(77) Analysis of the Total Internal Loss of Power in Direct Current Dynamos and Motors.

Introduction.—In test No. 82, p. 220, it is pointed out that the total internal loss of power in a direct current dynamo or motor is made up as follows:

(i) Copper Loss occurring in the armature and field coils, caused by heating due to the passage of the current.

This is at once easily calculable from the relations there given for finding the copper loss in either series, shunt, or compound machines, when the resistance of the several coils, and the respective currents which each carries, is known. The loss in each circuit varies as the square of the current.

(ii) Mechanical Friction due to air clashing or resistance, brush and bearing friction, each of which varies as the speed simply.

(iii) Eddy Current or Foucault Current loss occurring in the armature core, and also in the armature conductors, and varying as the square of the speed for the same excitation, since the eddy currents will be directly as speed at constant excitation while the watts used in producing them will vary as the square of these currents, or if \( W_f = \) Watts wasted in eddy currents
and \( n = \text{speed in revs. per min.} \), then loss from this cause is \( W_E \propto n^2 K_E \), where \( K_E \) is a coefficient depending on the eddy loss.

(iv) Magnetic Hysteresis in the core due to reversals of magnetization in it as it rotates and \( \propto \) to its speed. If \( W_H = \text{the loss from this cause and } K_H \text{ its co-efficient, then } W_H \propto n K_H \); hence the total iron loss \( W_T = W_H + W_E = n K_H + n^2 K_E \).

This equation has been made use of in several methods for separating these losses. Thus in Mr. Morley's method, which is applicable to determining the losses in an unexcited armature core as well as a wound one, the armature to be tested is driven, when in position between its own field poles, at different speeds \( (n) \), with its field \( (a) \) unexcited, \( (b) \) excited to a constant degree, \( (c) \) excited to various degrees, by an electromotor, and the power so required measured by a dynamometer or by knowing the efficiency of the motor accurately.

On plotting a curve between the speed \( (n) \) and the powers \( W \) required to drive at different speeds in a constant field, the constants \( K_H \) and \( K_E \) can be found from it.

Mr. Kapp's method is a slight modification of the preceding, and is only applicable to a ready-wound armature core. It consists in measuring the power \( W \) required to run the armature to be tested at different speeds in a constant field \( N \), by running the armature itself as a motor "light," and noting the corresponding voltage \( V \) and current \( A \) taken at each speed \( (n) \).

If then \( T_a = \text{total number of armature turns all round we have the fundamental relation } V = T_a N n 10^{-6} \).

\[
\begin{align*}
W = & AV = AT_a N n 10^{-6} = n K_H + n^2 K_E \\
\therefore A = & \frac{K_H}{T_a N 10^{-6}} + \frac{n K_E}{T_a N 10^{-6}} = \text{(a constant + n x a factor)}.
\end{align*}
\]

On plotting therefore the curve between \( A \) and \( n \) to the axes \( OF \) and \( OS \) with \( (n) \) along \( OS \), we shall obtain the straight line \( PQ \). The ordinate \( OP \) is the current required to overcome friction and hysteresis, while \( \tan \theta = \text{the eddy current effect}. \) If \( OP \) is plotted to a scale of current, then \( K_H = OP, NT_a 10^{-6} \), when \( P = \frac{W}{n} \) is also known.

We also have \( \frac{OP}{OS} = \frac{\text{Hysteresis + Friction}}{\text{Hysteresis + Friction + Eddy}} \).

Thus the three separate factors or losses are each determined.
The following graphical method of separating the various losses is a simple and convenient one, and independent of any mathematical treatment. It is due to Dr. G. H. Headan, and is as follows—

Separately excite the field magnets to the normal amount and keep this constant. Note the current and speed of the armature when running light as a motor for different rated voltages applied to it. Plotting current, which is torque with given field on the ordinates, and voltages which is speed with given field on the abscissæ, or Joules per revolution on the ordinates and revs. per second on abscissæ, the straight line PQ (Fig. 79) is obtained cutting the current axis in P.

If Q is any point on PQ and QS is parallel to OP, then the total loss for that speed OS is given by QS x SO. If PR is parallel to OS, then the area PS x OS x power lost in hysteresis and friction together, and area QR x RP x OS² x power lost in eddy currents where QP x RP x OS. Repeating the above with a different excitation will give a second line PR', usually parallel to PQ, showing that the eddy currents are constant for a given voltage.

It may be noticed that the total loss corresponding to any point such as Q on PQ = product of co-ordinates = OS x QS, and not the area of the Fig. PONQ. In other words, the Fig. represents the nature of a dynamic Characteristic rather than the indicator diagram of a steam-engine.

To obtain the total mechanical friction losses, run the armature with brushes down, field disconnected and unexcited by a direct coupled motor, and note the increase of current required to drive over that needed for the motor alone. Plotting this current OD on the ordinates and drawing DO parallel to OS, the area OD x total mechanical friction, and . . . DR must be O to the hysteresis loss alone. On noting this excess driving current with the brushes up, we get OD, and finally the area OE x bearing and wind friction only. DO being x the brush friction alone.
The total losses for a given voltage will be a minimum for a certain induction in the armature core, usually between 15,000 and 16,000 lines per square cm. Since the hysteresis losses increase rapidly with increase of field, while the frictional losses increase with decrease of field due to the higher speed needed to obtain the same voltage,

For high inductions up to 18,000 or 20,000 the eddy currents cause the curve to bend upwards, and also the angle θ to be greater. This is probably due to the eddy losses generated by the stray leakage field through the shaft, etc. If the line PQ bends, it shows that the eddy-current losses are producing perceptible demagnetization on the field. Since both the eddy and hysteresis losses increase with armature current, these losses should really be measured with full-load armature current flowing by using the method of Fig. 85, which with careful adjustment of excitation will give considerable range of speed for constant armature current.

This question of the separation of the various losses is of great importance to the dynamo maker, enabling him to see in what way a machine is faulty, i.e., whether the eddy-current loss is excessive due to insufficient lamination, or the hysteresis too great due to too hard or inferior quality of iron. We will now consider a complete experimental analysis in detail.

Apparatus.—Exactly the same as that prescribed for test 95, and in addition an auxiliary motor should be available for coupling direct to the machine to be tested.

Observations.—(1) Carry out observations 1-3, test 95.
(2) Repeat 1 for an excitation 25% above and 50% below the normal.
(3) Disconnect all apparatus from the machine tested, and also the field from the armature. Connect the instruments up with the auxiliary motor, so as to measure the power taken to drive it. Demagnetize the field magnets of the motor to be tested by sending round the field coils a gradually diminishing (to 0) alternating current.
(4) Measure the voltage and current needed to run the auxiliary motor at some ten different recorded speeds between 0 and the maximum allowable.
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(6) Direct couple the auxiliary motor to the armature tested, and with the brushes down, note the new power given to the auxiliary to drive the two machines at some ten different speeds, the field of the machine under test being entirely disconnected and unexcited.

(6) Raise the brushes and repeat 5, tabulating all your results as follows—

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor tested: No.</td>
<td>Resistance</td>
</tr>
</tbody>
</table>
| Total copper losses |...

<table>
<thead>
<tr>
<th>Speed in</th>
<th>Torque Motor coil driven (Volt)</th>
<th>Torque Motor coil driven (Volt)</th>
<th>Torque Motor coil driven (Volt)</th>
<th>Torque Motor coil driven (Volt)</th>
<th>Torque Motor coil driven (Volt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RPM</td>
<td>120 V.</td>
<td>240 V.</td>
<td>360 V.</td>
<td>480 V.</td>
<td>600 V.</td>
</tr>
<tr>
<td>Friction losses</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(7) Plot all your results to the same pair of axes, having in each case the speed in revolutions per second on the abscissa and the power in Watts required to be given to the shaft of the dynamo under test to produce these speeds under the various conditions mentioned in observations 1–6 on the ordinates.

(8) Calculate the various losses at normal speed as a percentage of the total loss in the whole machine at full load.

Inferences.—State very clearly all that can be inferred from your experimental results.

Note.—A variation of the preceding method for measuring the hysteresis and eddy current losses consists in measuring the watts absorbed by the armature in running the machine as a motor light at a series of excitations between 0 and the normal, the speed being kept constant at normal value by adjusting the voltage on the armature by means of a main circuit rheostat in series with it.

Plotting a curve with armature watts as ordinates, and excitation as abscissae, we find its lower portion to be nearly straight, and this part produced to cut the ordinates will give the watts which would be absorbed at zero excitation. Thus the differ-
ence between the watts at any given excitation, and at this zero value, will be the power lost in hysteresis, eddies, and mechanical frictions.

Again, if the curves are plotted between Joules per revolution as ordinates, and roe. per sec. as abscissae, the friction line BG separating frictional and electro-magnetic losses has a fixed position in the diagram; whereas, with the axes denoting current and volts, a different friction line has to be drawn for each excitation, thus making it difficult to see what proportion of the whole loss is electrical and what frictional, when more than one set of curves corresponding with different excitation is drawn on the same curve-sheet.

(78) Measurement of the Coefficient of Magnetic Leakage "μ" and of the Relative Distribution of the Waste Field of Dynamos and Motors. (Ballistic Method.)

Introduction.—The present test has a most important bearing on the design of the magnetic circuit of a dynamo or motor, for since only a fraction of the total number of lines of magnetic force, generated by the field magnets, are usefully employed in cutting the armature conductors and so generating the requisite H.M.F., the results of the test enable the designer to allow for this discrepancy, providing he knows the coefficient of magnetic leakage "μ" for the particular form and type of machine in question.

In addition to this, the relative distribution of the waste field around the machine enables defects in the design of the magnetic circuit to be seen and corrected, for at the best the magnetic circuit of a dynamo or motor is very imperfect.

It should be remembered that leakage of magnetic lines of force will take place across any two points between which there is a difference of magnetic potential, the magnitude of which leakage will depend directly on this potential difference, and inversely on the magnetic resistance of the path.

The following is a convenient method of measuring or comparing the relative amounts of leakage in different parts of a dynamo,
and therefore the static leakage coefficient \( k \) for the machine; the term static being here used to denote the value of \( k \) obtained when the armature is at rest, for it is well known that an armature delivering current exerts a demagnetizing action on the field which directly promotes leakage. Assuming the normal excitation constant, the leakage will increase with the output, and it will largely depend on the degree of saturation of the iron and on the relative magnetic reluctances of the various parts. The method depends on the measurement of induced currents produced by moving either (1) an exploring coil so as to cut the field to be tested, or (2) the field in such a way as to cut the coil, the latter method being here adopted. Either the relative or absolute numerical values of the steady and useful flux in the various parts can be found, the relative values being obtained with reference to that part in which the flux is a maximum which can be taken as unity. Knowing these, the absolute values can be obtained by running the armature at a known speed and measuring the H.M.F. without allowing it to develop current and thereby distort the field. The useful armature flux can now be at once calculated, and from it, that in each of the various parts, or thus—suppose we have a circuit consisting of a ballistic galvanometer, resistance box, earth inductor of \( N \) turns, mean area \( A \) square c.m., in series with an exploring coil of \( N \) turns, mean area \( A_2 \) square c.m., wound round the magnetic field to be tested. If now the inductor, with its plane vertical or horizontal, is rotated rapidly through 180°, cutting the earth's field of strength \( F_0 \), then the total quantity of electricity set up in the transient current is \( Q \) where \( F_0 = \frac{2N^2A_1A_2F_0}{R_0} = K \sin \frac{1}{2} \theta \) where \( K \) = ballistic constant, \( R_0 \) = total circuit resistance, \( \theta \) = angular throw in degrees. If the exploring coil is now made to cut the field to be tested of strength \( F \) by suddenly making, breaking, or reversing the exciting current, we get \( Q = \frac{N^2A_2F_0}{R_0} = K \sin \frac{1}{2} \theta \), where \( \theta \) and \( F_0 \) have the same meaning as before. Dividing we get

\[
F_0 = F_1 \frac{2N^2A_1A_2F_0}{N^2A_2R_1} \times d^2 \text{ lines per square c.m. in the loop or search coil (in absolute measure) where} \ d_1 \text{ and } d_2 \text{ = scale deflections corresponding to } \theta_1 \text{ and } \theta_2.
\]
As, however, it is the total field \((A_1F_1)\) which we really desire to obtain, and denoting this by \(F_1\), we have
\[
F_1 = \frac{2N_1A_1R_1}{d_1^2} \times d_2 \text{ lines.}
\]

**Apparatus.**—Earth inductor \(E\); resistance box \(R\); charge and short circuit key \(K\); ballistic mirror galvanometer \(G\) (p. 669), having a small log decrement and periodic time about 8 or 10 seconds, so that this may be large compared with the time of flow of \(Q_1\) and \(Q_2\), which can therefore pass through the coil before it begins to move. A shunt wound dynamo to be tested; ammeter \(A\); rheostat \(-\) (p. 669); quick break switch \(S\); and source of current \(B\).

**Fig. 89.**

**Observations.**—(1) Adjust the needle of \(G\) to zero, and wind a single complete turn of wire on the dynamo at position \(A\), connecting it up with the other apparatus as indicated in Fig. 89. The F.M. coils must be disconnected and excited separately from \(B\).

(2) Close \(S\) and adjust \((r)\), so as to get normal excitation through the F.M. coils.

(3) Close \(K\), open \(S\), and adjust \(R\) by trial so as to get a convenient throw on \(G\), then note its value \((D_1)\) on breaking, and \((D_2)\) on making circuit at \(K\), the excitation being that in \(a\). Repeat this twice and take the mean of each, calling it \((d_2)\).
(4) Repeat 1-3 for each of the positions of the exploring loop indicated by the letters B, C, D, E, F, G, H, I, J, K, and L, respectively.

(5) Repeat 4 for excitation 50% higher and 50% lower than the normal, and in each case calculate \( v \) from the formula,

\[ v = \frac{\text{Total Field}}{\text{Useful Field}} \]

(6) Let down the brushes and run the machine at a known speed, measuring the E.M.F. \( E \) at each of the three excitations used, and tabulate as follows—

<table>
<thead>
<tr>
<th>Date</th>
<th>March 2023</th>
</tr>
</thead>
<tbody>
<tr>
<td>June</td>
<td>2023</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E, ( \text{e.m.u.} )</th>
<th>( E ), ( \text{e.m.u.} )</th>
<th>( A ), ( \text{e.m.u.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gate resistance ( \theta )</td>
<td>( \text{ohms} )</td>
<td>Total No. armature conductors ( C )</td>
</tr>
<tr>
<td>Total resistance ( R )</td>
<td>( \text{ohms} )</td>
<td>Speed ( \text{rev. per min.} )</td>
</tr>
<tr>
<td>( A ), ( \text{e.m.u.} )</td>
<td>( \text{e.m.u.} )</td>
<td>( A ), ( \text{e.m.u.} )</td>
</tr>
</tbody>
</table>

**Inferences.**—State clearly all the inferences which can be drawn from the results of the above experiments, and point out their bearing on the design of field magnets for dynamos and motors.
(79) Magnetic Characteristic of a Dynamo with varying Air Gaps.

Introduction.—It is of considerable importance, especially in the design of dynamos, to know the effect which the length of air gap, between the field magnet (F.M.) pole faces and armature core has on the excitation required to force a given number of magnetic lines of force through the core. For convenience the curve showing the relation between the ampères (A.T.) or magnetomotive force (M.M.F.) which $= \frac{4\pi}{10}$ useful flux of lines ($N$) through the armature will be called the Magnetic Characteristic for the air gap used. The flux ($N$) can be found in two ways: (1) by using a ballistic galvanometer in series with a "search coil" temporarily wound on the armature and noting the throws produced on the galvanometer by making, breaking, or reversing known currents in the F.M. coils; (2) by running the armature mechanically and noting its R.M.S. speed, and number of conductors round periphery, $N$ being then calculated from the fundamental formula $F = NuC + 10$. This is the best and more practical method to employ, because the armature will now exert a slight demagnetising action on the F.M.s tending to increase leakage and approximate more nearly to actual working conditions. The Exp. is divided into three distinct parts, viz. the determination of the relation between—

(a) The M.M.F. and flux ($N$) through armature with constant air gap.

(b) The air gap and flux ($N$) through armature with constant M.M.F.

(c) The air gap and M.M.F. through armature with constant flux ($N$) in armature.

