 3}{6}\).
42. \(\frac{y - 1}{3} = \frac{2y + 1}{7}\).
43. \(\frac{2R + \frac{R}{5}}{5} = \frac{R - R}{3}\).
44. \(\frac{3x}{5} - \frac{x}{2} = \frac{1}{2}\).
45. \(1 + \frac{7a}{2} = a + 6\).
46. \(\frac{1}{2} + \frac{1}{3}z = \frac{1}{12}\).
47. \(R - 0.7R = 12\).
48. \(\frac{x}{0.3} = 4\).
49. \(\frac{y}{3} - 1\frac{1}{2} = 1\frac{1}{2}\).

[Note. For additional drill-examples, see Exercise E.P. 3, p. 135.]

General Procedure in the Solution of Problems

1. Read the question carefully. Do not start to try to work it out before you are sure that you understand what you are given and what you are asked to find out.

2. Take a letter to stand for some unknown number which the problem involves.

If the problem involves quantities, state clearly what the unit is.

Never say, let the length be \(x\); a clear statement would be, let the length of the room be \(x\) feet.

Never say, let \(x\) be the cost of eggs; a clear statement would be, suppose one dozen eggs cost \(x\) pence.
3. Check the answer by using the actual data of the problem. It is not sufficient to check by substituting in the equation, because your equation may be wrong.

Example 11. Share 10 shillings between two boys, A and B, so that A receives 1s. 6d. more than B.

Suppose that B's share is \( b \) shillings. Now A receives \( 1 \frac{1}{2} \) shillings more than B.

\[ \therefore \text{A's share is } (b + 1\frac{1}{2}) \text{ shillings.} \]

But the total sum shared out is 10 shillings;

\[ \therefore (b + 1\frac{1}{2}) + b = 10; \]
\[ \therefore 2b + 1\frac{1}{2} = 10; \]
\[ \therefore 2b = 10 - 1\frac{1}{2} = 8\frac{1}{2}; \]
\[ \therefore b = 4\frac{1}{2}. \]

\[ \therefore \text{B's share } = 4\frac{1}{2} \times 4 = 4s. 3d. \]
\[ \therefore \text{A's share } = 4s. 3d. + 1s. 6d. = 5s. 9d. \]

Check: The total sum shared out equals 4s. 3d. + 5s. 9d. = 10s.

**EXERCISE III. e**

Solve the following problems by Algebra, and check each answer.

1. I think of a number, divide it by 4 and add 11; the result is 17. What is the number?

2. I think of a number, add to it one-third of itself; the result is 28. What is the number?

3. The result of adding 42 to a certain number is the same as multiplying that number by 4. What is the number?

4. If I halve a certain number and add 1, the result is the same as dividing the number by 3 and adding 4. What is the number?

5. The sum of two consecutive numbers is 55. What are they?

6. What number exceeds 17 by the same amount as it falls short of 55?

7. The sum of three consecutive even numbers is 72; what are they?

8. From three-quarters of a certain number, 3 is subtracted; the result is two-thirds of that number. What is the number?

9. I think of a number, add 2 to it, multiply the sum by 5 and then subtract 7; the result is 23. What is the number?
10. I think of a number, double it, add 12, then divide by 2. I now subtract the number I first thought of; the result is 6. Can you find the original number?

11. In Fig. 41, find AP if AP is 3 inches longer than PB.

12. In Fig. 41, find AP if AP is twice the length of PB.

13. In Fig. 41, find AQ if AQ is \(2\frac{1}{2}\) times as long as BQ.

14. In Fig. 42, find \(\angle AOC\) if \(\angle BOC = 5\angle AOC\).

15. In Fig. 42, find \(\angle AOC\) if \(\angle BOC\) exceeds twice \(\angle AOC\) by 90°.

16. With the data of Fig. 43, find \(x\).

17. An excursion ticket is one-quarter of the ordinary fare. I save 5s. 6d. by taking an excursion ticket. What is the ordinary fare?

18. The flagstaff PQ in Fig. 44 is one-fifth of the height AB of a tower; Q is 90 feet above the ground. What is the height AB?

19. In Fig. 45, find \(\angle B\) if \(\angle B = \angle C = 4\angle A\).

20. In Fig. 46, find \(\angle P\) if \(\angle P = 3\angle Q\).
21. The sum of the angles of an n-sided polygon is \((2n - 4)\) right angles. How many sides has a polygon if its angle-sum is 20 right angles?

22. I buy a house, and have to spend one-third as much on repairs. The total cost is £3000. What did the house cost?

23. Fig. 47 represents a hurdle whose width is \(1\frac{1}{2}\) times its height. It is made of metal strips, whose total length is 36 feet. How high is the hurdle?

24. Fig. 48 represents a skeleton wire cage; \(AB\) is twice as long as \(BC\) and as \(CD\). The total length of wire is \(5\frac{1}{2}\) ft. What are the lengths of \(AB\) and \(BC\)?

25. The wind backs from direction \(AO\) to direction \(BO\), see Fig. 49, if the change of direction is \(100^\circ\), find its first direction.

[Note. For additional examples, see Appendix, Ex S. 4, p. 283.]

TEST PAPERS. A, 1-8

A. 1

1. Write more shortly:
   (i) \(2a \times 4\); (ii) \(3b \times 3bc\); (iii) \(\frac{3t}{4} \times 4\).

2. If \(y = 3x - 1\), find (i) the value of \(y\) if \(x = 4\);
   (ii) the value of \(x\) if \(y = 14\).

3. Solve the equations, (i) \(t + \frac{2}{3}t = 3\frac{1}{2}\);
   (ii) \(7 = \frac{1}{3}(2W + 3)\).

4. A boy is now \(k\) years old and his father is \(4k\) years old. How old will the father be when the boy is \(2k\) years old?
   What is \(k\) if the boy was 1 year old when his father was 25?
5. Fig. 50 represents a rectangle. Find the numerical value of (i) its perimeter, (ii) its area.

\[ 2(x + 1) \text{ in.} \]
\[ \frac{3x}{2} \text{ in.} \]
\[ (5x - 7) \text{ in.} \]

**Fig. 50.**

A. 2

1. Simplify:
   (i) \[ n - \frac{3n}{2} \]
   (ii) \[ 5y + y - 2y \]

2. (i) If \[ y = 2x^2 - 3x \] and if \[ x = 4 \], find \[ y \].
   (ii) Subtract 9\( r \) inches from 2\( r \) feet. Answer in yards.

3. (i) A clock loses \( n \) minutes per week; express this loss in seconds per hour.
   (ii) If a clock loses \( t \) seconds an hour, it loses \( T \) minutes a week, express \( t \) in terms of \( T \).

4. Solve the equations:
   (i) \[ \frac{3N}{8} = 9 \]
   (ii) \[ \frac{2r}{5} - \frac{r}{3} = 1 \]
   (iii) \[ p + 4 = q - 7 = 20 \]

5. With the data of Fig. 51, find \( x \) if \( x = 1\frac{1}{2}y \).

A. 3

1. Write more shortly:
   (i) \[ 3b \times 3ab \]
   (ii) \[ \frac{c}{2} \times 6c \]
   (iii) \[ 4p - \frac{1}{2} \]

2. A tram fare is 3d.; what is the cost of \( n \) journeys, (i) in pence, (ii) in shillings?

3. (i) Simplify \[ 2a(3a - 5b) + 5b(2a - 3b) \].
   (ii) What is the value of \( x \) if \[ 7y + 2y^2 = xy^2 + 2y \] when \( y = 3 \)?

4. Fig. 52 represents two people leaving A and B at 10 a.m.

\[ u \text{ m.p.h.} \]
\[ v \text{ m.p.h.} \]

\[ 10 \text{ miles} \]

**Fig. 52.**

How far apart are they at (i) 11 a.m., (ii) 10.30 a.m.?

What can you say about \( u \), \( v \) if they meet at 11.15 a.m.?
5. Find two consecutive numbers such that one-quarter of the smaller exceeds one-fifth of the larger by 4.

A. 4

1. If \( p = 1, q = 2, r = 3, s = 4 \), find the values of:
   
   (i) \( p + q(r + s) \); (ii) \( (p + q)r + s \).

2. The square of \( x + \frac{1}{x} \) is \( x^2 + \frac{1}{x^2} + 2 \). Show that this is true (i) when \( x = 1 \), (ii) when \( x = 3 \).

3. Solve the equations:
   
   (i) \( \frac{n}{4} - \frac{3}{5} \); (ii) \( 4 - z = 1.8 \); (iii) \( 17 - x = 2(2 + x) \).

4. The circumference of the wheel of a car is \( p \) feet; how many revolutions per minute does the wheel make when the car is travelling \( v \) yards per second?

5. A has £100, B has £50: but after B has paid A what he owes him, A has three times as much as B. What was the debt?

A. 5

1. (i) Simplify \( 7c - 2c + 3c \).
   
   (ii) If \( p = 2q \) and \( q = 5s \), find the value of \( pq \) when \( s = \frac{1}{2} \).

2. Express in florins the difference between \( £x \) and \( 3x \) half-crowns.

3. Solve the equations:
   
   (i) \( \frac{3}{n} = \frac{2}{7} \); (ii) \( \frac{3}{4}(y' + 5) = 9 \).

If \( 4(x + a) = 5(x - a) + 4 \), find \( a \) when \( x = 2 \).

4. A hotel bill for \( n \) days is \( 15n \) shillings if \( n < 4 \), and is \( 12(n + 1) \) shillings if \( n > 4 \). What is the bill for 3 days and for 5 days? What would you expect the bill to be for 4 days?

5. Fig. 53 shows the lengths of the sides of a rectangle in inches. What are their numerical values?

A. 6

1. If \( p = \frac{a}{b}, q = \frac{b}{c} \), find the values of
   
   (i) \( 2pq \); (ii) \( p^2 + q^2 \); (iii) \( \frac{p + q}{p - q} \).

2. I buy \( 6N \) eggs; how many are left after \( 6 \) have been eaten? What is \( N \) if one dozen are left?
3. Solve the equations:
   (i) \( l = 3(10 - l) \);
   (ii) \( 2.7x - 3.4 = 1.9x + 1.4 \).

4. Find in terms of \( x \) the third angle of the triangle in Fig. 54. Show that the triangle is isosceles if \( x = 10 \) or 34 or 40.

![Fig. 54.](image)

5. Find \( t \) if \( t^\circ \) Centigrade is the same temperature as \( 2t^\circ \) Fahrenheit. (Use the fact stated in Ex. II. c, No. 14, p. 23.)

A. 7

1. If \( b = 3 \), \( c = 0 \), \( d = 5 \), find the values of
   (i) \( \frac{d + 1}{b} \); (ii) \( bc - c \); (iii) \( (b + 1)(d - 1) \).

2. Simplify (i) \( t + 3t + t \); (ii) \( 3k \times 2k \); (iii) \( 9p \div 15 \).

3. Solve the equations:
   (i) \( \frac{5}{3} = \frac{2}{w} \); (ii) \( \frac{x - 7}{2} = \frac{3x + 1}{11} \).

   Prove that \( R^2 + 8r^2 = 6Rr \) if \( R = 6 \) and \( r = 3 \).

4. On a holiday, a man walks a certain distance the first day, and half as far again the second day. He walked \( s \) miles the second day; how far did he walk the first day?

5. A pyramid stands on an \( n \)-sided base. How many more edges than corners has it got?
   What is \( n \) if the sum of the number of edges and number of corners is 25?

A. 8

1. (i) Add \( 3N \) to \( 5N \) and divide the result by 6.
   (ii) Multiply \( 2p \) by \( 6p \).

2. (i) If \( W = 5w \), simplify \( \frac{W - w}{W + w} \).
   (ii) How many \( \frac{1}{2} \)d. stamps can be bought for \( P \) half-crowns?

3. Solve the equations:
   (i) \( \frac{p}{5} = 2.4 \); (ii) \( t - 0.3t = 3.5 \); (iii) \( \frac{1}{2}(3z - 7) = 4 \).
4. A closed tin box is 3c in. long, 2c in. broad, c in. high. What is the sum of the lengths of its edges (i) in inches (ii) in feet? What is the total surface area in sq. inches?

5. O is the centre of the circle in Fig. 55; the arcs AQB and APB are of lengths \((x - 2)\) inches and \((4x + 2)\) inches. Find the numerical value of the length of the circumference of the circle.
CHAPTER IV

ELEMENTARY PROCESSES

Like Terms

An expression of the form $5a - 3a + 4a$ is said to consist of like terms, because it can be reduced to a single term.

Just as $5$ dozen $- 3$ dozen $= (5 - 3)$ dozen $= 2$ dozen,

and $5$ dozen $- 3$ dozen $+ 2$ dozen $= 2$ dozen $+ 4$ dozen

$= 6$ dozen,

so $5a - 3a = (5 - 3)a = 2a$,

and $5a - 3a + 4a = 2a + 4a = (2 + 4)a = 6a$.

When simplifying, work from the left unless brackets or $\times$ and $\div$ signs show that operations must be performed in a different order.

Example 1. Simplify $4a - \frac{1}{2}a$.

$4a - \frac{1}{2}a = (4 - \frac{1}{2})a = 3\frac{1}{2}a,$

or $4a - \frac{1}{2}a = \frac{4a}{1} - \frac{a}{2} = \frac{8a - a}{2} = \frac{7a}{2}.

These two results are equivalent because $3\frac{1}{2}a = 6\frac{1}{3}a = \frac{7}{2} \times a = \frac{7a}{2}$.

Example 2. Simplify $b + \frac{2b}{3} - \frac{b}{6}$.

$b + \frac{2b}{3} - \frac{b}{6} = \frac{6b + 4b - b}{6} = \frac{9b}{6} = \frac{3b}{2}$.

EXERCISE IV. a

Simplify the following expressions:

1. $3a + 7a - 5a$.
2. $2b + 9b - b - 3b$.
3. $3c + 7c - 5c + c$.
4. $6d - 3d - 2d$.
5. $2e - 3e$.
6. $f - \frac{f}{2}$.
7. $\frac{1}{2}g + 2\frac{1}{2}g$.
8. $\frac{h}{2} + \frac{h}{3}$.
9. $\frac{k}{3} + \frac{k}{6}$.
10. $\frac{2l}{3} - \frac{l}{6}$.
11. $2m + m \times 3$.
12. $2n \times \frac{4}{3} - n$. 

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13. \(9p - 3 \times 2p\)  
14. \(3 \times 3q + q\)  
15. \(2r \times 3 - 3r \times 2\)  
16. \(6st - 3st - 2st\)  
17. \(5v^2 - 2v^2 + 3v^2\)  
18. \(3xy - xy\)  
19. \(4yz + 4xy\)  
20. \(6z^2 - 2z^2 - 4z^2\)  
21. \(\frac{2a}{5} + a + \frac{a}{10}\)  
22. \(\frac{9b}{12} - \frac{5}{4}\)  
23. \(\frac{2c}{5} + \frac{1}{2}c - \frac{1}{10}c\)  
24. \(3\frac{1}{2}d - 1\frac{1}{4}d + \frac{d}{4}\)  
25. \(10ef - 7fe - \frac{4ef}{3}\)  
26. \(\frac{1}{3}g^2 + g^2 - \frac{2g^2}{3}\)  
27. \(h + h + h + h + h + h + h\)  
28. \(l + l + l + l + \ldots\) fifteen terms.  
29. \(3m + 3m + 3m + \ldots\) ten terms.  
30. \(2p^2 + 2p^2 + 2p^2 + \ldots\) eight terms.

[Note. For additional drill-examples, see Exercise E.P. 4, p. 136.]

Unlike Terms

Example 3. A tourist walked at \(v\) miles an hour for 4 hours on the first day and at \(v\) miles an hour for \(t\) hours on the second day. How far did he walk in the two days?

On the first day, he walked \(4v\) miles.

On the second day, he walked \(tv\) miles.

\[\therefore\] the total distance is \((4v + tv)\) miles.

In this answer, \(4v\) and \(tv\) are unlike terms; we cannot simplify the expression unless the numerical value of \(t\) is given.

We can of course say that the tourist walked in the two days for \((4 + t)\) hours at \(v\) miles an hour, and therefore the total distance is \((4 + t)v\) miles or \(v(4 + t)\) miles. Here, we say that \(4\) and \(t\) are unlike terms, because \(4 + t\) cannot be written more simply unless the value of \(t\) is known.

The expressions \(4v + tv\), \((4 + t)v\) and \(v(4 + t)\) are equal, and each involves the sum of two unlike terms.

Example 4. Find the short-hand form of

\[3a + 2 + a + 8b - 2a - 3b.\]

In this expression, \(3a + a - 2a\) are like terms, also \(8b - 3b\) are like terms.

\[\therefore\] the expression = \(3a + a - 2a + 8b - 3b + 2\)

\[= 2a + 5b + 2.\]

There is no shorter way of writing this expression, if \(a\) and \(b\) represent any numbers whatever.
EXERCISE IV. b

1. What is the length of a fence formed by 10 hurdles each \( r \) feet long and 12 hurdles each \( s \) feet long?

2. What is the total bill for \( c \) lb. of tea at 2s. per lb. and \( p \) lb. of sugar at 4d. per lb.? Answer (i) in pence, (ii) in shillings.

3. Each mesh in Fig. 56 is \( l \) in. long and \( b \) in. broad. What is the total length of wire required for the network?

4. Fig. 57 represents a skeleton box with base \( x \) inches square and \( y \) inches high. What length of wire is used in making it?

5. A boy works for \( p \) hours on Sunday, \( q \) hours on each Wednesday and on each Saturday, and \( r \) hours on each other day. How many hours does he work each week?

6. From a rod \( (3l + 2m) \) inches long, a portion \( 2l \) inches long is cut off; what is the length of the remainder?

7. Subtract \( 7x \) pence from \( x \) shillings. Answer in pence.

8. Subtract \( k \) florins from \( £q \). Answer in shillings.

9. Fig. 58 shows the lengths of the sides of a triangle in inches. (i) What is the perimeter? (ii) By how much does \( AB + AC \) exceed \( BC \), and \( BA + BC \) exceed \( AC \)?

10. Fig. 59 shows the lengths of two adjacent sides of a rectangle in inches. (i) What is the perimeter? (ii) What is the length of \( AD + DC + CB \)?

11. If, in Fig. 59, \( r = 3s \), find the perimeter of the rectangle in terms of \( s \) (the units are inches).

12. A man starts with £\((x + y)\). He pays \( x \) bills of 5 shillings each and \( y \) bills of 15 shillings each. How many shillings has he left?
13. A body A weighing \((6x + 4)\) lb. is put in one scale pan of a weighing machine, and a body B weighing \(2x\) lb. is put in the other scale pan. What weight must be added to make them balance?

14. The outside measurements of a box without a lid are: length 3x in., breadth 2x in., height h in.; it is made of wood 1 inch thick. What are the inside measurements?

15. Equal holes, each \(x\) in. long, \(y\) in. wide are punched in the metal sheet, shown in Fig. 60. What is the surface-area of the sheet?

16. A rectangular garden is represented by the shaded area in Fig. 61; it is enlarged, as shown; the units of the given dimensions are yards. What is (i) the final area, (ii) the increase of area?

17. In Fig. 62, \(\angle ABC = (x + 2y)\) degrees, \(\angle BAC = (y + 2x)\) degrees. What is \(\angle ACD\) ?

18. In Fig. 62, \(\angle ACD = (3p + 4q)\) degrees, \(\angle ABC = (2p + q)\) degrees. What is \(\angle BAC\) ?

Write down (when possible) short-hand forms for the expressions in Nos. 19-52. If there is no shorter form, say so.

19. \(b + c + b\).
20. \(r + s + s + r\).
21. \(a + 2 + a\).

22. \(l + m + l + m + l\).
23. \(x + 2 + y + 3 + x\).
24. \(e + e + e + 3f\).

25. \(4 + 4c\).
26. \(3 + 2t + 1\).
27. \(z + 2 + z + r + l\).

28. \(R + r - R + r\).
29. \(10d - 7d - 2\).
30. \(3p - q + p\).

31. \(1 - t + t\).
32. \(2x + 2y\).
33. \(6b - 2b - 3\).

34. \(3a + 4a + 5a + 6\).
35. \(6p + q - 2p + q\).
36. \(3r - 3a\).

37. \(m - l\).
38. \(3e + 2f - e - f\).

39. \(2a \times 3 - 3b \times 2\).
40. \(3p + 5q - q - 2p\).
41. \( r + s + t + u - s + r \)  
42. \( x + y + z + x - y - z \)  
43. \( 3a + b + 4c - a - b \)  
44. \( p + 3q + 3 + 3q - 1 \)  
45. \( 3ab + 3ac \)  
46. \( 3rs + st - sr \)  
47. \( 3bc + 3cb \)  
48. \( pq + p + q \)  
49. \( 2xy + 3x - yx \)  
50. \( 5yz + 2zy - yz - 6yx \)  
51. \( ab + bc + abc \)  
52. \( x + 2y + 1 - x + 1 \)  
53. Add \( xy \) to \( x \)  
54. Subtract \( b \) from \( bc \)  
55. Subtract \( yz \) from \( xy \)  
56. Add \( xy \) to \( xz \)  
57. Increase \( p \) by 1  
58. Decrease \( 2q \) by 2  
59. Add \( R + r \) to \( 3R \)  
60. Add \( a \) to 0  
61. Simplify \( 11 \times 23 - 10 \times 23 \)  
62. Add \( 13 \times 7 \) to \( 13 \times 3 \)

[Note. For additional drill-examples, see Exercise E.P. 4, p. 136.]

**Powers**

The short-hand form of \( x \times x \times x \times x \) is \( x^4 \) and the short-hand form of \( 7 \times x \times x \times x \times x \) is \( 7x^4 \).

The numerical factor 7 in the term \( 7x^4 \) is called the coefficient of the term \( 7x^4 \) or more shortly the coefficient of \( x^4 \).