Apparatus.—The dynamo $D$ capable of being driven mechanically; tachometer; voltmeter $V$; ammeter $A$; switch $S$; rheostat $r$ (p. 599); supply of electricity.

The machine $D$ to be tested must be specially constructed in order to be able to operate this test. As shown in Fig. 61, the pole pieces are each capable of being made to approach or recede from the armature by turning a massive screw bolt fitted to
each, by means of a suitable key. The distance apart of the pole tips can be read off on a scale C fixed to the body of the machine.

Note.—The pole tips must never be closer together than the two zero scale divisions which will be termed their normal position in what follows, and must always be left at this distance after the test is over. To increase this distance turn the screws clockwise.

It will be noticed that the initial slopes of the curves in (a) are determined by the air gap, also that the air gap causes the curve to bend over.

All lubricators must be fed properly before the machinery is started.

Observations.—a.—(1) Connect up as shown in Fig. 81, and adjust the pointers of all the instruments to zero.

(2) Set the pole tips at exactly the normal distance apart and adjust the speed so that with the maximum excitation allowable in the E.M. coils 25% above normal, the B.M.F. can be read off on v.

(3) With air gap and speed constant, adjust the excitation to about 1/2 of the maximum allowable. Note this reading A and that on (e) viz. E.

(4) Repeat 3 for about eight ascending equal increments of current to about 25% above the normal excitation.

(5) Repeat 3 and 4 for the pole tips half-way and the farthest apart.

(6) Repeat 3–5 for the same current values descending.

(7) Plot curves in each case with M.M.F. as abscissa and \(N\) as ordinates.

\(\beta\)—(1) Adjust the exciting current to the normal value and the speed so that the B.M.F. can be read off on v.

(2) With M.M.F. (i.e. \(A\)) and speed constant and the pole tips at exactly the normal distance apart, note the reading (E) on v.
(3) Repeat 2 for eight different distances increasing by \( \frac{1}{3} \) at a time to the maximum possible.

(4) Repeat 2 and 3 for a return set of distances to the minimum (normal).

(5) Plot curves in each case with distances between iron of armature and pole face as abscissae and \( N \) as ordinates.

\[ \gamma \]  (1) Adjust the excitation to \( \frac{1}{3} \) maximum and the speed so that a suitable low reading of, say, \( \frac{1}{3} \) maximum voltage is obtained on a.

(2) With \( N \) (i.e., \( E \)) and speed constant and the pole tips at exactly the normal distance apart, note this distance \( (a) \) and the existing current \( A \).

(3) Repeat 2 for eight values of \( (a) \) rising by \( \frac{1}{3} \) of the maximum at a time to the maximum, noting \( A \) at each position, which is necessary to keep \( E \) constant.

(4) Repeat 2 and 3 for a return set of distances to the minimum (normal).

(5) Plot curves in each case with M.M.F. as abscissae and \( (a) \) as ordinates.

### Data

**Name:**

**No. Armature conductors \( C = \).**

**Total M.M.F. \( T \):**

**Magnet. (s) iron core =**

**Physical length \( L \):**

**Deductions.**—State very clearly all the inferences which you can draw from your results and point out their bearing on dynamo design.

### (50) Localization of Faults in Magnetizing Coils. (Induction-Ballistic Method.)

**Introduction.**—When a magnetizing coil of insulated wire is wound on a metallic bobbin, the latter is usually insulated on the inside by a thin strata of insulating material before winding on
the covered wire. Notwithstanding this, it may and does sometimes happen that the wire core becomes "shorted" to the metalwork of the bobbin, through the covering and insulation of the bobbin. This is particularly liable to be the case in shunt coils of dynamoelectric machines which are wound on metal "formers," insulated with vulcanized fibre tissue before winding.

Such a fault, through poor contact of, in many cases, a very uncertain nature, gives trouble in the ordinary methods of testing for its position, by giving unstable readings. Thus the ordinary resistance methods are extremely liable to be vitiated by variable contact resistance at the fault. The following method for localizing the position of the fault by means of induced currents, measured hysteretically, is often a more convenient and reliable one for the purpose.

**Apparatus.**—Metallic bobbin or former \( F \) to be tested, wound with the magnetizing coil (\( m \)) which is "shorted" to frame at the point (\( J \)); high resistance ballistic galvanometer \( G \); two-way key \( K \) (p 581); battery of secondary cells \( B \); switch \( S \); ammeter \( A \), and temporary primary magnetizing coil \( PP \) wound over the outside of the magnetizing coil \( m \) proper, which is to be tested; rheostat \( R \) (p. 906); known high resistance box \( P \).

N.B.—It will be noticed that, as represented in Fig. 82, the fault (\( J \)) is on the first layer of turns next to the frame \( F \), and we will suppose that the turn at (\( J \)) is making contact there on the metallic frame (\( F \)). Thus it will be seen that the point (\( J \)) divides the total number of turns on the whole bobbin into two parts between the leading out wires \( T_1 T_2 \) of the coil, so that total turns = turns between \( T_1 \) and \( J \) + turns between \( T_1 \) and \( J \).

**Observations.**—(1) Connect up as in Fig. 82, and adjust \( A \) and \( G \) to zero, the temporary coil \( PP \) having been previously wound on and a wire soldered to any point (\( P \)) on the metallic bobbin frame \( F \).
(2) With \( R \) full in, close \( S \) and adjust the current on \( A \) to some convenient amount. Next also close \( K \) to start \( I \) and adjust \( R \) to such a value as will give, say, \( \frac{1}{4} \) or \( \frac{1}{2} \) scale deflection \( d_4 \) on \( G \) when \( S \) is opened suddenly. Repeat two or three times with the same constant current, both made and broken in \( P \).

(3) Close \( K \) to start \( I \) and repeat 2 above with the same current, noting the new resistance out in \( r \) to give a suitable first throw on \( G \).

(4) Repeat 2 or 3 for about four or five current strengths \( d \) so as to obtain finally different throws on \( G \) which will check one another, and calculate the position of the fault \( (f) \), or the number of turns to be unwound, to reach it, from the relation

\[
\frac{N_1}{N_2} = \frac{\text{turns between } T_1}{\text{turns between } T_2} \quad \text{and} \quad f = \frac{1}{2} \times \frac{r_1}{r_2} \text{ approx.}
\]

where \( r, r_1 \) are the total resistances of \( r + G \) when obtaining \( d_1 \) and \( d_2 \) respectively, and which are assumed to be very large compared with the contact resistance at \( f \) and also the resistance of the turns between \( f \) and both \( T_1 \) and \( T_2 \). If the resistance of the coil \( (\omega) \) is from 5 to 20 ohms then \( r + G \) should if possible be at least 10,000 ohms.

**Table:**

<table>
<thead>
<tr>
<th>Checked for resistance only</th>
<th>1st throw on ( A )</th>
<th>Verified on ( F )</th>
<th>Check Break</th>
<th>Ratios of ( R/e )</th>
<th>Turns to unwind on ( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1 )</td>
<td>( d_2 )</td>
<td>( r )</td>
<td>( r + x )</td>
<td>( r' )</td>
<td>( r'' )</td>
</tr>
</tbody>
</table>

**Note:** It will be noticed from the formulas in 4 that if \( r \) is adjusted so that \( d_1 = d_2 \) then

\[
\frac{N_1}{N_2} = \frac{r_1}{r_2} \quad \text{or} \quad N_2 = \frac{r_2}{r_1 + r_2} N_1,
\]

or again if \( r \) is kept constant throughout,

\[
\frac{N_1}{N_2} = \frac{d_1}{d_2} \quad \text{or} \quad N_2 = \frac{d_2}{d_1 + d_2} N_1.
\]

If \( G \) is inductive an iron core may be inserted in the coil to form a closed circuit if possible; this will increase the flux for a given current made or broken in \( PP \), and therefore also the first throws \( d_1, d_2 \) on \( G \).
This has the further advantage that \( N_1 \) and \( N_2 \) will now enclose the same number of lines of force, which is only approximately true if there is no iron core and the coil long.

It should be observed in passing that even a simpler method still than the one described above, for finding the position of the fault \( (f) \), would be to employ a slide wire or meter bridge or other convenient form of potential divider in the following manner. Connect the ends \( T_1 \), \( T_2 \), Fig. 82, of the faulty field coil to the extremities of a meter bridge wire and also to two or three Leeds and Northrup cells; connect the galvanometer \( G \), which need not now be ballistic, but which must be sensitive, between the metallic former at \( p \) and the slider key of the bridge wire. Now move the key such that on tapping it \( G \) does not deflect. Then the lengths \( T_1 f \) and \( f T_2 \) of the faulty coil are in the proportion of the corresponding lengths of the stretched wire either side of the \( K \), and are therefore known if the gauge of winding and its resistance (which can be measured in the ordinary way) are known.

(81) Determination of the Rise of Temperature and Increase in Resistance of Magnet-windings.

Introduction.—Since every magnet coil has some resistance, which is usually considerable in shunt or pressure coils but small in series or current coils, it follows from Joule's law that heat must be generated in them when excited. The amount of heat developed per second by a current of \( I \) amperes flowing through, or a pressure of \( V \) volts across the terminals of, a coil of \( R \) ohms resistance is \( I^2 R \) or \( V^2 \). Any coil must therefore have such an external surface for radiation of heat relatively to the amount of heat developed in it, that the "steady" temperature attained when the rates of production and dissipation of heat become equal is not high enough to deteriorate the insulation of the winding. The maximum limit to this "steady" final temperature is usually fixed at about 60° C, for it is found that the commoner insulating materials used generally begin to deteriorate with temperatures exceeding 60 to 70° C.
Admiralty specifications, however, prescribe that after a six-hour run at full load, no accessible part of a machine may show a temperature of more than 70°F (= 32.2°C) above the surrounding air. This would seem unnecessarily low, but from remarks to follow may not actually be so.

In the case of dynamos and motors the rise of temperature and its final steady value is required for the armature, series or shunt coils, commutator, bearings and frame. Further, it has been shown that the radiating facility of a surface in contact with iron is nearly twice as good as when it is exposed to air.

Except in special measurements and research, when perhaps thermocouples and their equivalents may be used, the temperature rise of coils while energized is always obtained either (1) by thermometer, the bulb of which is placed on the coil and covered with a pad of cotton wool, or (2) by resistance measurement, obtained from the readings of an ammeter in series with, or a voltmeter across, the coil and the application of Ohm’s law. This latter method is the one usually employed in a test room, is the most accurate of the two, and the quickest method of finding the “true mean rise” of temperature, especially with series coils. With shunt coils this resistance method can be effected by switching the supply off and then quickly measuring the resistance of the coil by the Wheatstone Bridge method. Usually the true mean rise of temperature by resistance tools is found to be at least 1.4 to 1.6 times greater than the apparent mean rise by thermometer due to the temperature of the layers of winding increasing from the outer one to that situated about three-fourths of the thickness of coil from it, and then decreasing again to the inner layer next to the iron core.

If \( R_c \) is the resistance of the coil cold, and \( R_h \) that when hot, then

\[
R_h = R_c(1 + \alpha(t_h - t_c)) \quad \text{approximately},
\]

or

\[
R_h = \frac{1 + \alpha t_c}{1 + \alpha t_h} \quad \text{more accurately},
\]

where \( t_c \) and \( t_h \) = the temperature in deg. cent. of the coil, cold and hot respectively, and \( \alpha \) = the temperature coefficient of the material which for copper = 0.00428 ohm per ohm per 1° C.

\[
\frac{1}{2} \times 0.00428 = 0.00214 \text{ per °F.}
\]
the rise of temp. \(t_h - t_i = \frac{R_h - R_i}{u\cdot H_0}\)

\[ = 23\frac{R_h - R_i}{H_0} \text{ deg. cent.} = 420\frac{R_h - R_i}{H_0} \text{ deg. Fah.}\]

If now \(T = \text{final temp. rise above surrounding air,}\)

\(S = \text{total heat radiating surface in } \Omega^2 (\text{exclusive of end flanges and internal surface, if any}),\)

\(W = \text{total watts wasted in the coil at full load} = \text{total } P_h R_0,\)

then \(T = \frac{W}{S}\) or \(W = S\times T\)

where \(K = \text{a heating constant depending on the depth of winding, amount of seating by the armature, and whether the surrounding air is still or circulating, and may be taken as 75}\)

for the usual shape and size of field coils of dynamos and motors, especially of multipolar types, excepting when iron chok.

Hence

\[ T = \frac{75 W}{S} \text{ deg. cent.} \]

and since for shunt holding \(W = V I_{sh} = F_{sh} I_{sh},\)

\[ \text{max. shunt current } I_{sh} = \frac{\sqrt{2NI}}{2\pi R_{sh}} \text{ amperes.} \]

Apparatus.—Magnet coil \(F\) (of, say, a dynamo) to be tested; ammeter \(A\) and voltmeter \(P\) each capable of dealing with the full-rated current and voltage for the coil; switch \(S_1\); watch; small bulb thermometer and cotton wool; adjustable high resistance \(r\) for shunt coils, or low resistance for series coils;

![Fig. 63](image)

ammeter \(A\), voltmeter \(P\), switch \(S_1\) and adjustable load resistance for the main circuit to the armature \(M\). Separate means for driving \(M\).

Observations.—(1) [Armature \(M\) stationary]. Connect up as shown on the left half of Fig. 63 and adjust \(S_1\) and \(v\) to zero, if necessary. Note the temperature of the air of the room by the
thermometer, secure the thermometer with its bulb touching the outside of the coil, and cover the bulb with a pair of cotton wool.

(2) With r full in, close \( S_r \) and quickly adjust \( a \) so that \( v \) or \( o \) shows the normal value for the coil, and note the readings of both \( v \) and \( o \), the thermometer and the time.

(3) By adjusting \( r \) maintain \( v \) constant in testing a series coil, or \( o \) constant in testing a pressure coil, either at the above normal value, and note the readings of \( v \), \( o \), the thermometer and time.

and time, say every 10 minutes for the first 1½ hours, and then every 15 or 20 minutes, up to the condition when the variable quantity becomes constant. Then, again, take the temperature of the room and tabulate as follows—

<table>
<thead>
<tr>
<th>Time of Recruit</th>
<th>Maximum from start (A)</th>
<th>Voltage (V)</th>
<th>Calculated</th>
<th>Thermometer taking ( T ) at ( R_1 - R_2 )</th>
<th>With our Molder at Full Load and at Normal Speed</th>
</tr>
</thead>
</table>

(1) [A similar Moulder at Full Load and at Normal Speed].—

Repeat obs. 1 3 after the machine has cooled down to the temperature of the air.

(5) Plot curves to the same axes having time in “minutes from start” as abscissæ with values of \( R_1 \), \( T \), and \( t \) as ordinates; calculate the “heating constant” \( (K) \) from the relation

\[ K = \frac{T}{P} \]

and the maximum value of short current suitable for coil tested for the value of \( (K) \) found, and for a final temperature rise \( T \) of 60° C. above air.

Inferences.—Clearly state all that can be deduced from the results of the test, and point out their bearing on temperature testing.

(52) Efficiency of Direct Current Dynamos.

(Swinburne’s Electrical Method.)

Introduction.—This method, due to Mr. James Swinburne, has the advantage, firstly, in point of accuracy, of being solely an electrical one, and therefore far more accurate than a dynamo-
motor method in which the power required to drive is measured mechanically; secondly, of not requiring another similar machine for coupling to it, in addition to the one tested. The method, which is often termed the "Stray Power" method, is consequently very suitable for employment in workshop determinations, where usually no good transmission dynamometer is available for measuring the H.P. used in driving the generator under test, and is invariably used when Hopkins's method cannot be applied or, e.g., when no second similar machine is available. The principle of the present and all similar methods is based on the following, namely, that the total power put in = total power given out + total power lost internally or in symbols

\[ W_I = W_0 + W_L \]

where the suffixes I, O and L denote the input, output, and total losses in Watts (W) respectively.

Thus the commercial efficiency (η) of the dynamo is at once obtainable from the relation—

\[ \eta = \frac{W_O}{W_I} = \frac{W_O}{W_0 + W_L} \]

The output in Watts \( W_O \) developed by the dynamo is at once deducible from the product of the volts \( V \) and amperes \( I \) given out. The total loss \( W_L \) in Watts occurring internally in any dynamo is made up as follows—

(a) Copper losses \( L_a \) in armature and exciting coils due to heating by the passage of current, and which can easily be calculated when the currents and resistances are known.

(b) Friction losses \( L_f \) due to air churning, journal and brush friction.

(c) Magnetic frictions or iron losses \( L_m \) due to Eddy or Foucault currents and magnetic hysteresis. Hence the total internal loss \( W_L = L_a + L_f + L_m \) and to the quantity \( (L_f + L_m) \) Mr. Swinburne has given the somewhat appropriate name of "Stray Power."

The copper losses are calculable as follows—

Let \( I \) = the current given by the dynamo at its normal voltage \( V \) to some external circuit, and let \( R_a \) \( R_m \) \( R_s \) be the resistances of the armature series coils and shunt coils respectively of any dynamo to be tested, of which \( R_s \) can be measured by a Wheatstone Bridge and \( R_a \) \( R_m \) by the "Potential Difference" method (p. 84). We shall then have for a
Series dynamo \( L_s = C^2 (R_a + R_{sc}) \)

Shunt dynamo \( L_s = \frac{Y}{R_{sa}} + \left( C + \frac{V}{R_{sa}} \right) R_a \)

Compound dynamo (long shunt)
\[
L_s = \frac{Y}{R_{sa}} + \left( C + \frac{V}{R_{sa}} \right) (R_a + R_x)
\]

Compound dynamo (short shunt)
\[
L_s = C^2 R_{sa} + \left( \frac{V - CR_x}{R_{sa}} \right)^2 + \left( C + \frac{(V - CR_x)}{R_{sa}} \right) R_x
\]

The remaining losses, i.e., the stray power \((L_s + L_a)\), can readily be obtained by running the dynamo as a motor, the field magnets being separately excited so that the armature has the same magnetic induction as at full load, the E.M.F. supplied to it being at least equal to the total E.M.F. which the machine would develop when running on full load as a dynamo at normal speed. Thus the machine is running on no load other than its own friction, eddy currents, and hysteresis. If \( A \) = current flowing through the armature and \( V_a \) = the voltage across its terminals when the speed is up to normal, then we have

Stray power \((L_s + L_a)\) = \( AV_a - I_a \)

where \( L_a \) = copper loss in the armature for the current \( A \) in it.

Note.—Only a comparatively small current \( A \) at the proper E.M.F. mentioned above will be required to be furnished by the auxiliary source of current, and if \( R_a \) is very small, \( L_a \) can be neglected in comparison with \( AV_a \) in this last formula.

Apparatus.—Dynamo \( N \) to be tested, which for the purposes of discussion merely we will assume is shunt wound: voltmeter \( V \); low reading long scale ammeter \( A \); rheostat \( K \) (p. 506) and \( r \) (p. 509); tachometer; complete Wheatstone Bridge set \((W, B)\); two-way voltmeter key \( K \) (p. 587); switch \( S_2 \); source of constant \( E \) at a sufficiently high E.M.F.

Observations.—(1) Connect up as in Fig. 51, and adjust the instruments \( V \) and \( A \) to zero if necessary. Switch on \( E \), when the field should be then
excited to the normal amount, as can be seen by closing \( K \text{'1}, \) and observing whether the normal voltage which the machine would give at a dynamic at the proposed speed is indicated on \( V. \)

(2) With \( R \) at its maximum value (not less than about 10 ohms) close \( S \text{'2}, \) adjusting \( R \) so as to give the armature the full requisite E.M.F. \( E. \) It will still run under the normal speed, since with so small a current the armature produces no demagnetising action to quicken it up. Now adjust \( R \text{'2} \) so to bring it up to the normal speed, and note by closing \( K \text{'2} \) the volts \( V \text{'2} \) across the armature terminals and the current \( A \) amps, flowing through it.

(3) Repeat 2 at the same excitation for some ten different speeds in all, both below and above normal (by varying \( R\)), entering the readings in the small tabular form:

<table>
<thead>
<tr>
<th>Speed in R.P.M.</th>
<th>Motor Power readings</th>
<th>Total V.A.</th>
<th>Total Horse Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volt, ( V \text{'2} )</td>
<td>Amps, ( A \text{'2} )</td>
<td>( A \text{'2} - A \text{'1} )</td>
<td>W.H.P.</td>
</tr>
</tbody>
</table>

(4) Open \( R, S \text{'2}, \) and \( K \) and measure by means of W.H.B., the resistance \( R \text{'3}, \) of the armature and \( R \text{'4}, \) of the shunt, remembering of course to disconnect one from the other while measuring their respective resistances.

(5) Calculate the power supplied and the commercial or net efficiency of the dynamo for some ten different values of currents \( U \) (at, say, constant speed and voltage) taken from the machine, ranging from 0 to full load by about equal increments, and tabulate as follows:

<table>
<thead>
<tr>
<th>Power supplied through Dynamo, W.H.B.</th>
<th>Calculated Power to Drive ( R \text{'2}, ) W.H.P.</th>
<th>Commercial Efficiency, ( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total horse power</td>
<td>Total volts</td>
<td>Current</td>
</tr>
<tr>
<td>( V \text{'2} )</td>
<td>( A \text{'2} )</td>
<td>( A \text{'2} - A \text{'1} )</td>
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<tr>
<th>Power supplied through Dynamo, W.H.B.</th>
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</thead>
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<tr>
<th>Power supplied through Dynamo, W.H.B.</th>
<th>Calculated Power to Drive ( R \text{'2}, ) W.H.P.</th>
<th>Commercial Efficiency, ( % )</th>
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<tbody>
<tr>
<td>Total horse power</td>
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<td>Current</td>
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<td>( V \text{'2} )</td>
<td>( A \text{'2} )</td>
<td>( A \text{'2} - A \text{'1} )</td>
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</table>
Note.—There will be only one value, that corresponding to normal speed, in each of the columns 1, 2, 5, 6, and 7 (counting from left to right) in the last table, but as many values in the remaining columns as there are values of amperes C assumed between 0 and full load.

(6) Plot the following curves having—
(a) Efficiency as ordinates and Watts developed as abscissae.
(b) Stray power as ordinates and speed of armature as abscissae.
(c) Watts developed as ordinates and Watts to drive as abscissae.

Inferences.—State clearly all that can be inferred from your experimental results.

(83) Efficiency of Direct Current Generators. (Hopkinson's Electrical Method.)

Introduction.—The earlier methods of measuring the efficiency of direct current generators, in which the electrical output of the machine was obtained by the product of ammeter and voltmeter readings, while the total mechanical input was obtained by means of some suitable form of transmission dynamometer, are more or less limited in their application, from the fact that a reliable dynamometer is not always available. Even when it is, the method gives only an approximate result, for the error made in measuring the efficiency is proportional to the error made in measuring the input as given by the transmission dynamometer, and which is only too easy to make in an appliance such as this. It will therefore be evident that, given accurately calibrated instruments, any method of measuring the efficiency solely electrically will be capable of giving far more accurate results than could be obtained with any dynamometer.

The present method has this advantage, of being solely an electrical one, and requires two machines of nearly the same output as possible, the accuracy of the test practically depending on how nearly alike in this respect the two machines are.

They must be capable of being placed in alignment with their shafts coupled mechanically together. The test can be made with either series, shunt, or compound machines, but the shunt is much the simplest.
Apparatus.—Accurate ammeters $A$ and $a_1$, $a_2$; voltmeter $V_1$; rheostats $R_1$, $R_2$ for the field circuits (p. 509); change over voltmeter key $K$ (Fig. 264); dynamo (a) to be tested coupled both mechanically and electrically to a similar machine (β) which runs as a motor. An auxiliary source of current (γ), such as a storage battery, or another dynamo giving an E.M.F. about equal to the normal of α and β, and able to supply the losses occurring in the machines α and β; switch S; rheostat RA (p. 606, Fig. 274).

Observations.—(1) Connect up as in Fig. 256, and make sure that the E.M.F. of γ assists that of the dynamo α in driving the motor β in the right direction for self-exciting α.

(2) The respective fields $F_α$ and $F_β$, in series with rheostats $R_1$ and $R_2$ respectively, are excited from the terminals of γ, as shown, to the normal amount roughly, except that of β, which is weakened to enable it to run as a motor.

(3) With $R_β$ full in to start with, close S and adjust the auxiliary source of E.M.F. (γ) and the rheostat (RA) so that the machines get up speed, and if possible obtain the normal full load current of α through the circuit.

(4) Slightly re-adjust $R_1$ and $R_2$ to bring $aβ$ up to normal speed, then in quick succession measure the volts $V_1$ at the terminals of the dynamo α and the volts $V_β$ at the motor by means of the key $K_β$ at the same time noting the main current on $A$ and the exciting currents $a_1$ and $a_2$.

(5) If possible obtain three or four different load currents through $aβ$ from the normal downwards, and calculate the efficiency $S$ from the relation

$$S = \sqrt{\frac{V_β}{V_1}}$$

approximately,

and tabulate in a convenient manner.
Inferences.—Show how the above relation can be obtained, and state any assumptions made in obtaining it. What corrections would have to be applied to make it rigorously true? Obtain the true value of the efficiency $\eta$ by applying the correction in question.

The test, though simple, requires a certain amount of experimental skill, especially in the case of series and compound machines. Moreover, the starting is somewhat troublesome.

By a slight modification in the connections, the test is a little easier to carry out, and this is shown in Fig. 89. Like the preceding arrangement it involves the use of an auxiliary generator or a set of secondary cells having the same current capacity as the machines under test, and a voltage of from 3 to $3\%$ of that of the generator, according to their efficiencies. This, being, as before, in series with the generator and motor, takes the form of an added voltage to the system.

It is much better to exite the shunts from an independent supply instead of the auxiliary source.

In this arrangement the motor $\beta$ must have the strongest field, and in order to start, the field $F_\alpha$ of the generator $\alpha$ must either be broken or be made comparatively weak by means of the rheostat $R_1$.

Apparatus.—Similar to that for the preceding test; source of R.M.E. $E$ necessary to fully excite the shunts $F_\alpha$ and $F_\beta$, the Auxiliary Source $\gamma$ being as above mentioned.

Observations.—(1) Connect up as shown in Fig. 88, and adjust the ammeters $A$ and $a$, and voltmeter $V$ to zero, etc.
(2) With \( P_1 \) and \( R_1 \) full in and the voltage \( e \) of the source \( E \) at the required value, close \( N \), adjusting \( R_1 \) to obtain full load current \( A \) through \( a \) and \( \beta \), then simultaneously take the readings of \( a, e, A \) and the volts \( V_1 \) and \( V_2 \) across \( a \) and \( \gamma \) by means of the key \( K \).

(3) Calculate the efficiency of either machine from the relation

\[
\eta = \sqrt{\frac{A \cdot P_1}{V_1 (V_1 + P_1) + n \cdot w}}
\]

and tabulate your results in a convenient manner.

(84) Measurement of the “Nett” or “Commercial” Efficiency of Direct Current Dynamos. (Kapp’s Electrical Method.)

Introduction.—The following, being an electrical method entirely, has the advantage that all the measurements are electrical, thereby enabling the efficiency to be determined with far greater accuracy than would be possible with any mechanical transmission dynamometer.

The method consists in coupling the generator to be tested both mechanically (with their armatures in alignment) and electrically to a similar type machine of as nearly equal power as possible, and which latter is made to run as a motor, driving the other, by the weakening of its field, with a rheostat. A small auxiliary generator, giving the normal voltage of the machine to be tested, is required, and must be so connected that it can be placed in quick succession across the terminals of the two coupled machines. The auxiliary source therefore supplies the necessary exciting currents together with the difference of the currents flowing in the two coupled machines. The test, though simple, requires a certain amount of experimental skill, especially in the case of series and compounded machines.

Apparatus.—Dynamo \( A \) to be tested, assumed to be a shunt wound machine and having its field coils \( F_a \) across its terminals; another similar machine \( B \) to act as a motor and having a rheostat \( R_b \) in its field \( F_b \); “change over” switch \( C \) (Fig. 263);
main rheostat \( R_1 \) (p. 606); ammeter \( A \); voltmeter \( \mathcal{V} \); switch \( S_1 \) and auxiliary source of B.M.F. \( (y) \), which may consist of the town mains (if the supply is continuous current), secondary battery, or small dynamo giving the normal E.M.F. of the generator \( a \) to be tested.

Observations:—(1) Connect up as shown in Fig. 87, and adjust the point-ers of \( A \) and \( \mathcal{V} \) to zero, if necessary. Arrange the machines \( a \) and \( \beta \) in alignment and couple their shafts together by a suitable coupling.

(2) Turn the "change-over" switch \( C \) to \( a \), and with \( R_1 \) large, close \( S_1 \) and gradually adjust \( R_1 \), and consequently the current until the machines start. Then when they are running at a constant speed, with \( \mathcal{V} \) reading the normal voltage of \( a \), note the ammeter reading \( A_a \).

(3) Quickly "change over" \( C \) so as to place the auxiliary source \( y \) across \( \beta \) and note the ammeter reading \( A_\beta \) for the same voltage \( \mathcal{V} \) as before.

(4) Repeat 2 and 3 for some four or five different speeds, current, and voltages, and calculate the efficiency from the relation:

\[
\text{Combined efficiency of the two machines } = \frac{A_a}{A_\beta}
\]

Commercial efficiency of either machine = \( \sqrt{\frac{A_a}{A_\beta}} \)

Tabulate your results as follows:

<table>
<thead>
<tr>
<th>Name ......</th>
<th>Date ......</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generator tested: No. ... Type ... Maker ... Normal Volts ... Amps ... Speed ... Machine Co. Ltd. No. ... ... Normal ... a ...</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Speed in Revs. per min.</th>
<th>Voltage ( \mathcal{V} ) (Volts)</th>
<th>Currents in Amps.</th>
<th>Efficiency of Generator tested at the speed</th>
<th>( A_a )</th>
<th>( A_\beta )</th>
<th>( \frac{A_a}{A_\beta} )</th>
<th>( \sqrt{\frac{A_a}{A_\beta}} )</th>
</tr>
</thead>
</table>
Inferences. — Show how the expression for the efficiency can be obtained, and dilate on the advantages and disadvantages of the method.

The preceding method can be slightly simplified by the following modifications. As in the above test, the following one involves the use of an auxiliary generator or set of secondary cells, having the same voltage as the machines under test and a current output of about 8 to 25% of that in the armature of α or β. Being in parallel with the machine to be tested, it takes the form of an added current to the system at the same voltage as the combination under test. The present tests are more convenient, generally speaking, and much simpler as regards starting than those of No. 83. Fig. 86 shows the connections, and the apparatus required is much the same as in the preceding method, except that the change-over switch C, Fig. 87, is dispensed with.

The fields \( F_α \) and \( F_β \) can be connected as shown in Fig. 88 instead of as in Fig. 87 if preferred, and it will then be noticed that \( 87 \) and \( 88 \) are electrically the same when the change-over switch \( C \) is kept as shown, and an ammeter \( (a) \) inserted in one of the leads connected to it. The source of supply γ, whether power main or a third generator, must have a voltage at least equal to that of either α or β. Further, the losses in \( α \) and \( β \) are measured directly, and are small compared with the output of \( α \) and \( β \); hence a small percentage error made in measuring them will be very small compared with the output of \( α \) and \( β \), and will have but little effect on the resulting efficiencies. When two machines of the same size and type have to be tested, this method is almost always used in works for determining their efficiency and heating on a full-load time test.
Observations.—(1) Connect up as in Fig. 38 and set all the instruments to zero.

(2) To start up, put $H_1$ full in and cut out $R_2$ to short circuit, so that the fields $P_2$ and $P_3$ are as nearly as possible of equal and maximum strength. Then close $S$ and slowly cut out $R_0$, when the machines will start up as two similar motors in parallel on no load. The ammeter $A$ will now read about half that of (a) because a will be taking about half the supply current.

(3) Now weaken the field $F_4$ of the machine $\beta$ by slowly increasing $H_4$, which will cause it to run faster and act as a motor, driving $a$ as a generator. The reading of $A$ will simultaneously fall, while that of (a) will remain nearly constant; and when $A$ becomes zero, the voltage of $a$ will have reached a value just balancing that of the supply $P$, and (o) will indicate the current required to run $a$ and $\beta$ together at 0 load.

On still further increasing $H_4$ the current through (a) will be reversed, indicating that $a$ is now commencing to supply, instead of receive, current.