The term \( 7x^4 \) is said to be of degree 4 in \( x \) or of the 4th degree in \( x \). The symbol \( x^4 \) is read as "\( x \) to the power 4" and the 4 is called the index of \( x \).

Thus, in the expression \( 2x^5 + 4x^3 + x^2 + 3 \), the term of degree 3 is \( 4x^3 \) and its coefficient is 4, the coefficient of \( x^2 \) is 1, and the coefficient of \( x \) is 0, since this term is missing. The numerical term 3 is called the constant term or the term independent of \( x \), because its value does not depend on the value of \( x \), since it does not contain \( x \).

**Example 5.** A shed \( A \), 3x ft. long, 2x ft. wide, is built in a rectangular courtyard, leaving a passage 5 ft. wide along one side and 3 ft. wide along the other. What was the original area of the courtyard?

The courtyard can be divided into four rectangles: \( A, B, C, D \), as shown in Fig. 63.
The area of A is $2x \times 3x$ sq. ft. = $2 \times 3 \times x \times x$ sq. ft. = $6x^2$ sq. ft.
The area of B is $5 \times 2x$ sq. ft. = $10x$ sq. ft.
The area of C is $3 \times 3x$ sq. ft. = $9x$ sq. ft.
The area of D is $3 \times 5$ sq. ft. = $15$ sq. ft.

\[ \therefore \text{the total area is}\ (6x^2 + 10x + 9x + 15) \text{ sq. ft.} \]

Now $10x$ and $9x$ are like terms; \[ \therefore 10x + 9x = 19x. \]

\[ \therefore \text{the total area is}\ (6x^2 + 19x + 15) \text{ sq. ft.} \]

*Note.* This expression cannot be written more shortly, unless we know the value of $x$, because $6x^2$ and $19x$ are not like terms.

$6x^2$ means $6 \times x \times x$ or $6x \times x$.

$19x$ means $19 \times x$.

Just as $7x + 3x = (7 + 3)x$, so

\[ 6x^2 + 19x = 6x \times x + 19 \times x = (6x + 19) \times x, \]

but this is no shorter, and still contains two unlike terms, viz. $6x$ and $19$.

Expressions should always be written in an orderly way. They are usually arranged either in descending powers of the unknown, i.e. beginning with the highest power, then the next highest, and so on, or in ascending powers, i.e. beginning with the constant term, then the term of first degree, then the term of second degree, and so on.

Thus $2x^2 + 4x^3 + x^2 + 3$ is arranged in descending powers of $x$.
And $3 + x^2 + 4x^3 + 2x^5$ is arranged in ascending powers of $x$.

**Example 6.** Simplify $2y^2 + 3y + 2 - y^2 + 2y + 1$ and arrange it in (i) descending powers of $y$, (ii) ascending powers of $y$.

Take the like terms together.

\[ 2y^2 - y^2 = y^2; \ 3y + 2y = 5y; \ 2 + 1 = 3; \]

\[ \therefore \text{the expression} = y^2 + 5y + 3, \text{ descending powers} \]

\[ = 3 + 5y + y^2, \text{ ascending powers} \]
EXERCISE IV. c

Find the area of the figures in Nos. 1-9, all the corners being right-angled, the units of the given dimensions being inches.
Express the answers without brackets.

1. \[ \text{Fig. 64.} \]
2. \[ \text{Fig. 65.} \]
3. \[ \text{Fig. 66.} \]
4. \[ \text{Fig. 67.} \]
5. \[ \text{Fig. 68.} \]
6. \[ \text{Fig. 69.} \]
7. \[ \text{Fig. 70.} \]
8. \[ \text{Fig. 71.} \]
9. \[ \text{Fig. 72.} \]

Simplify, where possible, the following expressions, and arrange them in descending powers:

10. \(8a^2 - 4a^2\).
11. \(3b + b^2 + b\).
12. \(2c^3 + 6c - 3c - 3\).
13. \(p^3 + 2p^2 - 3p\).
14. \(t^2 - 1 + t - 1\).
15. \(1 + r + 2r^2 - r\).
16. \(5s^2 + 8s - 4s^2 - 8\).
17. \(x^2 + 2x + 6 + 3x\).
18. \(y^3 + y^2 + y + 1 + y + y^2\).
19. \(5 + 6z + 3z^2 - 5z\).
20. \(3 + 5k^2 - 2k - 1 + k^2\).
21. \(3e^2 + e^4\).
22. \(7b^6 - 3b - b^6\).
23. \(3 + t^2 + 3t^9\).
24. \(3x^3 - 5x + 7 - x^2 + 1\).
25. \(6 + 6y^3 - 5y^3 - 5y^3\).
26. $3a^3 - 3$. 
27. $b^4 + 4b^3 - b^2 - 4$.
28. $2c^3 - \frac{1}{2}c + 3c^3 + c$.
29. $5 + \frac{3t}{2} + \frac{t^4}{4} + \frac{1}{2}t - \frac{1}{3}$.

30. Arrange in ascending powers:
   (i) $8p + 3 + p^3 - p$;
   (ii) $\frac{c^3}{2} + 1\frac{1}{2} - \frac{c^3}{4} + 2c$.

31. Write down (i) the coefficient of $q^3$, (ii) the constant term, (iii) the term of degree 3 in
   (a) $4 + 2x^3 + 3x^3$;
   (b) $x^6 + x^2$;
   (c) $2x^3 - x + 5$;
   (d) $3x^4 + 2x^3 + 7x - 4$.

32. Write down (i) the term of highest degree, (ii) the coefficient of $p$, (iii) the constant term in
   (a) $12p^3 + 2p^4 + 7p + 2\frac{1}{2}$;
   (b) $p + 100p^2 + 3p^3$;
   (c) $12p^3 + 2p^4 + 5$;
   (d) $\frac{1}{2}p^3 - 8p^2$.

33. Write (i) in ascending powers, (ii) in descending powers
   (a) $4y^4 + 3 - 2y^2$;
   (b) $10t^3 + t^3 + 2t$;
   (c) $s^4 + 1 + 20s$;
   (d) $3z + z^2 - 2z + 3 + 5z^3$.

Multiplication and Division

Example 7. Multiply $x^2$ by $x^3$.

\[ x^2 \times x = x \times x \times x = x^3 \]

\[ \therefore x^2 \times x^3 = x \times x \times x \times x \times x = x^4. \]

Example 8. Multiply $3x$ by $4y$.

\[ 3x \times 4y = 3 \times 4 \times x \times y = 12 \times x \times y = 12xy. \]

Example 9. Multiply $12x^3$ by $\frac{3}{4}xy$.

\[ 12x^3 = 12 \times x^3 \text{ and } \frac{3}{4}xy = \frac{3}{4} \times x \times y. \]

\[ \therefore 12x^3 \times \frac{3}{4}xy = 12 \times x^3 \times \frac{3}{4} \times x \times y = 12 \times \frac{3}{4} \times x^4 \times x \times y = 8x^4y. \]

Example 10. Divide $a^6$ by $a^2$.

\[ \frac{a^6}{a^2} = \frac{a \times a \times a \times a \times a \times a}{a \times a} = a \times a \times a \times a \times a = a^4. \]

Example 11. Divide $12a^3bc$ by $9a^2b$.

\[ 12a^3bc \div 9a^2b = \frac{12 \times a \times a \times a \times b \times c}{9 \times a \times a \times b} = \frac{4}{3} \times a \times c = \frac{4ac}{3}. \]
Elementary Processes

Example 12. What is the square of \(a^4\)?

\[(a^4)^2 = a^4 \times a^4 = a^8.\]

Example 13. Find a value of \(\sqrt{x^6}\).

\[x^3 \times x^3 = x^6; \quad \therefore \text{the square of } x^3 \text{ is } x^6; \]

\[\therefore \text{one value of } \sqrt{x^6} \text{ is } x^3.\]

We shall see later that every number has two square roots.

Exercise IV. d

Simplify the expressions, Nos. 1-12, giving the reasons in full:

1. \(a^3 \times a^2\)
2. \(b^4 \times b^3\)
3. \(c^5 \div c^2\)
4. \(d^4 \div d\)
5. \(p^4 \times p^2\)
6. \(t^6 \div t^3\)
7. \((x^3)^2\)
8. \((y^2)^3\)
9. \(5x \times 2y\)
10. \(3x^2 \times 2x\)
11. \(\frac{1}{2}(2r)^3\)
12. \(3s^2 \times 3s^2\)

Simplify the following:

13. \(a^2 \times a^4\)
14. \(b^4 \times b^2\)
15. \(c^5 \div c^2\)
16. \(d^8 \div d^4\)
17. \(r \times r^3\)
18. \(s^5 \div s\)
19. \(t^4 \times t^4\)
20. \(x^8 \div x^4\)
21. \(4x^2 \times 2x\)
22. \(6y^2 \div 2y\)
23. \(z^3 \times 3z\)
24. \(2ab \times a\)
25. \(10a^4 \div 2a^2\)
26. \(3b \times 3b^3\)
27. \(4c \div c\)
28. \(2a^3 \times 4t^4\)
29. \(4d \times \frac{d}{2}\)
30. \(6 \times \frac{e}{4}\)
31. \(\frac{a^2 \times a^3}{2 \times 3}\)
32. \(\frac{2s}{3} \times \frac{3s}{2}\)
33. \(4d^2 \div \frac{d}{2}\)
34. \(10p^2 \div 2p^2\)
35. \(8r^2 \div 6\)
36. \(12xy^3 \div 3y\)
37. \(ab \times c^2\)
38. \(2b \times 4c\)
39. \(2c^2d \div cd\)
40. \(4m^4 \div 2\)
41. \((3d^2)^3\)
42. \((rs^4)^2\)
43. \(3pq \times 2pr\)
44. \(2r^2s \div 3s\)
45. \(2t^3 \times 3t^2\)
46. \(2t^2 \div 3t\)
47. \(6R^3 \div 4R\)
48. \(\sqrt{z^6}\)
49. \(4b^5c \div \frac{1}{2}bc\)
50. \(3rs \times 2rst\)
51. \(x^2 \times 2yz\)
52. \(6a^2b^2c^2 \div 2abc\)
53. \(6c^4d^2 \div 4c^2\)
54. \((4ef)^2\)
55. \((\frac{1}{2}rs)^3\)
56. \(4pq^3r^2\)
57. \((3r^2a^4)^2\)
58. \((x^3)^2 \div x^3\)
59. \((2x)^3 + (3x)^3\)
60. \((6y)^3 \div (2y)^3\)
61. \(\frac{a^4 \times a^4}{a^4}\)
62. \(\frac{b \times b^3 \times b^4}{b^3}\)
63. \(6cd \times 2ca^2\)
64. \(\frac{ab \times ac}{bc}\)
65. \(2rs \times (3r)^2\)
66. \(\sqrt{(16t^{10})}\)
67. \(6c^2d \times \frac{1}{4}cde\)
68. \((3s)^3 - 2(2s)^3\)
69. \((2pq)^2 \div pq^2\)
70. \(\frac{r^6}{r^2} + 3r^2\)
71. \(t^8 \times t^4 \div t^2\)
72. \(2n^2 \times 6n^2 \div 3n\)

[Note...For additional drill-examples, see Exercise E.P. 5, p. 137.]
H.C.F. and L.C.M.