Note.—For this reason $A$ should be either of the moving coil type of instrument, or of the moving coil permanent magnet type connected in circuit through a reversing switch, otherwise a central zero moving coil type must be used.

(4) Take a series of load currents, as indicated on (a), differing by about equal amounts between 0 and the full-load value for either machine by still further increasing $H_4$—noting the readings of all the instruments and the speed at each load, $V$ being constant at about normal voltage.

Note.—This circulating current $A$ between the machines $a$ and $\beta$ will increase with the difference between their field strengths; and the limit is reached when the combination of a large current in the motor armature, and its weak field and high speed, causes excessive sparking.

Tabulate your results as follows—
(3) Plot curves having values of $E$ and $F_1$ as ordinates, with $W$ as abscissa.

Inferences.—What errors, whether small or large, is the method liable to, and on what does the accuracy depend?


Introduction.—This method of measuring the efficiency of an electrical generator, namely, by means of a transmission dynamometer, can be applied to a direct current generator equally as well as to an alternating current one. As, therefore, the application of the method to each of these two great classes of machines, to form two separate tests, was considered superfluous, preference was given to its application with an alternator, in that the output of a direct current generator is at once given by the product of the volts and amperes, while that of an alternator may present some difficulty to obtain accurately, the reasons for which are carefully explained. The actual measurement of the driving power by the dynamometer is obtained in precisely the same manner no matter what generator is being tested.

There are many different methods of finding the commercial efficiency of an alternator, depending in some cases on whether the armature rotates or is stationary, on the capacity of the machine, and on the facilities at hand for testing. In all cases the commercial efficiency —"mean" useful power developed—total power absorbed by the alternator, the latter being—power
applied to the pulley to turn it, the power used in exciting.
The mean or true power developed is easily obtained if a non-
inductive resistance, such as a bank of glow lamps or water
resistance, is at hand which will carry the full load current of the
machine, for then the true power = amperes x volts. This will
not be true if the resistance is inductive owing to the "phase
difference" between the current and voltage. For such a case
the true power may be obtained by a non-inductive Wattmeter
or the S-voltmeter method (p. 379), etc. The power applied to
the alternator pulley to drive it is very commonly obtained by
indicating the engine, especially in large "sets." In the present
case a transmission dynamometer is used to measure this power.
It is of the spring type, and the means for recording the
readings of it were devised by Prof. W. Stroud. The indications,

![Diagram](image)

which are recorded electrically, represent the net pull, or
difference of tensions in the two sides of the belt in lbs. Then
knowing the speed of the alternator and the diameter of its
pulley, the H.P. can at once be deduced. For a full and
detailed description of the dynamometer, see Appendix, p. 625.

**Apparatus.**—Alternator B to be tested; transmission dynamo-
meter complete with its indicating galvanometer G (p. 625);
tachometer, a-c ammeter A and voltometer F; N-C am-
meter (a); switch X; and non-inductive resistance or bank of
lamps R (p. 629); exciting circuit containing ammeter (a),
rheostat r (p. 629), switch X, and exciting E.M.F. E.

**Observations.**—(1) Connect up as shown in Fig. 89, and see that
all connections in use are slowly. Adjust the secondary E.M.F.
(p. 629) for use with the dynamometer, so that when placed
directly across the terminals of G, a full scale deflection is
produced. Then insert it in its proper place.
(3) With the alternator belt on the loose pulley on the counter-shaft, start the motor which drives this shaft, and note the mean deflection on $S$ for different speeds. If this is appreciable it must be deducted from each of the readings which follow.

(3) Now throw the belt on to the first pulley so as to start $D$, and without $S$ being closed or the field excited, again note the mean deflection on $S$ for different speeds.

(4) Adjust the speed of $D$ to give normal frequency and the excitation to give normal voltage on $V$. Note the reading of $S$, with $S$ still open.

(5) Close $S$ and repeat 4 (keeping the speed constant) for about ten different load currents on $A$, rising by increments to the maximum permissible by varying ($N$).

(6) Repeat 4 and 5 for frequencies of 40, 50, and 60% of the normal respectively, and tabulate as follows:

<table>
<thead>
<tr>
<th>Name</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternator No.</td>
<td>Maker</td>
<td>Normal output</td>
</tr>
<tr>
<td>Resistance of exciting coils</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diameter of alternator pully</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Streke (bar)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(7) Plot curves for each speed having $A$ and useful H.P. developed as abscissae, and $V$ and efficiencies as ordinates respectively. Also between H.P. developed as ordinates, and H.P. required to drive as abscissae.

Note.—The nett pull of the belt in lbs. must be obtained from the deflection of $S$ with reference to the latest calibration curve of the dynamometer.

The Testing of Continuous and Alternating Current Electro-Motors.

General Introduction.—Since the production of the electro-motor in its more practical form within recent years, the use
to which it has been applied, for the electrical driving of workshops, haulage, electric traction, etc., etc., have assumed such proportions as to make the different forms and types of electric motor at the present day multimultiples. The systematic testing, therefore, of such machines becomes of considerable importance, in order that a comparison may be obtained and a judgment formed of the weak points of any particular type, together with its performance and qualities (whether good or bad) which it possesses.

No motor, least of all one intended for electric tram and railway work, should leave the makers' works or be installed in its proposed occupation without being first thoroughly tested for the following points—(a) Resistance, or conductivity of its electrical circuits; (b) Insulation resistance between earth or framework of the machine and the copper circuits both individually and collectively; (c) Brake horse-power; (d) Efficiency; (e) Heating, or rise of temperature of the various parts of the machine after a run at full load for a specified time. These tests we may now consider more in detail.

(a) Copper Resistance.—That of each of the copper circuits should be separately measured, by the Wheatstone Bridge in the ordinary way (p. 81) in the case of the shunt coils or other circuits of several ohms, and by the Potential Difference Method (p. 84) or voltmeter and ammeter method (p. 85) in the case of the armature and series coils or other low resistance.

(b) Insulation Resistance.—That of the various parts can be obtained by Tests Nos. 43 and 49 (pp. 113, 115) or other convenient method, at a pressure of something like three or four times the normal working pressure of the machine. The insulation resistance of the machine as a whole, when tested at the normal voltage, should not be less than 2000 ohms per volt, whereas, of course, that of the individual coils or circuits will be much higher. Some makers merely test the separate parts under a pressure of 2500 to 6000 volts alternating, and if they stand this they are passed as satisfactory. This pressure can conveniently be obtained by means of a small testing transformer, stepping up from, say, 100 to 5000 volts, carefully set fuses being placed in circuit to prevent damage should the insulation break down.

(c) Brake Horse Power.—This may be measured in one of
three ways, depending on the facilities at hand for testing; namely, by an absorption dynamometer, in other words, a modified form of Fray brake, by the "balance" or "cradle" method, or lastly by the electrical method. The last two methods will be described in conjunction with their application to tests which follow later on, but we will now consider the principle involved in the first-named method, reserving the description of some convenient forms of brakes until later. It will be sufficient if we consider the principle of the simplest form of brake, consisting of a rope or band abc, of diameter or thickness (a), lapped with any arc of contact b (in circular measure), from a fraction of a turn to more than one turn, over the face of the motor pulley P, having a radius (r) and which rotates we will suppose counter-clockwise, as indicated in Fig. 90. To one end c is attached a large weight W, and to the other (a) a small one w. Now when the pulley P is at rest, W = tension on the right-hand or "tight" side of the rope, while w = tension on the left-hand or "slack" part of the rope. Then, as P rotates, the couple or torque T, due to the force of friction between the rope and surface of the pulley, tending to resist motion, and against which the motor does work, is

\[ T = (W - w)(r + \frac{1}{2}d) \text{ pound feet}, \]

where \( W \) and \( w \) are in lbs. and \( r \) and \( d \) in feet, \( W - w \) being the difference in tensions or nett load on the brake in lbs., and \( r + \frac{1}{2}d \) the mean effective radius in feet of pulley and rope together at which the nett load acts. If \( n \) = number of revolutions per minute made by P, then \( 2\pi n \times 60 \times w \), the angular velocity of the pulley, and the work per second, or the rate at which work is done by the motor on the pulley = \( nT \).

Hence we have

\[ \text{H.P. developed} = \frac{(W - w)(r + \frac{1}{2}d)}{(2\pi n \times 60)} \times 33,000, \]

where 1 H.P. is equivalent to 33,000 foot-lbs. per minute.

All the power thus measured and appearing at the pulley is
wasted in heating this latter, and herein lies one of the chief difficulties in testing larger H.P.s, namely, the getting rid of the heat so generated by friction, for not only is the heat liable to burn the rope in two if the power of the motor is sufficient, but it also affects the coefficient of friction $\mu$ between the rubbing surfaces, thereby causing the brake to jerk and preventing any steady readings being taken.

To obviate this trouble, either the pulley must be water-cooled (see p. 633), or readings must be taken immediately after adding a weight, and then the weight released from the rope. The trouble is further intensified by the motor running at such fast speeds, which is common to this type of driving power. By a slight modification of this form of brake, viz. substituting a spring balance for $w$, the brake becomes automatically self-regulating for variations of $\mu$, for then if $\mu$ suddenly increases, $W$ rises, and $(\sigma)$, which now is the spring-balance reading, decreases, therefore $W-\sigma$ increases and restores the brake to its first position. The coefficient of friction $\mu$ can be calculated thus—

Let $\theta$ are of contact (in circular measure) between cord and pulley, then

$$W = \frac{c}{\theta}$$

where $c$ = base of the Napierian logarithms = 2.71828.

The friction surfaces (in contact) of the brake should be as large as possible, in order to readily dissipate the heat generated.

Mr. Maw gives the following rule for finding the smallest dimensions of a brake pulley: if H.P. = horse-power to be measured by the brake, and $v$ = peripheral velocity of the pulley in feet per minute, and $(\sigma)$ = width of rubbing surface in contact, measured axially, then $\frac{c}{\theta}$ must not be less than 700.

(c) FRACTION.—This can at once be obtained if the electrical H.P. absorbed by the motor for a given B.H.P. is known. If $A$ amperes are read off on the ammeter is passing into the machine at a 120$^\circ$ of $V$ volts read on the voltmeter placed across the terminals of the machine, then the input or E.H.P. $= \frac{AV}{115}$

where 1 H.P. = 746 Watts.

Hence the commercial efficiency $\eta = \frac{\text{B.H.P.}}{\text{E.H.P.}} = 100\%$. 

(c) Heating.—This may be limited by specification, or the question of safety to the conductors, and also considerations of overloading. It is not advisable that the rise of temperature of any part of the machine should exceed 40° C. above that of the external atmosphere after a six hours' run on full load. The temperature can be obtained by placing the bulb of a thermometer on the part to be tested and covering it over by some cotton wool. This can only be done to the armature at the moment of stopping, and it will here be noticed that a sudden rise of surface temperature occurs in the armature at the moment of stopping, due, of course, to the ceasing of the ventilating action which goes on while it is rotating (see p. 216).

(86) Variation of Speed with Voltage across the Armature of a D.C. Electro-Motor (at Constant Excitation).

Introduction.—This is an important test, in that it will familiarize the student with the fundamental principles underlying the regulation and control of motors. It can be carried out on a series, shunt or compound-wound motor, so long as the corresponding change in the connections and means for maintaining constant excitation are made. As, however, the same result is obtained with each type of motor, we shall operate the test with the simplest type, viz. the shunt motor.

Note.—In a series motor the field regulating resistance, at least equal in value to the resistance of the series coils, must be shunted across them; whereas in shunt and compound motors it is connected in series with the shunt coils, and has a resistance and current-carrying capacity at least equal to those of the shunt coils.

Apparatus.—Shunt motor, of which $M$ is the armature and $F$ the field; main circuit variable rheostat $R$, ammeter $A$, and switch $S$, each capable of dealing with the full-load current of $M$; voltmeter $V$ and supply mains $M_1 M_2$ of voltage $V$ each for the rated voltage of $M$; field rheostat ($e$) and low-reading ammeter ($a$); tachometer.

Observations.—(1) Connect up as shown in Fig. 91 and adjust the pointers of $V$, $a$, and $A$ to zero if necessary.
(3) With \( r \) all out and \( R \) full in, close \( S \) and gradually cut \( R \) out to short circuit as \( M \) gains speed, then adjust \( r \) to get normal speed \( (n) \). Note the readings of \( V, A, \) and \( u \), which last-named must now be kept constant throughout the test by varying \( r \).—(See that the lubrication of \( M \) is working).

(3) With the motor still running light as in (2) above, vary \( R \) so as to obtain some eight different speeds \( (n) \) in about equal steps between 0 and the normal, and note the corresponding readings of \( V, A, \) and \( u \) (\( u \) being kept constant throughout).

(4) Repeat (3) with the motor running at full load (if arrangements permit), and for the same value of constant field current \( (e) \) as before.

Note.—The leading-up of motor can most conveniently be effected by means of an oilly-current bank or by taking any desired output from a coupled generator.

![Diagram](image)

Fig. 91.

(5) Repeat (3) and (4) with, say, half the previous excitation maintained constant and tabulate all your readings as follows—

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>May 19</td>
<td>123</td>
<td>456</td>
<td>789</td>
<td>1011</td>
<td>2.5</td>
<td>3.2</td>
<td>0.8</td>
<td>0.5</td>
<td>1.2</td>
<td>0.9</td>
<td>0.6</td>
<td>0.7</td>
</tr>
</tbody>
</table>

(6) Plot to the same pair of axes, curves having speed \( (n) \) as ordinates with values of \( V, A, \) and \( (\text{supply voltage across } M_1 M_2) \) as abscissae.

Inferences.—State clearly what you can deduce from the table of results and curves, and show how they can be applied.
to the design of a main circuit current rheostat for controlling the speed of the motor.

(87) Variation of Speed with Excitation in a Direct Current Electro-Motor (with Constant Supply Voltage on Armature).

Introduction.—The reader should peruse the introduction of the last test, the remarks in which apply to the present test also. In addition, it may be pointed out that when the motor is running light the back E.M.F. will remain nearly constant, since the power required to drive is usually very small, and is \( \alpha (V - \phi) \), consequently the increase of speed will be almost inversely \( \alpha \) to decrease of field strength.

Apparatus.—That required for the present test is precisely as detailed for the last one, and need not be repeated again here.

Observations.—(1) Connect up exactly as shown in Fig. 91 of the last test, and adjust the pointers of \( V \), \( a \) and \( A \) to zero if necessary.

(2) With \( r \) all out and \( R \) full in, close \( S \) and gradually cut \( E \) out as the motor gains speed, until \( V \) reads the normal voltage of the motor; then adjust \( r \) to get normal speed. Note the readings of \( A \), \( a \) and \( V \), which last-named must now be kept constant throughout the test by varying \( R \) (see that the lubricating arrangements of the motor \( M \) (Fig. 91) are working).

(3) With the motor still running light, as in (2) above, vary \( r \) so as to obtain some 8 speeds \( \phi \), differing by about equal steps between 0 and the normal value, and note the corresponding readings of \( A \), \( a \) and \( V \) (\( V \) being kept constant throughout).

(4) Repeat (3) with the motor running at full load (if arrangements permit), and for the same constant value of \( V \) as before.

Note.—The most convenient way of winding up \( M \) (Fig. 91) is by means of an eddy current brake, or by taking the required output from a coupled generator.

(5) Repeat (3) and (4) with, say, half the previously normal value of supply voltage \( V \) across the armature, maintained constant, and tabulate as follows—
(b) Plot the same pair of axes, curves having speed \((a)\) as ordinates with values of \((A)\), \((a)\) and \((\text{supply voltage across } M_M)\) as abscissae, and between \(1/n\) as ordinates with \((a)\) as abscissae.

Inferences.—State clearly what can be deduced from the table of results and curves, and show how these can be applied to the design of a field regulating rheostat for controlling the speed of the motor.

(88) Variation of Voltage, Current, and Speed, with position of the Brushes around the Commutator of a D.C. Machine at Constant Excitation.

Introduction.—Although the usual practice now is to design D.C. generators and motors with a fixed diameter of commutation and immovable brush-bars, special cases are met with in which provision is made for moving the brushes through considerable angular space round the commutator. As is well known, the terminal voltage of a generator and the speed of a motor is each capable of variation by moving the brushes, while a motor can even be stopped and reversed by so doing. In fact, where the variation of voltage or speed required is not large, it can be obtained by brush movement without expensive field regulators and without altering therefore the field strength—a feature which is sometimes valuable, while most machines will admit of quite an appreciable brush movement without much sparking, when running light, such is not the case when they are running on load, so that the scope of this test may be limited by the amount of sparking. Further, it will be found easier to
apply the test to a motor than to a generator, and we shall therefore operate the present test on a motor.

**Apparatus.**—Shunt motor, of which $M$ is the armature and $F$ the field; main circuit variable rheostat ($R$); ammeter $A$, and switch $S$, each capable of dealing with the full-load current of $M$; voltmeter $V$ and supply mains $M_1 M_2$ of voltage $E$ at least equal to the normal for $M$; field rheostat $r$ and low reading ammeter ($a$); tachometer and, if possible, some scale for indicating the angular motion of the brushes round the commutator.

**Observations.**—(1) Connect up as in Fig. 12, adjusting ($V$) ($A$) and ($a$) to zero if necessary, and ($R$) to a value not less than the ratio of full-load current to the supply voltage, so as to prevent the full-load motor-current being exceeded if the armature comes to rest or its speed increasing too rapidly as the brushes are moved.

**Note.**—This value of $R$ will be given by blocking the armature and with normal excitation, noting the value necessary to give full-load current on $A$.

(2) Start the motor either by using the ordinary "Starter," or by ($R$) temporarily increased before closing $S$, and then gradually cut out $R$ until normal speed is reached—the brushes being in the normal full-load running position, and the excitation adjusted to normal full-load value.

Now note the values of $V$, $A$, and ($a$), and the speed.

(3) Next keeping ($a$) constant at the above value, adjust $R$ to the minimum value allowable and found in obs. (1), and note the readings of $V$, $A$, and speed for normal position of brushes (as in obs. 2), and for a series of different positions throughout an angular distance $= \frac{1}{2}$ the polar pitch either side of their normal position.

(4) Repeat (3) at the nearest B.H.P. load to full load which it is practicable to run at, and tabulate as follows—
Electrical Engineering Testing


data

Name . . . . . . . . .
Type . . . . . . . . .
Number . . . . . . .
B.H.P . . . . . . .
Armature Reactance . .
Vols . . . . . . .
R.p.m . . . . . . .
Normal Reaction . .

Function of Speed . .
Vols per min . .
Armature . . . . .
Back E.M.F . . .
Field Strength . .


Introduction.—The series motor in general possesses some characteristic features which it may be well here to note in view of the prominent place this type of motor has, and still is, taking in electric traction and power work generally. Since it can be shown that the torque $T$ of the motor is given by the relation

$$ T = \frac{C \cdot N \cdot A_0 \cdot (E - e)}{2\pi \cdot \frac{E}{\frac{v_a}{2\pi}} - \frac{e}{2\pi} - \frac{v_a}{2\pi}} $$

where $C =$ number of armature conductors all round,

$N =$ number of lines threading the armature or the useful flux,

$A_0 =$ number of armatures of current through armature,

$E$ and $e =$ impressed and back E.M.F.s of the mains and motor respectively,

$v_a =$ speed in revs. per second,

and $v_a =$ resistance of armature circuit.

It will be at once evident that the torque exerted is a maximum at starting, i.e., when $v_a = 0$, and that it varies as the armature current $A_0$, since $N$ also varies as $A_0$.

Again, when the motor is "running light" at its maximum speed $T = 0$ nearly, for then the back E.M.F. generated almost equals the voltage $E$.

Thus a series motor tends to race directly the load is
suddenly removed, which is an undesirable feature for workshop driving.

The fact that \( T = \text{maximum at starting, and that the motor} \) will start on full load, is a most valuable property for traction work on tram and railway lines.

In the following Fig. and all after it, the motor is represented symbolically, \((a)\) denoting the armature, commutator, and brushes, and \(FM\) the field magnet coils, which in this case, being series wound, are represented by a few curly lines.

Apparatus.—Electromotor (series wound) to be tested, fitted with an absorption dynamometer or brake (Fig. 295); ammeter \(A\); voltmeter \(V\); variable rheostat \(R\) (p. 606); switch \(S\); battery or dynamo \(B\), giving the requisite voltage needed for the motor, and speed indicator and set of half-pound and one pound weights for the brake, also a lubricant if necessary.

Observations.—(1) Connect up as indicated in Fig. 93, and adjust the pointers of \(A\), \(V\), and the tachometer to zero if necessary. See that all lubricating cups in use feed slowly and properly.

(2) See that \(R\) is at its full, then carefully remove the brake from the pulley and close \(S\). Take a series of gradually ascending and descending observations (by varying \(R\)) for about ten different speeds, ranging by about equal intervals between the lowest readable on the tachometer and the maximum safe speed for the motor, noting this speed and the corresponding values of \(A\) and \(V\) at each.

(3) Replace the brake and repeat 1 for no weight in the pan. From 2 and 3 the loss in watts in the brake can be found.

Note.—It will probably be necessary to exert a very small pressure by the finger on the brake in 3 to prevent it being carried round as the pulley rotates. No appreciable error need be introduced due to this. If a form of brake is used in which no loss of power can occur other than that incidental to its
use when actually measuring power, then omit obs. 3 and also the last seven columns in the next table, and substitute a column headed \( A_2 V_2 \) watts to run motor at no load.

Tabulate your results as follows—

<table>
<thead>
<tr>
<th>Speed,</th>
<th>Without Brake</th>
<th>With Brake</th>
</tr>
</thead>
<tbody>
<tr>
<td>tours, per min. (%)</td>
<td>Volts</td>
<td>Amperes</td>
</tr>
<tr>
<td>Accending</td>
<td>Decending</td>
<td>Accending</td>
</tr>
<tr>
<td>Mass</td>
<td>Loss in Brake</td>
<td>( W_B = (A_1 V_1 - A_2 V_2) )</td>
</tr>
</tbody>
</table>

(4) With the brake carefully replaced on the pulley and the smallest weight in the scale pan, close \( S \) and by varying \( R \) adjust the speed to the lowest convenient. Note this and the reading of \( V \) and \( A \) simultaneously, and the weight.

(5) Repeat 4 at the same constant speed for ten or twelve loads or weights in the pan, ranging from the smallest to that which will cause the current to rise to not more than 25% over normal.

(6) Repeat 4 and 5 for the maximum allowable speed and an intermediate one, each constant throughout, and tabulate your results as follows—

<table>
<thead>
<tr>
<th>Make...</th>
<th>Date...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor model...</td>
<td>No.....</td>
</tr>
<tr>
<td>Taps...</td>
<td>Type...</td>
</tr>
<tr>
<td>Voltage...</td>
<td>Number...</td>
</tr>
<tr>
<td>Speed...</td>
<td>Wires...</td>
</tr>
<tr>
<td>Normal H.P. =...</td>
<td>Amperes...</td>
</tr>
<tr>
<td>Weight...</td>
<td>Volts...</td>
</tr>
<tr>
<td>Speed = ...</td>
<td>Watts per min.</td>
</tr>
</tbody>
</table>

| Weight in Pan (lb.) | Lower | Newt | Weight or | Total H.P. | Development |
|---------------------|-------|------|Balance reading = (lb.) | Amperes | H.P. |
| \( (W - w) \) | Amperes | Total H.P. | \( \text{H.P.} = \frac{B_1}{220} \) | \( \text{H.P.} = \frac{R_1}{220} \) |

Note.—The true H.P. developed = H.P. calculated + H.P. lost in brake itself.

(7) With the brake removed from the pulley and \( R \) full in, close \( S \) and obtain the maximum speed allowable. Note this and also simultaneously the amperes \( A_1 \) and volts \( V_1 \).

(8) Replace the brake and add weights to the pan so as to obtain about ten different loads to the point when the largest load
stops the motor. Note the current $A_1$ and speed ($a$) at each, the volts $V_1$ having been kept constant by altering $R$ to suit the load.

Note.—The current should not exceed about 40% above the normal. Tabulate your results as above.

(9) From observations 3 and 3 plot the following curves between
(a) Volts $V_1$ as ordinates and the corresponding speeds ($a$) as abscissae.
(b) Brake loss $P_2$ as ordinates and the corresponding speeds ($a$) as abscissae.

From observations 1–6, plot for each speed curves between
(c) Efficiency and current as ordinates and corresponding B.H.P.s as abscissae.

From observations 7 and 8 plot the following curves between
(d) The speed in each case and B.H.P. input, B.H.P., and efficiency.
(e) The speed and current as ordinates and torque as abscissae.

(10) Calculate the coefficient of friction ($a$) between the brake band and pulley for various loads and for the arc of contact employed.

From the curves (d) deduce the relation between the speeds that give maximum efficiency and maximum B.H.P. respectively.

Inferences.—State very clearly all the inferences deducible from your experimental observations. Explain fully the curves obtained in (9) (d) above.

Note.—The general form of the curves (d) 9 above are shown
in Fig. 91. The diagram due to Mr. Kapp is an exceedingly useful one for seeing the relative H.P.s and efficiency. \( OaB \) is the B.H.P. curve, \( OaB \) is the efficiency curve, and \( Abh \) is the N.H.P. (input) curve. The shape of this last varies with the type of series motor run off constant potential mains. The ordinates, such as \((ad)\), of the efficiency curve \( OaB = \frac{cd}{bd} \) at this and similar points to any arbitrary scale of ordinates.


Introduction.—The particularly heavy and trying work which a tramway or railway motor has to perform renders it all important to subject the machine to the most searching tests for defects or other faults at the outset. Such tests are twofold—

1. A complete test of the motor at the works of the makers and again when fixed to the car.

2. A test of its performance when driving the car on some approved route on the system. With regard to this case, the worst route of the whole system is chosen, i.e., one having the steepest gradients and sharpest curves.

The car is loaded with an artificial load, such as sand bags, for instance, equal to the full load of passengers which it is intended to carry. It is then run as continuously as possible along that route with five-second stops every five minutes for one two hours.

This test is considered satisfactory, if, at the end of that time, all has gone on satisfactorily and the temperature of the armature and commutator of the motor has not risen above the prescribed limit.

Next, with regard to the "Works Tests." Besides the efficiency test at various loads, the motor should be run at the average speed it will run at in practice, say that corresponding to eight or nine miles per hour of the car, for four to six hours at the maximum load which the motor is intended for.

Except in the case of electric railways, where the car or engine axle is direct driven, single reduction gear between motor and car axles is almost universally employed of between 4.75 and 4.85 to 1.
The sizes of tramcar wheels are usually either 30 inches or 33 inches in diameter. The remarks mentioned in the Introduction of Test 29 should be carefully read and remembered, when the performance of the motor on test will at once be obvious.

Caution.—The operators of the controlling rheostat switch-gear, etc., must stand on the india-rubber mat provided, and must on no account touch any live metal work on the circuit of the 500 volt generator and tramway motor.

Great care must be taken to ensure that the rheostat in the main circuit of the motor is fully in before closing the main switch, and also that it is at once re-inverted before pulling out that switch on stopping.

The apparatus and connections of the preceding test are those now to be obtained, and the observations, as there given, to be carried out. In addition to curves 9, α—ε, plot two on the same curve sheet, having speed as abscissa, and both torque and amperes as ordinates.

(91) Relation between the Starting Torque and Current in a D.C. Electro-Motor.

Introduction.—It is very instructive to compare the results obtained in applying the principle of the present test to series, shunt, and compound wound motors, but it may also be applied to alternating current motors. The torque or turning effort by the armature on its shaft, measured in terms of a pull, acting at a given radius or leverage from the shaft centre, and tending to turn it, is expressed in pound-feet, usually, in this country. In a motor it results from the interaction of the field magnets and the field of the armature as set up by the currents flowing through it, and is α to the product of these two field strengths.

If \( C \) = the number of effective conductors on the armature;

\( N \) = the effective magnetic flux cut by them or flowing in the arm;

\( A \) = the armature current;

then it can be shown that the torque \( (T) \) is given by the relation

\[
T = \frac{CN}{8.52 \times 10^6} \times A. \quad \text{lb. ft.}
\]
an expression independent of the speed of the motor.

\[ T \propto \text{armature current} \times \text{field flux} \propto A N. \]

In a series motor, the field flux \( \Phi \) will vary as \( A \) varies, up to the point of magnetic saturation of the field magnets, when it will be practically constant. Any further increase in \( A \) will give the relation

\[ T \propto A. \]

This also holds for a shunt motor, in which the field excitation is practically constant and near saturation.

In a compound or differential motor, however, the shunt and series windings oppose each other magnetically, and hence on starting it may happen that they nearly balance, thus giving practically zero starting torque.

![Diagram of an electrical circuit](image)

**Fig. 95.**

In captain, windlass, and all traction work, maximum torque is required on starting; and since the series motor fulfills this condition and is exclusively used for the purpose in D.C. work, we will consider this type of motor as the one used in the present test.

**Apparatus.**—Series motor \( M \) to be tested on a suitable D.C. supply \( S \); brake-blocks \( B \) with yard-arm \( L \) and spring balance \( W \); ammeter \( A \); switch \( S \); and variable current rheostat \( R \).

**Observations.**—(1) Connect up as in Fig. 95, setting \( A \) and \( W \) to zero if necessary. Clamp \( B \) to the pulley of the motor so as to prevent slipping (rotation), and with the yard-arm horizontal. Attach the spring balance \( W \) at a measured distance \( L \) from the pulley centre.

(2) With \( R \) full in, close \( S \), and adjust \( A \) so as to give some ten currents through \( M \) and \( A \) differing by about equal amounts.
between 0 and the full-load current of M, noting the readings of W and A at each.

Note.—The connections of field F and armature of M must be such that the latter tends to turn in the direction shown. Also the yardarm may have an equal overhang as shown, or be otherwise balanced when horizontal.

Further, as there will be a good deal of static friction at the motor bearings, the armature should be rotated by hand a few revolutions before attaching B B, so as to well oil the journals; and even then the mean of several readings of W, taken for each value of A by disturbing the position at which (9) rests, by the hand.

(3) Take an ascending and descending series of values of A, and tabulate as follows—

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</tbody>
</table>

(4) Plot a curve having values of A as ordinates, and F as abscissae.

Inferences.—Point out all that can be deduced from the table of results and shape of the curve.


Introduction.—If a shunt motor, supplied at constant potential, has a very low armature resistance, high shunt coil resistance and field magnets giving a field relatively very much more powerful than that due to the armature, the variation of “load” of the brushes will be slight and the motor will be almost self-regulating in speed for wide variations of load, i.e. it would run at constant speed independent of the torque. The falling off of the speed in shunt motors as the torque increases will be the less as the field magnetism is the more powerful. The brushes, in two pole machines, should press at opposite ends of a diameter, and to ensure sparkless running must have a “backward lead.” In all
cases the efficiency = total power given out ÷ total power put in, both being reckoned in similar units. The input is easily deduced from ammeter and voltmeter readings, but the output is more difficult to obtain accurately. In the present test it is obtained by means of an "absorption dynamometer," which we will assume to be the modified form of Penn's brake introduced by Raffard. Such a brake wastes in heat all the power given out by the motor through friction, but at the same time forms a measure of this power. The arrangement is such that the brake automatically adjusts itself to variations of the coefficient of friction between the rubbing surfaces due to heat. In brake tests of this nature just sufficient lubrication (such as soap and water) and no more ensures smooth working without sudden jerks due to seizing, and this, together with experience in manipulation, is the secret of the success of such tests.

If \( r \) = mean effective radius of pulley and band together in ft., \( n \) = number of revs. per min., \( W \) = weight in lbs. in scale pan, and \( w \) = weight in lbs. at the slack side of the pulley; then the angular velocity of the pulley \( \omega = 2\pi n \) rad/sec., and the couple or torque resisting motion \( T = (W - w)r \); then the work done per sec. = \( \omega^2 T \), and the B.A.P. = \( (W - w) 2\pi n \) ft-lb. = 330000.

**Apparatus.**—Shunt motor \( M \) to be tested; voltmeter \( V \); ammeters \( A \) and \( a \); rheostats \( R_1 \) (p. 600) and \( R_2 \) (p. 600); switches \( S_1 \) and \( S_2 \); source of current \( B \); speed indicator. A set of \( \frac{1}{4} \) lb. and 1 lb. weights are provided with the brake, together with a pump and tank by means of which a slight dripping of lubricant may be allowed to fall into the central rotating pulley and band.