Factors and Multiples. Since $10ab^2c = 5ab \times 2bc$, we say that
$2bc$ is a factor of $10ab^2c$ and that $10ab^2c$ is a multiple of $2bc$.

Common Factors. Consider the two expressions, $10ab^2c$, $12a^2b^3$.

$10ab^2c = 2 \times 5 \times a \times b \times b \times c$; $12a^2b^3 = 2 \times 2 \times 3 \times a \times a \times b \times b$.

There are several different expressions which are factors of
both these expressions, i.e. common factors of $10ab^2c$, $12a^2b^3$.

$2, a, b, b^2, 2a, 2b, 2b^3, ab, ab^2, 2ab, 2ab^3$.

But each of these is a factor of the last one, $2ab^3$.

We therefore call $2ab^3$ the Highest Common Factor or the
H.C.F. of $10ab^2c$, $12a^2b^3$.

This method for finding the H.C.F. is the same as the method
used in Arithmetic (prime factors).

Common Multiples. Consider the two expressions, $10ab^2c$, $12a^2b^3$.

$10ab^2c \times 12a^2b^3 = 120a^3b^4c$; $120a^3b^4c$ is a common multiple
of $10ab^2c$, $12a^2b^3$. There are of course an unlimited number of
common multiples, e.g. $240a^4b^4c^4d$ is a common multiple.

Since $10ab^2c = 2 \cdot 5 \cdot a \cdot b^2 \cdot c$ and $12a^2b^3 = 2^2 \cdot 3 \cdot a^2 \cdot b^3$, the
smallest common multiple is $2^2 \cdot 3 \cdot 5 \cdot a^3 \cdot b^3 \cdot c = 60a^2b^3c$.

Every other common multiple is itself a multiple of $60a^2b^3c$.

We therefore call $60a^2b^3c$ the Least Common Multiple or the
L.C.M. of $10ab^2c$, $12a^2b^3$.

This method for finding the L.C.M. is the same as the method
used in Arithmetic (prime factors).

EXERCISE IV. e

1. Is 12 a multiple of (i) 4, (ii) 48 ?

Is 6ab a multiple of (i) 2a, (ii) 6a^2b^3 ?

2. Is 18 a multiple of (i) 180, (ii) 6 ?

Is a^2 a multiple of (i) a^2, (ii) 2a^4 ?

3. Is 24 a factor of (i) 12, (ii) 48 ?

Is 6xy a factor of (i) 3x, (ii) 18xy^2 ?

4. Is 24 a common multiple of (i) 2 and 3, (ii) 48 and 240 ?

Is 6x^2y a common multiple of (i) 3x, 2xy, (ii) 6x^2y^2, 12x^2y^3 ?

5. Is 12 a common factor of (i) 2 and 4, (ii) 24 and 36 ?

Is 6ab a common factor of (i) 2a, 3b, (ii) 12a^2b, 18ab^2 ?
6. Is $10x^4y^2$ a multiple of $5x^2y^2$? Simplify $\frac{10x^4y^2}{5x^2y^2}$.

7. Is $3b^3c$ a factor of $6bc^3$?

8. Is $2y^2z$ a common factor of $6y^2z$ and $4y^2z^2$?

9. Is $12x^2y^3$ a common multiple of any of the following pairs: (i) $3x^3y$, $4xy^2$; (ii) $12x^2$, $y^3$; (iii) $10x$, $4y$; (iv) $3x^2$, $y^4$.

10. Which of the pairs in No. 9 have a common factor, other than unity, and what is it?

Find the following:

11. L.C.M. of $3x^2, 2xy$.

12. H.C.F. of $3a^2, 2ab^3$.


15. L.C.M. of $4r^2s, 5rs^2$.


17. L.C.M. of $a, bca^2$.

18. H.C.F. of $2ab, 3c$.

19. L.C.M. of $3a^2b^2, 2a^3b^3$.

20. H.C.F. of $2a^2, 6ab, 4ac$.

21. L.C.M. of $6x^2, 3xy, 6y^2$.

22. L.C.M. of $10x^2, 15x^4$.

23. H.C.F. and L.C.M. of $24, 3x, 3xy, 6xz$.


[Note. For additional drill-examples, see Exercise E.P. 6, p. 138.]

Expressions involving Fractions

Example 14. (i) On a farm of 480 acres, there are 190 acres of arable land and the rest is pasture. What fraction of the farm-land is pasture?

(ii) On a farm of $n$ acres, there are $p$ acres of arable land and the rest is pasture. What fraction of the farm-land is pasture?

(i) Out of 480 ac., there are 190 ac. of arable land;
   \[ \therefore \text{there are } (480 - 190) \text{ ac. } = 290 \text{ ac. pasture}; \]
   \[ \therefore \text{the fraction of the farm under grass is } \frac{290}{480} = \frac{29}{48}. \]

(ii) Out of $n$ ac., there are $p$ ac. of arable land;
   \[ \therefore \text{there are } (n - p) \text{ ac. pasture}; \]
   \[ \therefore \text{the fraction of the farm under grass is } \frac{n - p}{n}. \]

EXERCISE IV. f

1. What fraction is (i) 4 pence of 6 pence, (ii) $c$ pence of $n$ pence, (iii) $p$ pence of $s$ shillings, (iv) $r$ shillings of £1?

2. Express (i) 8 inches in feet, (ii) $l$ inches in feet, (iii) $w$ inches in yards, (iv) $k$ yards in feet, (v) $p$ feet in yards.

3. Express $p$ shillings $q$ pence (i) in pence, (ii) in s., (iii) in £.
4. (i) A school contains 400 pupils; of these, 240 are boys; what fraction of the school are girls?
(ii) A school contains $p$ pupils; of these, $b$ are boys; what fraction of the school are girls?
(iii) In a school, there are $b$ boys and $g$ girls; what fraction of the school are boys?

5. From a stick $l$ inches long, a portion $p$ inches long is cut off. What fraction of the stick remains?

6. After the $n$th day of January, what fraction of January remains?

7. A boy sleeps $k$ hours a day; what fraction of the day is he awake?

8. After motoring $s$ miles, I have gone $\frac{1}{n}$th of the total distance. What is the total distance? How much further have I to go?

9. After $\frac{3}{4}$ of a chest of tea has been used, $p$ lb. of tea remain. What did the chest hold originally?

10. The Sun rises at $x$ a.m. and sets at $y$ p.m.; how many hours long is the day-light? What fraction is this of 24 hours?

11. Find with the data of Fig. 73, the values of

\[
\begin{align*}
(i) \quad & \frac{AB}{BC}, & (ii) \quad & \frac{BC}{AC}, & (iii) \quad & \frac{AC}{AB}.
\end{align*}
\]

\[
\begin{array}{c}
A \quad \text{p in.} \quad \text{q in.} \quad C
\end{array}
\]

\[\text{Fig. 73.}\]

12. How many 1s$d.$ stamps can be bought for (i) 1s., (ii) ps.? What fraction is 1s$d.$ of $q$ shillings?

13. Taking 8 km. as equal to 5 miles, express (i) $l$ km. in miles, (ii) $s$ miles in km.

14. Sweets are sold at 5d. for 2 oz. What is the cost of $n$ oz.? What amount is obtained for $k$ shillings?

15. What fraction is (i) 5 inches of 5 feet, (ii) $l$ inches of $l$ feet, (iii) 3$l$ inches of $l$ feet?

16. A shed $l$ ft. long, $b$ ft. wide, see Fig. 74, is built in the corner of a rectangular enclosure $x$ ft. long, $y$ ft. wide. What fraction of the total area does the shed occupy? What is the value of this fraction if

(i) $x = 2l, y = 2b$, (ii) $x = 2l, y = 3b$.
17. (i) If a man walks 4 miles an hour, how long does he take to walk \( p \) miles, \( (p + q) \) miles?

(ii) If a man cycles 12 miles an hour, how long does he take to cycle \( 3p \) miles? Simplify \( \frac{3p}{12} \); \( \frac{5p}{20} \).

18. Simplify (i) \( \frac{70}{80} \); (ii) \( \frac{7p}{8p} \); (iii) \( \frac{6p}{8} \); (iv) \( \frac{9p}{12q} \).

19. Simplify (i) \( \frac{1}{5} + \frac{3}{4} \); (ii) \( \frac{p}{5} + \frac{p}{4} \); (iii) \( \frac{a}{5} + \frac{b}{4} \).

20. Simplify (i) \( \frac{3}{4} - \frac{1}{5} \); (ii) \( \frac{p}{4} - \frac{p}{5} \); (iii) \( \frac{a}{4} - \frac{b}{5} \).

21. If in Fig. 73 \( AB \) is \( x \) yards, \( BC \) is \( y \) yards, express the lengths of \( AB \) and \( BC \) also in feet and in inches. Then express the fraction \( \frac{AB}{BC} \) in three different ways.

22. How many parts, each \( \frac{2}{3} \) inch long, can be cut off along a line (i) 4 inches long, (ii) \( b \) inches long? Simplify \( b \div \frac{4}{3} \).

23. How many \( \frac{3}{4} \) lb. packets can be made up from \( n \) lb. of tea?

24. The price of coal rises from \( x \)s. per ton to \( y \)s. per ton; how many fewer tons can now be bought for \( \mathbf{L} p \)?

25. A tap can fill a bath in \( t \) minutes. What fraction of the bath is filled in (i) 3 minutes, (ii) half a minute?

26. One tap can fill a bath in \( p \) minutes and another tap can fill it in \( q \) minutes. What fraction of the bath is filled by both taps together, (i) in 1 minute, (ii) in 4 minutes?

27. Fine parallel lines are engraved on a bar at intervals of \( \frac{1}{t} \) of an inch. There are \( n \) lines in all. What is the distance between the first line and the last? [Invent a numerical example.]

28. A packet of paper 1 inch high contains \( n \) sheets. How thick is each sheet?

29. A packet of paper \( \frac{1}{2} \) inch high contains \( 2p \) sheets. How thick is each sheet?

30. (i) A sheet of paper is \( \frac{1}{100} \) inch thick. How many sheets are there in a pile 3 inches high?

(ii) A sheet of paper is \( \frac{1}{n} \) inch thick. How many sheets are there in a pile \( h \) inches high?
Simplification of Fractions

Example 15. Simplify \( \frac{6a^2b^3}{15ab^3} \).