**Note.**—For further remarks on the testing of motors see the "General Introduction" on the subject, p. 233, et seq.

**Observations.**—(1) Connect up as indicated in Fig. 96 and adjust the pointers of all the instruments to zero where necessary. See that all lubricating cups in use are filled slowly and properly.

(2) Uncouple the absorption dynamometer from the motor shaft. Set \( R_1 R_2 \) at their maximum values, and close \( S_2 \), adjusting the exciting current \( (a) \) to the normal value by means of \( R_2 \).

(3) Close \( S_1 \) and take a series of gradually increasing and decreasing observations (by varying \( R_1 \)) for about ten different
speeds ranging by about equal intervals between the lowest readable on the tachometer and the maximum allowable for the motor in question, at constant normal excitation, noting the speed and corresponding values of $A$ and $V$ at each.

4. Repeat 2 and 3 for exciting currents (a) 50% below and 20% above normal respectively.

5. Re-couple the brake and motor together and repeat 2-4 for no weight in the pan. From 2-5 the loss in Watts in the brake can be found.

N.B.—It will probably be necessary to exert a very small pressure by the finger on the brake in 5 to prevent it being carried round as the pulley rotates. No appreciable error need be introduced due to this. Tabulate your results as follows—

![Diagram]

FIG. 96.

1. If a form of brake is used in which no loss of power can occur other than that incidental to its use when measuring power, they may be omitted. It is also the last column in the above table and subtitute a column headed $A_2/V_2$. Watts to the motor are fixed.

5. Adjust both the exciting current (a) and the speed (b) to the normal for the motor being tested and keep both constant, then take a series of readings of $A_2$ and $V_2$ for some ten different loads varying from the smallest weight in the pan to the one which will give an armature current not exceeding 25% over normal.
(7) Repeat 6 for a 50% smaller excitation at the same speed.
(8) Repeat 6 and 7 for a 50% smaller speed.

(9) For a constant voltage across the armature maintained by means of \( R_i \), load the brake with different weights, and note these and the corresponding speeds and currents through the armature at constant normal excitation, and tabulate as follows—

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAME</td>
<td>DATE</td>
</tr>
<tr>
<td>Resistance—Shunt coils ( r_0 ) = ... ohms.</td>
<td>Armature ( r_a ) = ... ohms.</td>
</tr>
<tr>
<td>Effective radius of brake pulley and load ( r ) = ... in.</td>
<td>Current in shunt coils ( i_a ) = ... amperes.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Speed in</th>
<th>Weight in</th>
<th>Nett ( E ) volts</th>
<th>Total H.P. developed</th>
<th>Armature volts</th>
<th>Total H.P.</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r.p.m.)</td>
<td>(lbm.)</td>
<td>(r.p.m.)</td>
<td>volts</td>
<td>Horsepower</td>
<td>volts</td>
<td>volts</td>
</tr>
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<td></td>
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</table>

**Note.**—The true H.P. developed = H.P. calculated + H.P. lost in brake itself.

(10) From experiments 2—5 plot curves between, volts \( E \) as ordinates, and speed (r) as abscissæ also with brake loss as ordinates and speed as abscissæ.

From experiments 6—8 plot curves for each speed, having efficiency as ordinates and total H.P. developed as abscissæ, also between the latter and speed as ordinates.

From experiment 9 plot the mechanical characteristic curve, having speed and current as ordinates and torque as abscissæ.

**Inferences.**—What can you deduce from the results of your experiments, especially from observation 9?


**Introduction.**—The Compound Wound motor is an automatically self-regulating one for maintaining constant speed independently of the magnitude of the load.

Without considering the theory of this regulation, which is outside the province of the present work, and for which the reader should refer to standard theoretical works, it may be
remarked that the desired result is obtained by employing the series and shunt coils to magnetize the field magnets differentially, i.e. while the shunt magnetizes, the series coils de-magnetize. This differential compounding results in the production of a net field at any particular load sufficiently greater than what would be given by an equivalent pure shunt motor to cause the back E.M.F. to rise sufficiently to maintain the speed constant. The efficiency of such motors cannot manifestly be so high as one of the same size which is not wound in this way, since an extra amount of power is used up in producing the demagnetizing force which actually destroys part of the field.

**Apparatus.**—Precisely similar to that required for the shunt motor test (p. 248).

**Observations.**—These are the same as for the above-mentioned shunt motor test, and will not consequently be repeated here.

The experimenter should refer to and carry the present test out in the same way, but in coupling up at the outset, care must be taken to connect so that the series coils oppose the shunt and tend to demagnetize the magnets. Exactly similar tabulation of results and plotting of curves must be carried out with the inferences deducible.

N.B.—The applied E.M.F. to the motor should be maintained constant.

(94) **Efficiency and B.H.P. of Small Direct or Alternating Current Electro-Motors.**

(Cradle-Balance Method.)

**Introduction.**—In testing small motors, such as from $\frac{1}{2}$ to $\frac{1}{4}$ of a H.P., difficulties present themselves in measuring the power developed by them or the work which they will do, owing to the relatively large amounts of extraneous friction introduced in applying the usual brake tests. In fact, in the case of the smaller power motor, this source of friction would entirely vitiate the results and make them worthless. The following method practically gets over this difficulty entirely, and may be carried out in one of two ways—

(a) The motor to be tested is suspended freely with its armature spindle in centres, or on friction wheels, the field magnet
system with its bed-plate, etc., being carefully balanced by counterpoise weights so as to bring the centre of gravity of the system in a line with the spindle. On the motor being supplied with electrical energy, and made to rotate and do work against the friction introduced at the face of its pulley by a stretched cord passing once round, the armature rests on the field magnets tending to rotate them in the opposite direction with a certain force.

If then this action is resisted by a weight or force W attached to the arm of the system at a leverage L, then the moment of this force resisting the tendency, i.e. the torque, = WL.

Thus the arrangement is practically an electro-magnetic dynamometer in which the magnetic friction between armature and field magnets takes the place of mechanical friction in the ordinary dynamometer.

The preceding arrangement of the method has the disadvantage that the weight of the heaviest portion of the machine is resting on the shaft, and consequently there will be a bearing friction resisting the magnetic pull of the armature on the field.

A better arrangement in this respect is one devised by Prof. C. F. Bracket, which is merely a slight modification of the preceding one. It consists in fixing the motor in a "cradle" supported freely by knife edges resting on steel or agate planes, or on friction rollers, carried in a suitable fixed frame. The whole suspended system is very carefully balanced by means of counterpoise weights so that the centre of gravity lies in the axis of the motor shaft, this latter having been set in a line joining the knife edges of the cradle.

A horizontal balanced lever controls the cradle, the end of it either supporting weights or being attached to a spring balance. Thus it will be seen that now the weight of the armature only is on the bearings, and it is being used under ordinary conditions. The balanced lever might be graduated and a sliding weight used to run along it to balance the torque, but the arrangement of the spring balance shown in Fig. 286 is the simplest and easiest to manipulate. This method has a further advantage that the friction at the journals of the motor does not affect or vitiate the measurement, but in the case of the application to a dynamo it should be remembered that it does. It should be borne in mind
that a fruitful source of error may arise due to loss of power in driving the speed indicator. When small motors are being tested, care should be taken to choose an indicator that is very easily driven, and to drive it by means of a spiral or helical spring of, say, thin harddrawn brass. Any eccentricity between the two shafts does not then matter so much as it would if they were direct coupled.

Apparatus.—That required for this test is precisely similar to what is detailed under one or other of the preceding methods of testing series, shunt, or compound wound direct current motors or alternating current single phase motor, according to which of these types of motors the one being tested belongs. In addition the cradle absorption dynamometer is needed, for a complete description of which see p. 631.

Observations.—As, with the exception of the somewhat different type of brake herein to be manipulated, the whole test will be precisely similar to one of the foregoing motor tests, depending on which kind of electromotor is to be tested, the rationale of this present test will not be repeated here. The experimenter should refer to the proper corresponding test and carry out the present one in an exactly similar manner, tabulating and plotting the results in just the same way.

Though the following expression will be found in connection with the description of the cradle dynamometer on p. 631, we may repeat that if \( W \) = weight or force applied at the end of the cradle lever in order to keep the same at zero when the motor is doing work, and if \( L \) = distance between its point of application and the fulcrum of the cradle, then the torque exerted by the motor = \( WL = T \), and the work it does per sec. = \( \omega T = 2\pi nT \).

Where \( n \) = speed in revs. per sec.\-

\[ \therefore \text{H.P. developed} = \frac{2\pi n T}{550}. \]

Consequently if different tensions are applied on the cord wrapped round the motor pulley, causing it to do various amounts of work, thereby taking in different currents \( (d) \) amps. at different voltages \( V \), the efficiency of the motor at each load is---

\[ \text{Efficiency} = \frac{\text{H.P. developed}}{\text{H.P. absorbed}} = \frac{2\pi n T \times 746}{550 \times AV}. \]

Introduction.—In the usual brake tests it is difficult and often impossible to obtain very accurate results, owing to variation of the coefficient of friction between the rubbing surfaces and the resulting jerky behaviour of the brake. The advantage of any method, therefore, of measuring the input and output of a motor by solely electrical means will at once be apparent, as it is possible to obtain much more accurate results with such a method.

The present method, which is purely an electrical one, is due to Mr. James Swinburne, and is sometimes termed the "Strong Power" method. The principle of it and all similar methods is based on the fact that

Total Power given out = Total Power put in - Power lost internally, or in symbols, \[ W_o = W_i - W_L \]; where the suffixes \( O \), \( I \), and \( L \) denote the output, input, and total losses in Watts (\( W \)) respectively.

We thus at once obtain the commercial efficiency of the motor to be

\[ \eta = \frac{W_o}{W_i}. \]

The input in Watts \( W_i \) given to the motor is at once obtained by the product of the volts and amperes of the supply. The total loss \( W_L \) in Watts we will consider now more in detail, and which in any machine is made up as follows: (a) the copper losses \( L_c \) in armatures and exciting coils due to heating by the passage of current, and which can easily be calculated when the currents and resistances are known; (b) the friction losses \( L_f \) due to air churning, journal and brush friction; (c) magnetic frictions or iron losses \( L_m \) due to eddy or Foucault currents and magnetic hysteresis.

Hence total internal loss \( W_L = L_f + L_c + L_m \) and to the sum \((L_f + L_m)\) Mr. Swinburne has given the somewhat appropriate name of "Strong Power." The copper losses are calculable as follows —

Let \( C \) = total current flowing into the motor from the supply, and let \( R_a \), \( R_{es} \), and \( R_{ms} \) be the resistances of the armature, series
coils, and shunt coils respectively of any motor of which $R_m$ can be measured by a Wheatstone Bridge, and $R_a, R_m$ by the "Potential Difference" method (p. 81) or ammeter and voltmeter method, p. 86. Then we shall have for a

Series motor: $I_G = C (R_a + R_m)$,

Shunt motor: $I_G = \frac{V^2}{R_m} + \left( C - \frac{V}{R_m} \right) R_a$, where $V =$ normal working voltage.

Compound motor (long shunt):

$C = \frac{V}{R_a} + \left( C - \frac{V}{R_m} \right) (R_a + R_m)$,

Compound motor (short shunt):

$C = \frac{V}{R_m} + \left( C - \frac{V}{R_m} \right) R_a$.

The remaining inflexes, i.e. the stray power $(I_G + I_m)$, can readily be obtained by running the motor at no load, i.e. with no other load than its own friction, eddy currents and hysteresis, at normal excitation of the field. Then we have

$\left( I_G + I_m \right) = A V_a - A V' = S$.

where $A$ now = eurrent flowing into the motor armature at voltage $V_a$ across the armature, and $A V'$ is the copper loss in the armature occurring for this current and voltage.

Note.—Only quite a small current at the normal voltage of the motor is required to be furnished by an auxiliary source of E.M.F., and if $R_m$ is very small, $A V'$ can be neglected in comparison with $A V$, in this last formula.

Apparatus.—Motor $M$ to be tested, which for purposes of discussion merely we will assume is shunt wound; voltmeter $V$; low reading long scale ammeter $A$; rheostat $R$ (p. 666); tachometer; complete Wheatstone Bridge set (W.I.); two-way voltmeter key $K$ (p. 587); switch $S_2$, source of E.M.F. $E$ at least equal to that for which the motor was built; rheostat $\nu$ (p. 590) in the field coil circuit.

Observations.—(1) Connect up as in Fig. 97, and adjust the instruments $V$ and $A$ to
zero if necessary. Insert E, when the field should then be excited to the normal amount, which can be seen by closing K1 and noting whether the normal voltage is read off on V.<p>(2) With E at its maximum value (not less than about 10 ohms), close K2, adjusting E and if necessary the excitation by the rheostat, so that the machine runs at its normal speed. Now note, by closing K3, the volts V, across the armature terminals and the current A amps. flowing through it.</p>(3) Repeat 2 at the same excitation for some ten different speeds in all, both below and above normal, and tabulate as shown.</p>(4) Open E, S1, and K, and measure by suitable means the resistance R of the armature and R of the shunt, remembering of course to disconnect one from the other at the time.

(5) Calculate the B.H.P. and commercial efficiency of the motor at normal voltage V for some ten different assumed values of current C supplied to the machine ranging from 0 to full load by about equal increments, and tabulate as shown in the larger table.<p>(6) Plot the following curves having:</p>(a) Efficiency as ordinates and output W, as abscissas.<p>(b) Input W as abscissas and speed as ordinates.<p>(c) Output W, as abscissas and input W, as ordinates.<p>Conclusions—State very clearly all that you can infer from the above experimental results.

2 See note in larger table, p. 222-3.
(96) Efficiency of Direct Current Electro-Motors by Poole’s Electrical Method.

Introduction.—This method, due to Mr. Cecil P. Poole, is an electrical one entirely, and enables the efficiency of an electromotor to be obtained without using an absorption brake. The rated B.H.P. of the machine is assumed for the purposes of calculation, and the whole essence of the test consists in obtaining the armature current at which this rated output is obtained.

Let $A$ be the full load armature current.

$V =$ the normal voltage which the motor should have.

$W =$ the normal rated B.H.P. of motor, reckoned in Watts.

$v$ and $a =$ the measured quantities as detailed below.

Then the armature core friction losses $w = (V - v) a$ Watts,

and the armature resistance $r = \frac{V}{a}$ ohms.

Hence $A = \frac{V - \sqrt{V^2 - 4(V + w)v}}{2v} = \frac{V}{r} \left(0.5 - \sqrt{1 - \frac{(V - w)^2}{r^2}}\right)$

This value for $A$ is based on the assumption that the core losses, armature friction, windage, and eddy currents in the pole pieces all remain constant from 0 to full load. While this is not strictly the case, the error introduced is practically negligible.

Apparatus.—Suitable source of supply of slightly higher voltage than the normal required for the motor to be tested; rheostat p. 590; low-reading long scale ammeter (a); voltmeter $V$ to read the normal voltage; low-reading long scale voltmeter (v); and if the motor is shunt or compound wound, an ammeter to measure the normal shunt current $e_{sh}$; switch.

N.B.—If the resistance of the shunt $e_{sh}$ be known, then the last-named ammeter may be dispensed with for $e_{sh} = \frac{V}{r_a}$ amperes.

Observations.—(1) Connect up the above apparatus so that the rheostat and ammeter (a) are in series with the armature alone, and the switch, so that it cuts off the supply entirely from motor and all apparatus, the voltmeter $V$ being across the armature terminals, the shunt coils of the motor being across the mains.

1 The results of the method first appeared in the American Electrical, to which the author is indebted for it.
(2) With the rheostat fully in, close the main switch when the shunt will at once be fully excited. Now gradually cut out resistance in the armature circuit, thereby running up the speed, until $V$ reads the normal voltage across the armature, then running light. Now note the small armature current $(a)$ appears flowing.

(3) Switch off the shunt circuit and block the armature to prevent it moving. With the rheostat fully in, close the main switch and again pass the same current $(a)$, as in observation (2) above, through the armature while stationary, noting the corresponding fall of potential $(e)$ volts across the armature terminals by means of the low-reading voltmeter, and switch off.

(4) Repeat observation (2 and 3) twice or three times and take the mean of the respective values of $(e)$ and $(a)$, calculating the efficiency $\varepsilon$ of the motor on full load from the relation

$$\varepsilon = \frac{100 V}{V(\lambda + \sigma a) \theta}$$

and tabulate your results as follows—

**Inferences.**—State clearly any advantages or disadvantages which you consider the method possesses.
General Considerations Relative to the Testing of Asynchronous Alternating Current Induction Motors.

While it is not proposed to discuss either the construction or the theory of action of such machines, certain considerations relative to the testing of both single and polyphase induction motors may with advantage be noted. In all cases they are self-starting by reason of the rotating magnetic field set up by the supply current, whether single or polyphase, flowing in the windings of the stator or fixed portion of the motor. It is, however, only in single-phase types that after reaching full speed the rotating field (produced only during the starting-up period) is changed by switching to a simple alternating, or reversing, or pulsating field.

The speed attained at the end of the starting period with no pulley load is called "full" or "asynchronous" speed, but in all induction motors the speed of the rotor decreases as the load increases.

If \( \frac{f_1}{p} \) = the periodicity of the supply in cycles per sec.,
\( n_1 \) = the speed of the rotor in revs. per sec.,
\( p \) = the number of pairs of poles in the stator,
then synchronous or full speed is the speed of the rotating field = \( \frac{f_1}{p} \) revs. per sec., while the difference between the speeds of the field and rotor, called the "slip", \( \frac{n_1 - \frac{n_1}{p}}{p} \) revs. per sec., and \( \left( \frac{f_1 - \frac{n_1}{p}}{p} \right) \times 100 = \left( \frac{f_1 - \frac{n_1}{p}}{f_1} \right) 100 \) = the slip in percentage of full speed, which varies from about two or three in large motors to as much as twelve in very small ones. We therefore see that the slip equals the periodicity of the rotor currents.

Measurement of Slip.—The last-named fact is made use of in the following method of measuring slip, but is applicable only in the case of induction motors with slip-ring rotors. Connect preferably a moving-coil permanent magnet D.C. ammeter in one
of the leads between rotor and starter, then since such an instrument indicates for currents in one direction only, the number of impulses given to, or kicks ($K$) of, the pointer per min. in the same direction will directly equal the number of complete cycles per min. of the induced slow period rotor currents—in other words the slip. If ($f$) = periodicity of the supply to the stator, then the percentage slip = \( \frac{K}{60 \times f} \times 100 \).

Thus, if $K = 120$ kicks per min., the slip = \( \frac{120}{60 \times 50} \times 100 \) = 4% with a 50 ~per sec. supply.

If a dead-beat moving soft iron needle A.C. ammeter is used, the number of kicks per min. would be doubled, for the same value of $f$ and slip, and would, even if the ammeter was sufficiently dead-beat to indicate with such rapidity, be impossible to count. With even a very dead-beat moving coil D.C. ammeter, a 5 or 6% slip is about the maximum measurable by this method. A slight variation of the above method consists in counting the oscillations of a light pivoted compass needle placed above or below one of the leads between rotor and starter, the lead having a direction N. and S. so that the needle lies parallel to it when no current is flowing. The slip is then obtainable as before.

The above are direct methods of measuring slip, but if a long-range accurate tachometer is available, the slip can be obtained usually with sufficient accuracy by reading the rotor speed ($n_2$) running light, and ($n_1$) at any load when the slip is given by

\[ \text{slip} = \frac{n_1 - n_2}{n_1} \times 100\% \]

**Determination of Slip by Calculation.**—If an induction motor has a three-phase wound rotor and both the rotor current ($I_2$) and resistance ($R_0$) per phase in each case are known, the slip ($S$) in cycles per sec., or in percentage of the supply frequency ($f_1$), or in revs. per min. or per sec., can be calculated for the corresponding load as follows—

If $f_2$ = frequency of the rotor currents,

$W_2$ = mechanical output from the rotor of the motor (in watts),

$p$ = number of pairs of stator poles,
\[ W_1 = \text{power (in watts) transmitted electro-magnetically by the rotating field in the stator to the rotor,} \]
\[ w = \text{power (in watts) lost in the rotor } \approx 3A_4^2R_4 \]

Then briefly
\[ \frac{7}{5} \frac{W_2}{W_3} = \frac{W_3}{W_2} \]

and hence
\[ \frac{2V_1}{V_2} = W_2 + w \]

for a 3-phase stator supply.

Thus
\[ \frac{V_1}{V_2} = \frac{W_2 + w}{W_3} \]

and by a well-known rule in proportion we therefore have
\[ \frac{V_1}{V_3} = \frac{W_2 + w}{(W_3 + W_2) - w} \]

where \[ W_2 + W_3 = w \]

\[ \therefore \frac{V_1}{V_3} = \frac{W_2 + w}{w} = \frac{3A_4^2R_4}{w} \]

Determination of Frequency, Slip, and Speed (Stroboscopic Method).

Introduction.—Although the measurement of such quantities as those mentioned above—by this method—is by no means common, it probably enjoys the advantages and accuracy of the method to be realised in order to bring it into much more general use.

Measurement of Frequency and Slip.—The phenomenon and principles of stroboscopy can be applied in the measurement of either the frequency of an alternating current supply from an alternator, or the slip of an induction motor, as follows: A black disc having white radial lines is fixed concentrically on the shaft of an A.C. motor run from the supply, and is illuminated by an A.C. arc lamp fed from the same supply. Now the illumination from the lamp will vary periodically and flicker with the supply frequency, and when the speed of the stroboscopic disc corresponds with this supply frequency, i.e. when the angular velocities of the two are equal, the white lines will always be illuminated in the same place and appear to be at rest. If the
speed of the disc is greater than that corresponding to the frequency of supply, the white lines will appear to slowly rotate in the same direction as the disc; whereas if the speed has a smaller value, the lines will appear to rotate in the opposite direction. The last condition will obtain with an induction motor, and if the number of white lines equals the number of pairs of stator poles, they will rotate in the opposite direction to that of the disc with the same number of revolutions per min, as those lost by the motor, i.e. on the slips.

For example: the rotating field in a 2-pole stator on a 60-cycle supply will make one revolution in the periodic time of the current, or will rotate with a speed of 60 × 60 = 3600 revs. per min. If the slip between rotor and field is 5% (= 5 × 30 = 150 revs. per min.), a single white line on the black disc will appear to rotate backwards at a speed of 150 revs. per min., and will also make one complete revolution in the periodic time of the current.

With a 4-pole motor and the same slip and supply frequency, the speed of the rotating field equals 1800 revs. per min., slip equals 75 revs. per min., and each of the two white lines will appear to rotate at 75 revs. per min., which can be counted against time, and the slip thereby at once obtained. Strips, alternately white and black, can be painted on the disc or even the pulley, and used instead of the black disc with radial white lines if so desired.

Measurement of the Resistance of Single and Polyphase Windings.—This is usually effected by the ammeter-ammeter method with direct current (see p. 86) applied to a single phase-winding in the case of a single-phase generator or motor, and to each of the phase-windings separately of 2-phase machines.

Thus, if \( r \) equals resistance of each phase-winding, we see that the total copper loss in a single phase machine equals \( A^2r \), and in a 2-phase machine equals \( A_1^2r_1 + A_2^2r_2 \); or if in the latter case the resistances of the two windings are equal as they should be, and usually are, we have \( r_1 = r_2 = r \), and if \( A_1 = A_2 \) then the total copper loss \( \frac{1}{2}A^2 = A^2 \times 2r \), where \( A \) is the current in either phase, and \((2r)\) the so-called equivalent resistance of the machine.

In 3-phase windings, the resistance between any two terminals
is, with star connection, that of 2 phase-windings in series (as seen from Fig. 143 a), and therefore = 2\(r\); while with mesh connection (Fig. 143 b) we see that between any two terminals there are two circuits in parallel, composed of 1 phase-winding in parallel with the other 2 phase-windings in series, or (1) in parallel with (2\(r\)), i.e., a terminal resistance of

\[
\frac{1}{r} + \frac{1}{2r} = \frac{2}{3r}.
\]

Now, if without troubling to trace the connections in order to see whether they are star or mesh, the resistance between the three pairs of phase-terminals are measured and added together, the sum \(\div 2\) will equal the equivalent resistance of the whole stator or rotor windings, and the total copper loss in the stator or rotor = (line-current)\(^2\) x equivalent resistance.

(97) No-Load “Open Circuit” Test of an Induction Motor on a varying Voltage, constant Normal Frequency Supply. (Rotor running Light at No Load.)

Introduction.—Under these conditions the motor will run at its maximum possible speed, namely that corresponding almost, but not quite, to true synchronism, and therefore with an almost zero slip—the small difference being necessary for overcoming the small losses due to windings, mechanical and magnetic frictions, and copper loss due to the no-load running current. The test can be operated, of course, on single-, two-, or three-phase motors, but we shall assume the use of a three-phase motor here on account of the connections being slightly more complex.

Such a motor may have either a squirrel-cage (short circuited) rotor or a wound rotor with slip rings. If the former, it may be started up from full voltage mains either by a star-delta switch or through an auto-transformer or sectioned choker, depending on its size. If it has a wound rotor, the starting rheostat connected to this is put to “full in” and the stator then switched directly to the supply. The starter is then gradually cut out to short circuit as the speed increases. If
now, with the motor running at full speed, normal voltage, and frequency, the voltage is gradually decreased, the speed will remain practically constant until the lower voltages are reached, when it will fall off rapidly.

Both the stator and rotor current will also decrease gradually, the former owing to a decrease in magnetizing and core-loss current producing the stator flux and depending on the voltage, the latter in an inverse proportion to the strength of the rotating field and voltage.

As the voltage falls the idle or magnetizing component of the current will also decrease, while the energy component overcoming friction will remain much the same in value, hence

![Diagram](image)

Fig. 98.

the ratio of idle to energy current will decrease and the power factor will in consequence rise.

If \( V_s \) = normal voltage per phase on the stator
and \( V_r \) = the corresponding maximum voltage per phase of the rotor indicated when this is turned through a polar pitch with slip rings open-circuited. Then \( \frac{V_s}{V_r} \) is called the ratio of transformation, which is not equal to the ratio of the stator and rotor currents owing to the magnetizing current of the rotor.

The induced E.M.F. in the rotor circuits varies directly as the slip, and has a frequency \( f_s = f_r \) — the slip. Thus the reactance of the rotor circuits \( -2\pi f_s R_0 \) bears a constant ratio to the slip.

As the rotor current is not usually measured, the ratio of transformation enables it, and also the most suitable starting resistance, to be calculated, knowing the stator current at any load.

Apparatus.—Source \( E \) of three-phase alternating current, preferably a motor-driven alternator, the speed and field of
which are each variable between wide limits; three-phase switch \( S_2 \); voltmeters \( V_a \), \( V_b \), \( V_c \); ammeters \( A_a \), \( A_b \), \( A_c \); frequency meter \( f \); wattmeter \( W \); reversing key \( K_1 \) (p. 285); and two-way key \( K_2 \) (p. 287) for line-wire circuit of \( W \); three-phase induction motor, of which \( A \) is the stator and \( R \) the rotor and \( S \) the starter.

**Note.**—By the use of only one wattmeter, with its pressure-coil connected through \( K_2 \) to the remaining two mains in rapid succession, we assume the motor to be electromagnetically balanced, or equally loaded, in the three phases (see p. 389). On no load, and in a case, for the current \( A_1 \) is small, and therefore any inequality in the ampere turns, resistance, or reactance of the windings is nearly negligible in effect compared with what it would be on load. With phase-windings unequally loaded or balanced, two wattmeters must be used to obtain the true power absorbed (see p. 392).

The ammeter \( A_2 \) in the rotor circuit should be very dead beat and of the moving needle type, and should be of low resistance so as not to throw out the balance of the rotor currents. If the rotor \( K \) is of the squirrel-cage type, \( I_a \) \( A_2 \) and \( r \) cannot be used.

**Observations.**—(1) Connect up as in Fig. 88, leveling and adjusting to zero, such instruments as are used. On starting any machine always see that its oiling arrangements are working properly before doing anything else.

(2) With \( R \) running light at no load, adjust the voltage \( V_a \) and frequency \( f \) of the supply \( E \) to the normal values for the motor, and note the readings of all the instruments under this supply condition, and also (with the same frequency kept constant) for a series of values of \( V_a \) (by field regulation) decreasing by about equal amounts to the point where the speed begins to decrease, and from this point by smaller and more gradual decrements of \( V_a \) until the speed decreases too rapidly to read.

**Note.**—After the speed begins to fall, sudden changes in \( V_a \) must be avoided, while the simultaneous readings of all the instruments must be taken rapidly.

If \( V \) must be read, first with its vol coil across \( ab \) and then with it across \( bc \) at each voltage \( V_a \), by using the key \( K_p \). Further, in some cases (not all), this change of connection will be accompanied by a reversal of direction of the deflection of \( V \) depending on the magnitude of the power factor. The re-
versing key K1 must in such cases be turned through 90°, so as to bring the deflection on to the scale again; but the reading must now be considered as and subtracted from the other to give the total watts (see p. 383), otherwise when the readings across ab and be are both on the scale and therefore both positive, their sum gives the total watts.

(c) With the frequency (f) constant at normal value for the motor, raise the voltage $V_a$ from 0 by small and very gradual steps until the speed begins to increase too rapidly to enable readings to be taken. The simultaneous reading of all instruments at each voltage must be done rapidly. Tabulate all your results as follows:

<table>
<thead>
<tr>
<th>Induction Motor No.</th>
<th>Motor ...</th>
<th>Induction motor ...</th>
<th>Rotor ...</th>
<th>Type ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposed end ( ppl.) of Rotor Windings</td>
<td>...</td>
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<tr>
<td>Motor ...</td>
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<td>Rotor ...</td>
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<td>Type ...</td>
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</tbody>
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<thead>
<tr>
<th>Motor ...</th>
<th>Induction motor ...</th>
<th>Rotor ...</th>
<th>Type ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposed end ( pple.) of Rotor Windings</td>
<td>...</td>
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<tr>
<td>Motor ...</td>
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<td>Rotor ...</td>
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</tr>
<tr>
<td>Type ...</td>
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<td>...</td>
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</tbody>
</table>

(1) Plot the following curves (from obs. 2 and 3) having values of motor volts $V_a$ as abscissa with (1) speed, (2) intake watts $W$, (3) static torque, $A_n$, (4) ex $\phi$, (5) ratio $\frac{V}{f}$, as ordinates in each case.

**Inferences.**—From a careful study of the shapes and dispositions of the curves relative to the axes and of the tabular results state all that can be deduced.

(98) **No-Load "Short-circuit" Test of an Induction Motor on a varying Voltage, constant Normal Frequency Supply. (Rotor kept stationary and short-circuited.)**

**Introduction.**—Under these conditions the slip will be 100% since the speed is zero, while the power absorbed will almost...
wholly consist of copper loss \( \propto \) to the square of the current. The only remaining source of loss is that due to hysteresis and eddy currents in the iron which will be small, owing to the low induction density reached with even the maximum voltage it is possible to use in this test. Further, it will be noticed, from a reference to test No. 137, that the motor approximates to a static transformer with a stator primary and rotor secondary under the conditions for maximum magnetic leakage which the stator windings maintain in the air gap between stator and rotor.

Under stationary conditions the ratio of stator to rotor current is practically a constant and approximates to the ratio of transformation of the motor. Under running conditions the ratio of currents departs from constancy, due to the no-load current taken by the motor at normal voltage, and no longer approximates to the ratio of transformation.

**Apparatus.**—That indicated for the preceding test (p. 206), but without \( K_1, K_2 \), an additional wattmeter now being used with its current coil in main \( a \) or \( c \) (Fig. 98), one end of each of the volt circuits of the two wattmeters, to be denoted by \( w_1 \) and \( w_2 \), being connected to the third main as indicated in Fig. 143a. The reason for now using two wattmeters is that the heavier stator currents to be used in this test, which will depend mostly on the resistances of the windings, will show up any slight want of symmetry, and may (or may not) result in the unequal current loadings of the three phases—a condition necessitating the use of two wattmeters (p. 302). \( A_3 A_3 w_1 \) and \( w_2 \) must also now be capable of taking the heavier currents used in the present test.