Divide the numerator and denominator by every common factor.

\[
\frac{6a^2b^3}{15ab^3} = \frac{6 \times a \times b^3}{15} \times \frac{a}{b^3} \times \frac{2a}{5b} = \frac{3 \times 1 \times b}{5}.
\]

Or, proceed as follows:

The H.C.F. of \( 6a^2b^3 \) and \( 15ab^3 \) is \( 3ab^2 \).
Divide the numerator and denominator by \( 3ab^2 \).

\[
\frac{6a^2b^3}{15ab^3} = \frac{3ab^2 \times 2a}{3ab^2 \times 5b} = \frac{2a}{5b}.
\]

EXERCISE IV. g

Simplify the following:

1. \( \frac{3a}{3c} \)
2. \( \frac{3a}{5a} \)
3. \( \frac{6c}{6} \)
4. \( \frac{3d}{d} \)
5. \( \frac{2ab}{3ac} \)
6. \( \frac{2a^2}{5a} \)
7. \( \frac{2bc}{c^3} \)
8. \( \frac{2p}{2p} \)
9. \( \frac{r^2}{2r} \)
10. \( \frac{3s^3}{6st} \)
11. \( \frac{4bc}{6cd} \)
12. \( \frac{8de}{6d^2e} \)
13. \( \frac{r}{rs} \)
14. \( \frac{st}{st} \)
15. \( \frac{2s^2t^2}{st} \)
16. \( \frac{12a}{12ab} \)
17. \( \frac{4a^4}{5a^4} \)
18. \( \frac{a^2xy}{ayz} \)
19. \( \frac{xy^2}{y^3x} \)
20. \( \frac{3cd}{6c^3d^3} \)
21. \( \frac{4a^2b}{2a^2c} \)
22. \( \frac{a^4}{q^8} \)
23. \( \frac{12x^2y^3}{8xyz} \)
24. \( \frac{a^6c^6}{a^3c^2} \)
25. \( \frac{a^2b^2c^3}{abc} \)
26. \( \frac{3pq}{p^2} \)
27. \( \frac{x^2 \cdot (xy)}{(xy)^2} \)
28. \( \frac{3ab^2}{6b^3a^4} \)
29. \( \frac{x^2y^2}{2xy} \)
30. \( \frac{4a^3b^2c^3}{2a^5bc} \)
31. \( \frac{b^{10}}{2b^4} \)
32. \( \frac{(3xy)^3}{3xy^3} \)
33. \( \frac{a^2b \cdot ac^3}{c^q \cdot ba} \)

Example 16. Express \( \frac{3x}{y} \) in the form \( \frac{2y^2z}{y} \).

\[
2y^2z = y \times 2yz; \quad \therefore \quad \frac{3x}{y} = \frac{3x \times 2yz}{y \times 2yz} = \frac{6xyz}{2y^2z}.
\]
Example 17. Simplify \( \frac{2p}{3} - \frac{p}{4} + \frac{5p}{6} \).

The L.C.M. of 3, 4, 6 is 12.

\[
\frac{2p}{3} - \frac{p}{4} + \frac{5p}{6} = \frac{8p}{12} - \frac{3p}{12} + \frac{10p}{12} = \frac{8p - 3p + 10p}{12} = \frac{15p}{12} = \frac{5p}{4}.
\]

Example 18. Simplify \( \frac{7}{10a} + \frac{2}{15a} \).

The L.C.M. of 10a, 15a is 30a.

\[
\frac{7}{10a} + \frac{2}{15a} = \frac{21}{30a} + \frac{4}{30a} = \frac{21 + 4}{30a} = \frac{25}{30a} = \frac{5}{6a}.
\]

Example 19. Simplify \( \frac{b}{6c} - \frac{c}{4b} \).

The L.C.M. of the denominators 6c, 4b is 12bc.

\[
\frac{b}{6c} - \frac{c}{4b} = \frac{b \times 2b}{6c \times 2b} - \frac{c \times 3c}{4b \times 3c} = \frac{2b^2 - 3c^2}{12bc}.
\]

Example 20. Express as a single fraction \( \frac{r^2}{s} - t \).

Since \( t = \frac{t}{1} \), the L.C.M. of the denominators s, 1 is s.

\[
\therefore \frac{r^2}{s} - t = \frac{r^2 - ts}{s}.
\]

Note. As soon as the method is understood, the two intermediate steps in Example 19 should not be written down.

EXERCISE IV. h

Copy and complete the following, Nos. 1-14:

1. \( \frac{3}{4} = \frac{12}{16} = \frac{28}{32} = \frac{52}{64} \)
2. \( \frac{a}{b} = \frac{2a}{2b} = \frac{3a}{3b} \)
3. \( \frac{8}{5} = \frac{24}{6} \)
4. \( \frac{c}{d} = \frac{5c}{5d} = \frac{ac}{bc} \)
5. \( \frac{3r}{4s} = \frac{8r}{8s} = \frac{8r}{8st} = \frac{3r}{3} \).

6. \( b = \frac{3}{3} = \frac{a}{a} = b \).

7. \( \frac{l}{m} = \frac{m^2}{3lm} = \frac{kl}{l} \).

8. \( \frac{2t^2}{4tz} = \frac{3t}{2z} = \frac{3t}{t^2} \).

9. \( \frac{x}{yz} = \frac{x}{y} \).

10. \( \frac{1}{yz} = \frac{y}{z} \).

11. \( \frac{1}{e^3} = e^{-3} \).

12. \( f = \frac{1}{2g} \).

13. \( h^2 = \frac{3h^3}{3} \).

14. \( m^2 = \frac{m^3}{m} \).

Simplify the following:

15. \( \frac{2}{3} + \frac{1}{7} \cdot \frac{a}{3} + \frac{a}{7} = \frac{2a}{3} + \frac{a}{7} \).

16. \( \frac{5}{7} \cdot \frac{b}{5} = \frac{b}{7} \).

17. \( \frac{3}{8} \cdot \frac{4}{5} = \frac{3}{8} \cdot \frac{4}{5} = \frac{1}{2} \).

18. \( \frac{5}{9} \cdot \frac{7}{9} = \frac{5}{9} \cdot \frac{7}{9} = \frac{5}{9} \).

19. \( \frac{1}{4} + \frac{1}{10} = \frac{1}{2c} + \frac{5}{5c} = \frac{2c}{5c} \).

20. \( \frac{1}{7} + \frac{r}{s} = \frac{1}{7} + \frac{r}{2s} = \frac{r}{2s} + 1 \).

21. \( 2 - \frac{7}{5} + \frac{2}{y} = \frac{1}{2} - \frac{x}{2y} = \frac{1}{2} - \frac{x}{3y} \).

22. \( \frac{1}{3t} \cdot \frac{6t}{6} \).

23. \( \frac{1}{g} \cdot \frac{1}{4g} \).

24. \( \frac{b}{2c} = \frac{x}{6c} \).

25. \( \frac{m}{n} - 1 \).

26. \( \frac{1}{m} + \frac{1}{p} \).

27. \( \frac{1}{2u} + \frac{1}{3v} \).

28. \( \frac{r + s}{s} \).

29. \( \frac{2r}{s} - t \).

30. \( \frac{1}{ab} \).

31. \( \frac{1}{c} - \frac{c}{2} \).

32. \( \frac{3}{5d} - \frac{1}{10d} \).

33. \( \frac{2x}{3y} - \frac{x}{6y} \).

34. \( \frac{e}{y} - \frac{e^2}{f} \).

35. \( \frac{r + s}{4s + 6r} \).

36. \( t - \frac{2}{5} \).

37. \( \frac{a}{4ab} + \frac{c}{6bc} \).

38. \( \frac{x}{2y} + \frac{y}{2} - 2 \).

39. \( \frac{3c}{2d} \cdot \frac{2c}{3d} + \frac{5c}{6d} \).

40. \( \frac{b}{c} - \frac{c}{d} + \frac{d}{b} \).

41. \( \frac{p + q}{qr} \).

42. \( \frac{ab}{bc} - \frac{ab}{ac} - 1 \).

43. \( \frac{1}{2} \) to \( \frac{1}{2} \).

44. Subtract \( \frac{1}{2} \) from \( \frac{2}{3} \).
45. Subtract $\frac{1}{15n}$ from $\frac{2}{5n}$.

46. Add $\frac{u}{3v}$ to $\frac{u^2}{6uv}$.

47. Add $\frac{r}{4s}$ to $\frac{r}{12s}$.

48. Subtract $\frac{1}{z}$ from $z$.

49. $\frac{7}{20} \times 60$; $\frac{a}{2b} \times 6b$.

50. $\frac{15}{14} \div 3$; $\frac{3d}{2c} \div 3$.

51. $\frac{5}{11} \times 2$; $e \times 2$.

52. $1 \div \frac{3}{4}$; $1 \div \frac{x}{2y}$.

53. $p \times \frac{q}{r}$.

54. $p \div \frac{q}{r}$.

55. $2b \times \frac{c^2}{bq}$.

56. $\frac{l}{m} \div \frac{r}{c}$.

57. $\frac{x}{v} \times \frac{u}{y}$.

58. $\frac{y^2}{z} \times \frac{z^2}{y}$.

59. $a \div \frac{a^2}{b^2}$.

60. $\frac{x}{y} \times \frac{x}{y}$.

61. $p \div \frac{1}{q}$.

62. $\frac{1}{6c^2} \div \frac{1}{4cd}$.

63. $3m \times \frac{3n}{m^2}$.

64. $ab^2 \times \frac{1}{b^2a}$.

65. $\frac{2ab}{15c^2} \times \frac{5bc}{4a^2}$.

66. $\frac{1}{d^2} \div \frac{d^2}{a^2}$.

67. $\frac{a^2}{b^2} \times \frac{b^2}{c^2}$.

68. $\frac{4a^2}{6bc} \div \frac{3ab}{5ac}$.

69. $\left(\frac{u}{3v}\right)^2 \times 3v$.

70. $\frac{ab}{c} \div \frac{b}{c^2}$.

[Note. For additional drill-examples, see Exercise E.P. 7, p. 138.]

Miscellaneous Examples

EXERCISE IV. j

1. How many inches are there in (i) $\frac{8}{3}$ feet, (ii) $\frac{7}{4}$ yards?

2. How many yards are there in (i) $4\frac{1}{2}$ feet, (ii) $8\frac{1}{2}$ inches?

3. What is the length in yards of a train which has $n$ coaches, each coach is $c$ feet long and the engine is $6p$ feet long?

4. A clerk writes $n$ letters an hour, how many minutes does he take to write $\frac{7}{5}$ letters?

5. A rug is 6 feet wide, $l$ feet long; what is its area in (i) sq. ft., (ii) sq. yd.?

6. A man smokes $\frac{3}{4}$ lb. of tobacco a week; how long does W lb. of tobacco last him?

7. The length of fence for a square enclosure is $2p$ yd.; what is the area enclosed in (i) sq. yd., (ii) sq. ft.?
8. Fig. 75 represents the floor of a hall. What is its floor area in terms of \( l \), if the longer side is \( 3l \) feet?

9. Fig. 75 represents the floor of a hall. Find in sq. feet its floor area in terms of \( p \), if its perimeter is \( p \) yards.

10. In Fig. 76, what fraction is the shortest side of the perimeter of the triangle?

11. In Fig. 76, the longest side is \( 2d \) inches. Find in terms of \( d \) the perimeter of the triangle.

12. In Fig. 76, the perimeter is \( p \) feet; find in terms of \( p \), the longest side.

13. What relation connects the areas of the three squares whose sides are those of the triangle in Fig. 76?

14. A pencil costs \( \frac{3p}{2} \) pence; how many pencils can I buy for half-a-crown?

15. I bicycle at \( \frac{1}{2}v \) miles an hour; how far do I go in 40 minutes?

16. The average speed of a train is \( 10u \) miles an hour; how long does it take to go 50 miles? How long would it take if the speed was 10 miles an hour less?

17. Take the number \( N \) and from half of it subtract one-third of it. By what must the result be multiplied to give \( N \)?

18. From a square sheet of cardboard of side \( 3t \) inches a square of side \( 2t \) inches is cut away. What area remains?

19. A train has \( 11n \) compartments with 8 seats each and \( 2n \) compartments with 6 seats each; how many seats are there on the train?

20. With the data of No. 19, find the number of such trains required for 2000 passengers if each has a seat.

21. Fig. 77 shows the dimensions of a brick in inches. What is the sum of its edges in feet?

22. Find in terms of \( t \) the total area of the surface of the brick in Fig. 77, if \( h = \frac{t}{3} \); the units of the given dimensions being inches.

23. A handkerchief costs \( k \) pence and a pair of socks costs one shilling more. What is the total cost of 8 handkerchiefs and 4 pairs of socks (i) in pence, (ii) in shillings?

24. It takes \( x \) men \( t \) days to repair a certain road; how long should it take \( 2x \) men?
25. A tank is 4p ft. long, 2p ft. wide and contains water to a depth of 3p ft.; what is the area of the wetted surface?

26. How many tiles, measuring 8 in. by 6 in., are required for the floor of a hall 2b ft. long, b ft. broad?

27. A photograph l in. long, w in. wide is printed on a sheet of paper \( \frac{5l}{4} \) in. long, 2w in. wide. What area of the paper is not used?

28. A certain milestone on the Winchester-Southampton road reads \((n + 1)\) miles to Winchester, 2n miles to Southampton. How far is Winchester from Southampton? What is \( n \) if this distance is 13 miles?

29. A shopkeeper would make a profit of \( £P \) by selling a table for \( £S \). For what must he sell it to make a profit of (i) \( £(2P) \), (ii) \( £(\frac{1}{2}P) \), (iii) \( £R \).

30. At a shooting gallery, you pay 2d. if you miss and receive 6d. if you hit the target. What is the result if your score is \( \frac{h}{3} \) hits and \( \frac{m}{2} \) misses?

[Note. For additional examples, see Appendix, Ex. 3.5, p. 285. For a revision exercise on Ch. III-IV, see Appendix, Ex. R. 2, p. 259.]

**TEST PAPERS, A. 9-15**

**A. 9**

1. (i) Add \( 6ab - 3a - 2b \) to \( 6a + 4b - 2ab \).

   (ii) For what value of \( W \) are the expressions \( \frac{W}{2} + 1 \) and \( \frac{2W}{3} - 1 \) equal?

2. Copy and complete the following table, if \( 4x + 3y = 10 \):

\[
\begin{array}{c|c|c|c}
   x & 0 & 1 & 2 \\
\hline
   y & 0 & 1 & 3 \\
\end{array}
\]

3. A car uses a gallon of petrol every 18 miles. How far can the car run on \( k \) gallons?

   The car travels at \( v \) m.p.h.; how much petrol is used in half an hour?
4. The relation between the thickness \( T \) inches and the diameter \( D \) inches of a steam engine cylinder is

\[ T = \frac{3}{8} \sqrt{D} + 0.015D. \]

What is \( T \) when \( D = 16 \)?

What is the increase in \( T \) if \( D \) is now increased by 9?

5. The base of a prism is an \( n \)-sided figure. What is the number of (i) its edges, (ii) its corners, (iii) its faces, including the two ends?

What is \( n \) if the number of edges is \( 2\frac{1}{2} \) times the total number of faces?

\[ A \cdot 10 \]

1. (i) Square \( 2x \) and halve the result.
   (ii) If \( r = \frac{3}{4}R \) and if \( r = 9 \), what is the value of \( R - r \)?

2. (i) Find the H.C.F. of \( 4abc^2 \), \( 6a^2bc \), \( 8a^3c^3 \).
   (ii) Simplify \( \frac{10xyz^2}{6x^2yz} \).

3. Solve the equations,
   (i) \( 0.6l = 9 \);
   (ii) \( \frac{n}{3} + \frac{n}{5} = 0 \);
   (iii) \( 3(a + 4) + 2(a - 2) = 20 \).

4. Houses along a side road are built in pairs (semi-detached) with \( s \) yards between each pair; the front width of each pair is \( l \) feet. What length of road, in yards, is needed for 12 houses?

5. A man walks in the direction \( x^\circ \) E. of N. along AB, see Fig. 78, he turns clockwise through \( \left( \frac{x}{2} + 5 \right) \) degrees at B and walks along BC, and then turns anti-clockwise through \( (2x - 15) \) degrees at C; he is now walking due North; find \( x \).

\[ A \cdot 11 \]

1. (i) Divide \( 2p \) by \( 3p \); subtract the result from \( p \).
   (ii) If \( u = 1\frac{3}{4} \) and \( v = 2 \), find the value of \( \frac{1}{u} + \frac{1}{v} \).

2. (i) A lift is licensed to carry \( n \) persons, on the assumption that the average weight of a person is less than 16 stone. What is regarded as the maximum safe load in tons?
   (ii) How many eggs at 2 for 3\( r \) can I buy with \( k \) florins?

3. Solve the equations,
   (i) \( \frac{h}{3} + 12 = h \);
   (ii) \( 3(2y - 1) + 5(3y + 2) = 8(3y - 1) \).

4. A is 6 years old and B is 29; in how many years' time will A be just half as old as B?
5. The stretched length of a spiral spring is \((21 + 2\frac{1}{2}W)\) inches, when it is carrying a load of \(W\) lb. What is the natural (unstretched) length of the spring? What load is required to stretch it to twice its natural length?

A. 12

1. If \(p = 3, q = 5, r = 7\), find the values of
   (i) \(p + r(p + q)\); (ii) \((p + r)q + r\); (iii) \((p + q) + (p + r)\).

2. (i) Add \(3a - b + 2c\) to \(3c - b - 2a\); (ii) express the sum in terms of \(b\) if \(a + 1 = b = c - 1\).

3. (i) Solve the equation, \(\frac{1}{4}(z - 3) = \frac{1}{2}\).
   (ii) When the day is \(t\) hours long, the night is \(\frac{3}{2}t\) hours long. What is \(t\)?

4. A hawser, \(C\) inches in circumference, breaks under a load of \(W\) tons, where \(W = \frac{5}{4}C^2\); the working load is not allowed to exceed one-sixth of the breaking load. What is the maximum working load for a hawser whose circumference is 3 inches?

5. Share £1.10s. between A, B, C so that A gets 6 shillings more than B, and C gets \(\frac{1}{3}\) of what A and B together get.

A. 13

1. Simplify, and write in ascending powers of \(x\),
   \[4x^4 + 5x^3 - 3x + 7 + 4x - 4x^3 - x - 1\.
   
   What is (i) the coefficient of \(x^2\), (ii) the coefficient of \(x\), (iii) the constant term?

2. Simplify (i) \(\frac{1}{15n} + \frac{1}{30n}\); (ii) \(2t^2 \times 3t^3 \div 5t^5\).

3. A knife costs \(p\) pence and a fork costs half as much again; what is the cost in shillings of a dozen forks?

4. Solve the equations,
   \(\frac{1}{u} + \frac{1}{5} = \frac{1}{3};\) (ii) \(n + \frac{5n}{7} = 0.36\).

Prove that the equation \(x^2(11 - x) = 36(x - 1)\) is satisfied by \(x = 2\) and by \(x = 3\) and by \(x = 6\).

5. The length of wire required to make the rectangular grid in Fig. 79 is 20 inches. Find the length and breadth of the grid.
A NEW ALGEBRA

A. 14

1. (i) If \( R = 3r \), simplify \( \frac{R^2 - r^2}{R^2 + r^2} \).

(ii) If \( \frac{x}{2} = \frac{3}{4} = \frac{5}{y} \), find the value of \( 3y(x - 1) \).

2. A man stays 10 days at a hotel; he is charged 15s. a day for the first 2n days and 12s. 6d. a day for the rest of the time. What is his total bill (i) in shillings, (ii) in £? Assume \( n < 5 \).

3. (i) Add \( \frac{r}{4s} \) to \( \frac{r^2}{12rs} \); (ii) Subtract 1 from \( \frac{a + b}{2b} \).

4. Solve the equations,
   (i) \( l - 0.3l = 14 \); (ii) \( \frac{3}{5}(W - 1) = \frac{1}{2}(W + 5) \).

5. A has three times as much money as B. If A gives B four shillings, he will then have just twice as much as B; how much have they between them?
   How much must B give A in order that A may have five times as much as B?

A. 15

1. If \( r = 5 \), \( s = 0 \), \( t = 3 \), find the values of
   (i) \( (r + s)t \); (ii) \( 2s(r + t) \); (iii) \( (r^2 - t^2) - (r - t)^2 \).

2. A rectangle is 7k inches long, 3k inches wide; find the area of a square whose perimeter is the same as that of the rectangle. Which has the larger area, the rectangle or the square, and by how much?

3. A workman is paid 2s. an hour ordinary time and 4s. an hour overtime. He receives £P for a 50-hour week. How many of the 50 hours are counted as overtime? What is the least possible value of \( P \)?

4. (i) Solve the equation, \( \frac{1}{t} + \frac{1}{2t} = \frac{1}{6} \).

(ii) If \( \frac{4}{5}(n + 1) \) and \( \frac{3}{5}n + 1 \) are equal, prove that each of them must equal \( n \).

A starts at a salary of £250 a year and receives an annual increase of £15 a year. B starts at £320 a year and receives an annual increase of £16 a year. After how many years does A receive a larger salary than B?

[For additional test papers on Ch. I, IV, see Appendix, P. 1-5, p. 312.]
CHAPTER V

THE A B C OF GRAPHS

Graphs on Plain Paper

The following example is intended for oral work.

Example 1. A motor-car is fitted with a gauge which shows the number of gallons of petrol in the tank. When full, the petrol gauge readings.

![Diagram showing petrol gauge readings over time](image)

Fig. 80.

tank holds 5 gallons. A motorist starts out at 10 a.m. and notes the readings on the gauge at hourly intervals. The result is shown in Fig. 80.

EXERCISE V. a (Oral)

Use the data and figure of Example 1, above, to answer the following questions:

1. What is the length in inches of the line which represents (i) 2 gallons, (ii) 5 gallons?

2. How much petrol is represented by an upright line of length (i) 1 inch, (ii) 1/2 inch, (iii) 1.5 inches?

3. How much petrol did he start with? How much had he at 1 p.m. and at 4 p.m.?

4. During what time is it probable that the car was not running?

5. About what time did he fill up the tank?
6. If the car averages 24 miles to the gallon, estimate the distances travelled in successive hours.

7. Can you give a meaning to an upright inserted midway between the first and second uprights; if so, what meaning?

8. Can you insert with fair accuracy an upright for 2.30 p.m.?

**EXERCISE V: b**

Draw on plain paper, as in Fig. 80, diagrams to represent the records tabulated below. State in each case (i) whether any meaning can be given to intermediate upright lines, (ii) whether interpolation is possible with fair accuracy without further data.

*Give each diagram a title, and write along each axis what that axis represents and, where suitable, how it is graduated.*

1. The distribution over the world of the chief languages:

<table>
<thead>
<tr>
<th>Language</th>
<th>English</th>
<th>German</th>
<th>Russian</th>
<th>French</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number who speak it, in millions</td>
<td>160</td>
<td>100</td>
<td>100</td>
<td>70</td>
<td>50</td>
</tr>
</tbody>
</table>

2. The average diameter of oak trees of different ages:

<table>
<thead>
<tr>
<th>Age in years</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
<th>70</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter in inches</td>
<td>5</td>
<td>10</td>
<td>14</td>
<td>23</td>
<td>32</td>
<td>41</td>
<td>54</td>
<td>64</td>
</tr>
</tbody>
</table>

3. The average daily receipts of a certain grocer:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Receipts in £</td>
<td>13</td>
<td>11</td>
<td>14</td>
<td>6</td>
<td>12</td>
<td>18</td>
</tr>
</tbody>
</table>

4. The average weight of boys of different ages:

<table>
<thead>
<tr>
<th>Age in years</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight in lb.</td>
<td>79</td>
<td>85</td>
<td>92</td>
<td>102</td>
<td>114</td>
<td>129</td>
<td>142</td>
<td>146</td>
</tr>
</tbody>
</table>

**Use of Squared Paper**

*Both time and trouble are saved by drawing graphs on squared paper.*
Example 2. Full marks each week in a class is 100; the following table shows the marks obtained by a boy in successive weeks of a term:

<table>
<thead>
<tr>
<th>Number of week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marks obtained</td>
<td>56</td>
<td>62</td>
<td>81</td>
<td>64</td>
<td>60</td>
<td>52</td>
<td>78</td>
<td>83</td>
<td>65</td>
<td>70</td>
<td>72</td>
</tr>
</tbody>
</table>

Represent this table by a graph.

First draw a line across the paper and mark points at convenient intervals along it to represent the 1st week, 2nd week, 3rd week, etc.

Next draw a line up the page and graduate this to show the marks obtained. Since no mark is less than 50 or greater than 90, it is unnecessary to show any graduation outside these limits.

These two lines are called the axes of reference and the graduations show the chosen scales.

Lastly draw lines up the page whose lengths represent the marks obtained at the times indicated.

Weekly Marks.

Oral Work, using Fig. 81.

(i) In how many weeks did he obtain more than 70 marks?
(ii) In how many weeks did he obtain less than 60 marks?
(iii) In which week did he obtain (a) most marks, (b) fewest marks?
(iv) Would an upright midway between the 3rd and 4th uprights have any meaning?

Axes and Scales

Axes. A graph records how one quantity varies in size when another quantity, on which it depends, varies. We select values of the latter quantity and then find by observation, or measurement, or calculation, the corresponding values of the former quantity. The axis for the quantity, whose values we select, should always be drawn across the page, and the axis for the quantity, whose values are observed or calculated, should be drawn up the page. Thus, in the example on p. 65, we select special times and then observe the height in the gauge at these times. The time-axis is therefore, in this case, drawn across the page.

The quantity whose values we select is called the independent variable, the quantity whose values we observe or calculate is called the dependent variable.

Scales. Great care is needed in the choice of scales: a bad scale may make the graph worthless.

First, see what range of values is to be represented; then see what length of axis is available for graduation. Thus in Example 2, p. 67, the marks obtained are never less than 50 or more than 90. The lowest graduation on the upright axis is therefore taken as 50 and the highest as 90; also 90 – 50 = 40; . . . for an upright axis, 2 inches long, we take 1 inch to represent 20 marks. If the axis had been graduated from 0 to 100, it would have been necessary to take 1 inch to represent 50 marks; this scale would be inconveniently small.

Choose a scale which makes plotting and reading easy. 1 inch may be taken to represent 1, 10, 100, etc., or 0.1, 0.01, etc., or 2, 20, etc., or 5, 50, etc., or 0.2, 0.5, etc. Occasionally, 1 inch may be taken to represent 4, 40, etc., but this is not so easy a scale to work with; 1 inch should not be taken to represent 3 or 7, etc.
THE ABC OF GRAPHS

The object in representing facts graphically is to convey information rapidly. To make a graph easy to understand, the following instructions must always be carried out.

(i) Write above the graph a title or a brief explanatory heading.
(ii) The quantity whose values are selected must be measured along the axis across the page; the quantity whose values are observed, or calculated, along the axis up the page.
(iii) Write along each axis what that axis represents.
(iv) Choose as large a scale as the paper will allow, but it must be a scale which makes plotting and reading easy.
(v) Graduate each axis so as to show clearly the scale for that axis.

EXERCISE V. C

State which quantity should be measured along the axis drawn across the page for the following graphs:

1. Postage on parcels of various weights.
2. Number of passengers on the Underground at different times of day.
3. A boy's age and height.
4. The H.P. of a motor-car and the tax on it.
5. A travel graph: distance from home and time of day.
6. A man's age and his expectation of life.
7. Temperature of a patient during the day.
8. Record times for races of various lengths.
9. Rainfall and time of year.
10. Age and cost of immediate annuity.
11. Depth of sea at various distances westwards from Land's End.
12. Stretch of a spiral spring under various loads.
13. A mark reducer: original marks and scaled marks.
14. A graph to convert degrees Fahrenheit to Centigrade.
15. A graph to convert miles to kilometres.

What scales would you choose and what would be the smallest and largest graduations, to represent the following ranges of values, for the given lengths of axes?
16. Length 10 inches; from 7 to 5s.
17. Length 10 inches; from 135 to 1050.
18. Length 10 inches; from 5·6 to 23·8.
19. Length 6 inches; from 45 to 100.
20. Length 8 inches; from 100 to 250.
21. Length 10 inches; from 0 to 1.
22. Length 5 inches; from 65 to 295.
23. Length 6 inches; from 2·75 to 3·25.
24. Length 7 inches; from 0·28 to 0·54.
25. Length 6 inches; from 500,000 to 800,000.

Represent on squared paper the following statistics, as in Fig. 81. State in each case (i) whether any meaning can be attached to intermediate upright lines, (ii) whether interpolation is possible with fair accuracy without further data.

26. The number of motor-cars sold by a firm in successive quarters is as follows:

<table>
<thead>
<tr>
<th>Period -</th>
<th>1928</th>
<th>1929</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cars</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>-</td>
<td>420</td>
<td>560</td>
</tr>
</tbody>
</table>

What is (i) the best time of year, (ii) the worst time of year for selling cars?

27. The annual premium for a Life Assurance of £1000 varies with the age of the insurer at the time of the first payment, as follows:

<table>
<thead>
<tr>
<th>Age</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premium</td>
<td>£14. 5s.</td>
<td>£16. 10s.</td>
<td>£19. 5s.</td>
<td>£23</td>
<td>£27. 15s.</td>
</tr>
</tbody>
</table>

Estimate the premiums for starting at the ages of (i) 32, (ii) 38.

28. The number of fatal street motor accidents in England and Wales is shown in the following table:

<table>
<thead>
<tr>
<th>Year</th>
<th>1920</th>
<th>1921</th>
<th>1922</th>
<th>1923</th>
<th>1924</th>
<th>1925</th>
<th>1926</th>
<th>1927</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatal Accidents</td>
<td>1812</td>
<td>1825</td>
<td>1958</td>
<td>2205</td>
<td>2750</td>
<td>3032</td>
<td>3662</td>
<td>3947</td>
</tr>
</tbody>
</table>
29. The following table shows the length of the longest day in different latitudes:

<table>
<thead>
<tr>
<th>Latitude in degrees</th>
<th>15</th>
<th>25</th>
<th>35</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of day in hours</td>
<td>12.9</td>
<td>13.8</td>
<td>14.4</td>
<td>15.4</td>
</tr>
<tr>
<td>Latitude in degrees</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>Length of day in hours</td>
<td>16.1</td>
<td>17.1</td>
<td>18.5</td>
<td>21.2</td>
</tr>
</tbody>
</table>

Estimate the length of the longest day in latitudes 30°, 52°.

In representing a set of statistics by a graph, it is not necessary to draw in the whole of the uprights; the usual custom is to mark only the top point of the upright, and to leave the rest to the imagination.

Example 3. The graph in Fig. 82 shows the number of passengers per hour throughout the day on the London Underground.

The Underground
Number of Passengers per hour, during the day

Oral work on Fig. 82.
(i) The "workers" are travelling between 7 a.m. and 8 a.m. about how many?
A NEW ALGEBRA

(ii) The business men between 8 a.m. and 10 a.m.; about how many? And late-comers between 10 a.m. and 11 a.m.; about how many?

(iii) When is it most comfortable to travel?

(iv) When are the "rush" hours?

(v) Who are travelling between 5 p.m. and 6 p.m.?

(vi) Doors open at 5 a.m. and close at 1.30 a.m. How could this be shown on the graph?

(vii) This graph does not apply to Saturday or Sunday. What would be the chief differences (a) for Saturday; (b) for Sunday?

EXERCISE V. d

Represent on squared paper, as in Fig. 82, the following statistics:

State in each case (i) whether any meaning can be attached to intermediate points, (ii) whether any intermediate points can be inserted with fair accuracy without further data.

1. The income-tax paid by a man in various years.

<table>
<thead>
<tr>
<th>Year</th>
<th>1921</th>
<th>1922</th>
<th>1923</th>
<th>1924</th>
<th>1925</th>
<th>1926</th>
<th>1927</th>
<th>1928</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax in £</td>
<td>212</td>
<td>225</td>
<td>247</td>
<td>203</td>
<td>235</td>
<td>192</td>
<td>208</td>
<td>195</td>
</tr>
</tbody>
</table>

2. In 1926, the Civil Service annual salaries were supplemented by a bonus on the following scale:

| Bonus in £ | 99 | 123 | 147 | 176 | 195 | 205 |
| Salary in £ | 200 | 300 | 400 | 600 | 800 | 1000 |

Is the bonus or the salary the independent variable?

3. The number of pupils who left and entered a school during successive years was as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>1922</th>
<th>1923</th>
<th>1924</th>
<th>1925</th>
<th>1926</th>
<th>1927</th>
<th>1928</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number leaving</td>
<td>164</td>
<td>147</td>
<td>173</td>
<td>169</td>
<td>152</td>
<td>144</td>
<td>177</td>
</tr>
<tr>
<td>Number entering</td>
<td>153</td>
<td>165</td>
<td>171</td>
<td>156</td>
<td>174</td>
<td>168</td>
<td>180</td>
</tr>
</tbody>
</table>

Draw both graphs on the same figure; mark points on the graph of "numbers leaving" by crosses and points on the graph of "numbers entering" by dots enclosed in circles.

4. Using the data of No. 3, and the fact that there were 810 pupils in the school on Dec. 31, 1921, show graphically the total numbers of the school at the end of each of the years 1922 to 1928.
5. The population of Australia is recorded as follows:

<table>
<thead>
<tr>
<th>Year</th>
<th>1871</th>
<th>1881</th>
<th>1891</th>
<th>1901</th>
<th>1911</th>
<th>1921</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population in millions</td>
<td>1.66</td>
<td>2.25</td>
<td>3.17</td>
<td>3.77</td>
<td>4.45</td>
<td>5.44</td>
</tr>
</tbody>
</table>

Estimate roughly the population in 1896 and 1907.

6. The following table compares the death-rate of first-class cricketers with the general (male) death-rate, by giving the number per 1000 who die between various ages:

<table>
<thead>
<tr>
<th>Age</th>
<th>25-35</th>
<th>30-40</th>
<th>35-45</th>
<th>40-50</th>
<th>45-55</th>
<th>50-60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cricketers</td>
<td>39</td>
<td>46</td>
<td>61</td>
<td>83</td>
<td>117</td>
<td>122</td>
</tr>
<tr>
<td>General</td>
<td>47</td>
<td>60</td>
<td>78</td>
<td>102</td>
<td>138</td>
<td>190</td>
</tr>
</tbody>
</table>

Draw both graphs on the same figure, see No. 3.

**Locus Graphs**

Fig. 83 shows the temperature, at stated times, of a boy with a feverish cold. If his temperature had been taken more frequently,
A NEW ALGEBRA

Draw in pencil a smooth curve through the marked points in Fig. 83. This curve probably represents with fair accuracy the locus of the top points of the uprights which correspond to the temperatures at intermediate times. We therefore call it a locus-graph. It shows at a glance the boy's temperature (approximately) at any time between 8 a.m. and 8 p.m.

Oral work on Fig. 83.
(i) What is approximately his temperature at 3 a.m., 5 p.m.? 
(ii) At what time approximately is his temperature 101°, 101.7°?
(iii) At what times approximately is his temperature 100.4°?

The top points of successive uprights are often joined by straight lines in order to guide the eye rapidly from one point to the next; in such cases the intermediate points on the lines do not represent intermediate observations.

Similarly, we may join the top points of the uprights in Fig. 81, p. 67, by straight lines in order to show clearly the ups and downs in successive weeks, although here the intermediate points on the lines have practically no meaning.

Example 4. The height of the barometer in inches is recorded at hourly intervals on a certain day as follows:

<table>
<thead>
<tr>
<th>Time</th>
<th>9 a.m.</th>
<th>10 a.m.</th>
<th>11 a.m.</th>
<th>12</th>
<th>1 p.m.</th>
<th>2 p.m.</th>
<th>3 p.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height in inches</td>
<td>29.55</td>
<td>29.70</td>
<td>29.77</td>
<td>29.70</td>
<td>29.90</td>
<td>29.72</td>
<td>29.15</td>
</tr>
</tbody>
</table>

We select the times at which the height is observed, the time-axis is therefore taken across the page; unit, 1 inch represents 2 hours, the first graduation is 9 a.m.

All readings lie between 29 in. and 30 in.; the lowest graduation on the axis up the page may be taken as 29 in.; scale, 1 inch along axis represents 0.5 in., height of barometer.

The given observations are represented by the points, marked by crosses, in Fig. 84.

If an automatic recording machine had been employed, the pointer of the machine would have marked not only these isolated
points but also a continuous curve passing through them, thus forming a locus-graph; this is shown in Fig. 84.

**Fig. 84.**

*Oral work on Fig. 84.*

(i) What is the height of the barometer at 9.36 a.m., 12.24 p.m., 1.24 p.m., 2.48 p.m.?

(ii) At what times is the height of the barometer 29.65 in., 29.80 in., 29.45 in.?

(iii) Between what times was the barometer rising?

(iv) Between what times was the barometer above 29.65 in.?

(v) How much did the barometer fall between 1.30 p.m. and 2.30 p.m.?

(vi) How much did the barometer rise between 9.30 a.m. and 10.30 a.m.?

(vii) What inferences can you draw from noticing that a special part of the graph *slopes downwards* and that one part slopes downwards *more steeply* than another part?

**Exercise V. e**

1. The following table gives the distance *d* yards in which a train running at *V* miles per hour can be stopped:

<table>
<thead>
<tr>
<th><em>V</em></th>
<th>36</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>d</em></td>
<td>100</td>
<td>176</td>
<td>223</td>
<td>276</td>
<td>400</td>
</tr>
</tbody>
</table>
Find from a graph (i) how much further a train runs after the brakes are put hard on when the speed is 35 m.p.h., 55 m.p.h.; (ii) how fast a train is travelling if it can be stopped in 200 yards. Compare the extra distances that must be allowed, for an increase of velocity of 1 mile an hour, for two trains travelling at 35 m.p.h. and 55 m.p.h. respectively.

2. If £1 is allowed to accumulate at 4 per cent. per annum compound interest, the amount is as follows:

<table>
<thead>
<tr>
<th>Number of years</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount in £</td>
<td>1</td>
<td>1.22</td>
<td>1.48</td>
<td>2.19</td>
<td>3.24</td>
<td>3.95</td>
</tr>
</tbody>
</table>

Find from a graph the amount after (i) 15 years, (ii) 25 years, (iii) 33 years.

After what time will £1 amount to £3. 10s.?

Draw on the same figure a graph showing the amount of £1 if allowed to accumulate at 4 per cent. per annum, simple interest, for the same period.

3. Expectation of life of an Englishman at different ages.

<table>
<thead>
<tr>
<th>Age in years</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expectation</td>
<td>33.2</td>
<td>28.5</td>
<td>26.9</td>
<td>19.6</td>
<td>13.6</td>
<td>8.6</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Find from a graph (i) how much longer an Englishman may expect to live at the age of 34, 53, 66; (ii) at what age the expectation of life is 22, 16, 11.

4. The time of a complete oscillation for pendulums of different lengths, in London, is as follows:

<table>
<thead>
<tr>
<th>Length in ft.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in sec.</td>
<td>1.11</td>
<td>1.57</td>
<td>1.92</td>
<td>2.21</td>
<td>2.48</td>
<td>2.71</td>
</tr>
</tbody>
</table>

Find from a graph (i) the time for lengths 2 ft. 6 in., 4 ft. 9 in.; (ii) the length to give a time, 2 sec.

The pendulum of a clock should make complete oscillations every 2 seconds if the clock is keeping time. How should the length be corrected if a complete oscillation takes 2.1 seconds? What alteration in length is required to reduce the time of a complete oscillation from 1.3 sec. to 1.2 sec.? Is it the same as before?

5-8. Describe in general terms the following rough graphs, Nos. 5-8, and explain any peculiar features.
9. Fig. 89 shows some points on the travel-graph of a steamer. How far did the steamer travel in (i) 2 hours, (ii) 4 hours, (iii) 6 hours? Use your ruler to show that all the marked points lie
on a straight line. What does this mean? What is the speed of the steamer?

Draw, in pencil, travel-graphs for speeds of 50 m.p.h., 25 m.p.h., 5 m.p.h.

§ 10. Interpret the travel-graph in Fig. 90, stating the different speeds in miles per hour. What is the average speed for the whole journey?

How can you tell without any calculation which part of the graph corresponds to the greatest speed?
11. Fig. 91 shows two travel-graphs. The graph OCG corresponds to a man who successively cycles, motors, walks, takes a bus and a train; the graph OQS to a man who motors and then walks. Both men start from the same house at 10 a.m.

(i) Take the graph OCG and describe the journey in detail, giving the various speeds.

(a) Can you tell at a glance when he is moving fastest and when he is walking?

(b) How far does he go altogether? When does he start to come home?

(ii) Repeat (i) for the graph OQS.

(iii) If the two men pass one another, when and where does it happen?

12. A motorist leaves home at 9 a.m.; the mileage recorded by his cyclograph, originally set at zero, is as follows:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mileage</td>
<td>2</td>
<td>5</td>
<td>9</td>
<td>12</td>
<td>16</td>
<td>19</td>
<td>24</td>
<td>31</td>
<td>37</td>
</tr>
</tbody>
</table>

Find from a graph (i) the readings at 9.36, 9.54, 10.15 a.m.; (ii) the distance travelled between 9.35 a.m. and 10.05 a.m.; (iii) the time when the distance travelled is 14 mi., 21 mi., 33 mi.; (iv) what is his approximate speed at 9.40 a.m., 10.10 a.m.?

13. A spiral spring is suspended from one end and its length is measured when different weights are attached to the other end.

<table>
<thead>
<tr>
<th>Weight in gm.</th>
<th>10</th>
<th>15</th>
<th>30</th>
<th>50</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length in cm.</td>
<td>22</td>
<td>24</td>
<td>30</td>
<td>38</td>
<td>48</td>
</tr>
</tbody>
</table>

Represent graphically the relation between the length and the load.

What is the length if the load is 20 gm., 40 gm., 65 gm.?
What is the load if the length is 23 cm., 28 cm., 42 cm.?
What is its natural length, i.e. the length when there is no load?
Is the graph a straight line? If so, what does this mean?
How can you interpret the slope of the line?

14. The British amateur running records are as follows:

<table>
<thead>
<tr>
<th>Distance in yd.</th>
<th>100</th>
<th>200</th>
<th>440</th>
<th>600</th>
<th>880</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time in sec.</td>
<td>14.6</td>
<td>19.4</td>
<td>47.0</td>
<td>70.8</td>
<td>112.2</td>
<td>132.4</td>
</tr>
</tbody>
</table>

Represent this table by a graph. [P.T.O.]
What would be the probable record for 500 yd., 750 yd. ?
The American record for 300 metres (1 m. = 1.09 yd.) is 33.2 sec.; how does this compare with British records?
Should this graph be a straight line?

Draw rough graphs to illustrate the following, Nos. 15-18:

15. A travel-graph: a boy walks for 5 minutes, runs for 2 minutes, stands still for 3 minutes, and then returns to his starting-point in a car.

16. The inland postage for letters of various weights: charge, up to 2 oz., 1/3d.; for each additional 2 oz. (or part of it), 1/4d. more.

17. The change of depth of water in a tank, into which the rain-water from the roof of a house runs: fine early, a heavy shower at breakfast, fine between breakfast and lunch, a drizzle after lunch and a steady downpour after tea which fills the tank by 7 p.m.

18. The time a person takes to run a hundred yards at different ages.

19. Draw on an enlarged scale the travel-graph in Fig. 92. State the distance travelled in the first 4 minutes, in the first 6 minutes, in the first 8 minutes. Find in miles per hour (i) the average speed for the first 4 minutes, (ii) the approximate speed after 3 minutes, (iii) the approximate speed after 8 minutes.

After what time is the speed greatest?

20. The amount of water in a tank, t seconds after the escape pipe is opened, is n gallons where t, n are related as follows:

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>64</td>
<td>56.2</td>
<td>49</td>
<td>42.3</td>
<td>36</td>
<td>25</td>
<td>16</td>
<td>9</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Find from a graph (i) how many gallons run out in the first 25 seconds, and in the first 55 seconds; (ii) the approximate rate at which the water is running out, in gallons per second, after 15 seconds and after 30 seconds.