**Observations.**—(1) Connect up as in Fig. 98, levelling and adjusting to zero such instruments as need it. See that the lubricating arrangements of the supply set are working properly.

(2) With the rotor short-circuited and prevented from rotating and the supply frequency \( f \) constant at normal value, take the readings of all the instruments at each of a series of supply voltages \( V_3 \), increasing from zero to a value which will produce a stator current \( A_3 \), say, 50% in excess of that of full load, and tabulate as for the last test (p. 207).

Introduction.—The somewhat rapid development of the distribution of electrical energy by single phase alternating currents in recent years has brought with it the introduction of single phase alternating current motors, of which, up to comparatively recently, there has been no practical commercial instance. Now, however, there are several forms, but none of them are able to compete with the direct current motor in the matter of efficiency and powers of starting under load with the amount of electrical power absorbed in doing so. There are two classes of alternating single-phase motors, known as the Synchronous and Asynchronous types. The former cannot start themselves but have to be run up by a separate source of power into synchronism with the periodicity of the supply current; then, on being switched into circuit, they run perfectly synchronously with the generators, i.e., at constant speed, for wide variations of load from 0 to considerably over full load, and are of course separately excited. The latter class are self-excitation and self-starting (on very small loads) by using suitable means, but are non-synchronous, and the difference between the speeds of rotation of the magnetic field and the rotating armature is called the "Magnetic Slip" or "Slip" simply. This generally only amounts to a small percentage at full load.

The self-starting property is obtained by producing a rotary magnetic field at starting, caused by diphasing the current in two separate circuits by means of the suitable use of either self-induction or capacity, one circuit being cut out when the motor gets up speed.

The fixed portion of the motor (i.e., field magnets) through the
winding of which the supply current flows is usually termed the "Stator." The rotating portion (i.e., the armature) is termed the "Rotor," and usually consists of short-circuited conductors carried on a well-baded drum. There is no electric connection in most cases to the rotor, or between rotor and stator. It will also often be found that the best efficiency is not at normal full load, which is analogous to the series wound direct current motor in this respect. Speaking broadly, it may be said that single-phase motors should be self-starting, and this on a current certainly not exceeding that taken at full load. The power factor should be high.

A motor built for a given periodicity will not give as a rule its full power when supplied with a current of a much higher periodicity, while it will take too much current with a lower periodicity.

The efficiency of any motor = the total power given out + the total "mean power" absorbed, both being reckoned in equivalent units. In the present and similar tests the true mean input cannot be obtained by the product of the amperes and volts, as in the case of direct current motors, owing to the "phase difference" between the current and pressure, but must be obtained by means of a non-inductive Wattmeter. The output, or B.H.P., is obtained by an absorption dynamometer, which is a modified form of I'Hony brake. Such brakes waste, in heat, all the power developed by the motor from friction, but at the same time give a measure of this power. No lubrication is usually needed, but a little black lead may be applied to the pulley if the brake is sticky.

Apparatus.—Alternating current motor M to be tested; brake complete with weights; non-inductive Wattmeter W; alternating current ammeter A and voltmeter V; switch S; rheostat R (p. 677); tachometer; source of alternating current E; preferably one that can be varied.

Test.—(1) Connect up as indicated, and adjust the pointers of all the instruments to zero, loading such as need it. See that all instruments in use read slowly, and that the resistance switch (S) is open.

(2) Adjust the speed and excitation of the alternator so as to give the normal voltage and frequency required for M, and remove the brake.

(3) Make E a maximum; let a. In the present case put resistance
Switch $S$ to start, and when the motor has got up speed throw $S$
over from start to full.

Then, when the speed has become steady, note it and the readings of $A$, $V$, and $W$.

(4) Place the brake in position, and with no weight in the pan, again note the motor speed and readings of $A$, $V$, and $W$.

(5) Repeat 4 for about ten loads, rising by about equal increments of weight in the maximum, the voltage and frequency being kept constant.

(6) Repeat 4 and 5 for a higher and lower frequency than the normal.

(7) Determine the power required to just start $M$ by removing the brake, turning $S$ to start, and adjusting the speed of the alternator to give normal frequency (to be kept constant).

(8) Carefully and gradually raise the voltage (at constant speed) by means of the excitation until $M$ just starts, then instantly note the readings of $A$, $V$, and $W$. Repeat this three or four times and take the mean.

(9) Repeat 7 and 8 for about five different frequencies, rising by about $= 4$ increments to about $20\%$ above normal.

(10) Determine the relation between the speed of $M$ and frequency of supply by removing the brake, and when the motor has got up speed, turning the switch (3) to "full." Then for constant normal voltage note the speed of $M$ and readings of $A$, $V$, and $W$ for about ten different frequencies, rising up to about $20\%$ above normal.

(11) Determine the effect of variation of voltage at constant normal periodicity with the motor running light by altering $R$, and noting the speed and readings of all the instruments. Tabulate all your results as shown.
**ELECTRICAL ENGINEERING TESTING**

**Data**

Motor: No. ... Type: ... Make: ... Mfr's Stamp: ... Nozzle Mass of Cylinder and Details = P.
Nominal B.H.P. = ..., volts and ... amps, per minute, and frequency = ..., per sec.
Alterations per revolutions of Dynamo X = ...

<table>
<thead>
<tr>
<th>Alternating</th>
<th>Current</th>
<th>Frequency</th>
<th>Power Absorbed</th>
<th>Voltage</th>
<th>Frequency</th>
<th>Power Factor</th>
<th>Power Factor</th>
<th>Power Factor</th>
<th>Power Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>24 A</td>
<td>50 Hz</td>
<td>3.5 HP</td>
<td>77 V</td>
<td>50 Hz</td>
<td>0.85</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note.** The nett weight on brake = (weight of scale pan and weights) + (reading of spring balance).

(12) In experiments 1-6 plot curves having values of (a) power factor, (b) efficiency, (c) true power absorbed respectively as ordinates, and B.H.P. developed as abscissa, for each frequency.

(13) In experiments 7-10 plot curves having (d) true power required to start, (e) speed of motor, as ordinates and frequency as abscissa, in each case.

**Inference.** What can you deduce from your experimental results? Taking the cost of electrical energy for power purposes at 2d. per B.T.U., find the cost per B.H.P. per hour, and also when \( M \) is running on no load at normal voltage and frequency.

(100) **Efficiency-Load Test of a Polyphase Induction Motor.** (Absorption - Brake Method.)

**Introduction.** The efficiency of any electric motor

\[
\text{B.H.P. developed} = \frac{\text{B.H.P.}}{\text{B.H.P. absorbed}} = \text{B.H.P.} - \text{internal losses}
\]

The internal losses in an induction motor comprise (1) copper losses in stator and rotor windings, (2) iron losses (hysteresis and eddy currents) in stator and rotor cores, (3) mechanical friction due to windage, journals, and bushings, if it possesses a slip-ring rotor.

If \( I_a \) and \( r_a \) = the current and resistance respectively, per phase of stator winding and \( I_a \) and \( r_a \) = the current and
resistance, respectively, per phase of rotor winding; then, if this latter is of a three-phase type, the stator copper losses are—

\[ A_{a}^{2} \] for single-phase; \( 2A_{a}^{2} \) for two-phase; and \( 3A_{a}^{2} \) for three-phase induction motors, while the rotor copper loss is \( 3A_{r}^{2} \).

For a given iron core, we have seen (p. 351) that the expression for the iron losses contains two variables only, namely, the frequency and the induction density \( \Phi \) to flux, and dependent solely on the supply voltage and number of stator turns. Hence the iron loss is independent of load. Further, since the frequency of the rotor currents and consequently of the flux in the rotor core equals the slip, which is only some 5% of the speed of the stator field, it follows that the iron loss in the rotor core will be small compared with that in the stator core and the other losses, and will increase slightly with speed. The friction losses being \( \alpha \) to speed, will be sensibly constant at all loads in an induction motor, since the speed of such a motor has a variation of some 5% only. It will therefore be at once realized that the copper losses (increasing as the square of the current) are mostly responsible for the rapid increase of the total internal losses as the load increases.

The supply current to the stator of an induction motor is composed of two components—

(a) One which may be termed the no-load or magnetizing component, producing the rotating magnetic field, and which is not only in quadrature with the supply voltage but nearly constant at all loads.

(b) Owing to the air-gap between stator and rotor cores, the ampere turns of excitation, and hence the magnetizing component necessary to produce a given flux, is much higher, and the power factor much lower, than if the magnetic circuit was a closed one, and therefore an induction motor takes a considerable no-load current which may be from a quarter to one-third of full-load current. The smaller the air-gap the smaller this current, the greater the power factor and output of the motor for a given size. For this reason the air-gap of such motors is reduced to a mere clearance for rotation.

(c) The other, which may be called the load-component, out
of phase with the voltage, but producing a field in the stator
equal and opposite to that produced by the rotor currents in
the stator, and hence balancing the demagnetizing effect of the
rotor-induced currents on the stator field.

This load component increases directly with the B.H.P. output
of the motor and = \( \frac{N_2 \times \text{turns on rotor}}{N_1 \times \text{turns on stator}} \)

Thus it will be seen that for the rotating field to have a
constant strength, the stator current taken at no load will just
suffice to produce this requisite field strength and provide for
the iron and friction losses. As the load increases, the increase
in the rotor ampere turns is balanced by an equal and opposite
increase in stator ampere turns, and we have the following
relation, viz. that the

Total stator amp. turns

\[ = \text{total rotor amp. turns} + \text{no load amp. turns} \]

or Total stator current

\[ = (\text{rotor current} \times \text{ratio of transformation}) + \text{no load current} \]

The line above denoting that the sum is vectorial and not
algebraic.

The total power given out, i.e., the B.H.P., can be measured
either by means of an absorption dynamometer brake in the
manner already clearly defined in the previous tests, or by
making the motor to be tested drive a direct current dynamo,
the commercial efficiency of which is accurately known at various
loads. This method should be adopted whenever possible, as it
has the advantage, when carried out properly, of being more
accurate than the ordinary brake methods. The method consists
in suitably driving the dynamo from the motor to be tested either
by means of a thin supply (pliable) belt or by the direct coupling
of their shafts (placed accurately in alignment), through a flexible
coupling or helical spring sufficiently strong for the purpose,
thus avoiding the difficulty of getting their shafts exactly in true
alignment. The belt arrangement also obviates the same diffi-
culty, but it must be very pliable, otherwise errors will be introdu-
ced due to the extra power absorbed by the slipping and
bending of this belt round the pulleys.

Thus measuring the electrical power developed by the dynamo
which is at once given by the current \( \times \) voltage, and knowing
its commercial efficiency \( n \), the B.H.P. of the motor can easily
be calculated as $\frac{1.11 \times \text{D.C. output}}{740} \times n \approx e$. Further, if $n$ = the speed in revs. per min., the torque $T$ of the motor is given by the relation $T = \frac{\text{B.I.P.} \times 33000}{\text{rpm}}$ lb. ft.

In all cases the efficiency of any motor is total power given out at its pulley + total power absorbed, both being reckoned in equivalent units. A multiphase alternating current motor is self-exciting, self-starting, but synchronous as regards speed and the periodicity of the supply. The starting torque can be made equal to that of the best direct current motor without an excessive percentage over load in the current taken.

They can be wound to run direct on 8000 volt circuits and

over without much fear of the insulation breaking down, and their great advantage, except in the larger sizes, lies in the fact that there are no rubbing contacts of any kind to get out of order, and consequently there is no sparking. In the present case we will assume that the motor to be tested is of the three-phase type, as perhaps the measurements of input are not so obvious as in the two-phase system.

Apparatus.—Source of three-phase alternating current $E$; three-phase motor $SR$ to be tested either coupled mechanically to a direct current dynamo $D$ of known commercial efficiency, or fitted with a Pony dynamo $P$, Wattmeter $W$; alternating current ammeters $A_g A_p$ and voltmeters $V_a V_b$; triple pole switch $S_1 S_2 S_3$; tachometer; and if a coupled dynamo load is used, direct current ammeter $A$ and voltmeter $V$; rheostat $R$ (p. 606); switch $S$.

Note.—For a detailed description of power measurements in multiphase circuits, see pp. 588-400.

Observations.—(1) Connect up as in Fig. 100, and adjust all the instruments to zero, levelling such as require it. See that
all lubricating arrangements in use feed properly on starting the motor in the usual way.

(2) With the motor quite free, take readings on all the instruments concerned when M thus runs "light" at its normal frequency and voltage, noting the speed.

(3) If a dynamo load is used stop DA, couple the shafts of D and SA together and start the combination up again, with A at its maximum when B is closed; or if a Parny brake is used take a series of about ten different loads from D or on the brake, varying from the smallest to the largest permissible corresponding to the maximum current allowed for SA. Note simultaneously the readings of all the instruments at each load and also the speed, the supply voltage and frequency being constant throughout.

(4) Repeat 3 for speeds 20% above and 20% below normal respectively, if possible, by varying the speed of the generator.

(5) Adjust the direct current load to a convenient amount, then, keeping $V_A$ constant, alter the speed of the three-phase generator by successive steps, and note the corresponding effect on $W_A$, $W_B$, and the speed of the motor.

(6) Keeping the speed of the three-phase generator constant, alter $V_A$ by successive steps and note the effect on $W_A$, $W_B$ and the speed of the motor, using the same load. Tabulate all your results as follows—

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make</td>
<td>Notes</td>
</tr>
<tr>
<td>Model Motor</td>
<td>No.</td>
</tr>
<tr>
<td>Normal Value</td>
<td>A.A.A.</td>
</tr>
<tr>
<td>Rev. Watts per Phase Winding</td>
<td>Motor(s)</td>
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<tr>
<td>Diameter D. Bele</td>
<td>No.</td>
</tr>
<tr>
<td>Normal Value</td>
<td>A.A.A.</td>
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<tr>
<td>Diameter of Brake Pulley</td>
<td>f. f.</td>
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<table>
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<tr>
<th>Speed</th>
<th>Per min.</th>
<th>Volts</th>
<th>Amperes</th>
<th>Horse Power</th>
<th>Efficiency</th>
<th>Horse Power</th>
<th>Efficiency</th>
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<tbody>
<tr>
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<td>1600</td>
<td>1800</td>
<td>2000</td>
<td>2200</td>
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<td>1000</td>
<td>1250</td>
<td>1500</td>
<td>1750</td>
<td>2000</td>
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<tr>
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<td>750</td>
<td>875</td>
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<td>11.71875</td>
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<td>0.724531</td>
<td>0.86809523</td>
<td>0.96</td>
<td>1.0397</td>
</tr>
</tbody>
</table>

For Use with Dynamic Load.

For Use with a Parny Brake.
(7) Plot the following curves from observations 3 and 4 for each speed: efficiency, power factor, slip, speed, current, and intake Watts as ordinates and B.H.P. as abscissas, also curves having Torque as abscissas with \( A_x \) \( A_x \) and slip as ordinates.

And from observations 5 and 6, curves between voltage \( V_x \) and supply frequency as ordinates with speed of the motor as abscissas in each case.

Inferences.—State very clearly all the inferences which can be drawn from your experimental results, and point out their bearing on electrical driving by multiphase current motors.

(101) Determination of the performance of an Induction Motor at all loads without loading it at all. (Heyland's Method.)

Introduction.—It sometimes happens to be inconvenient or even impossible to brake, or otherwise absorb the B.H.P. of large induction motors, and in other cases to supply the large amount of electrical power required by them at full load from a generating plant which may be already running nearly at full load. The difficulty is met fortunately in such cases by the following method entailing the construction of the well-known "Heyland Diagram" from very simple "no load and short circuit" readings on the motor. The intake current and power, the output, the power factor, and the slip, etc., and hence efficiency, can then be deduced for all B.H.P.'s and a complete set of curves drawn showing the performance of the motor. The temperature rise at any load cannot however be obtained with this method, and the best and most economical way of determining it is to let the motor drive a generator which is capable of absorbing the required B.H.P. and simultaneously return the output of this generator to the motor supply. Thus in running a 6 hours' temperature test on, say, a 500 B.H.P. motor having an efficiency of 95%, the power wasted would only
be some 10% or about 50 H.P. as against over 500 H.P. if the output of the generator had been taken up in rheostats.

Apparatus.—Three-phase induction motor \( M \) with phases equally balanced (presumably) complete with stator. A tachometer; a voltmeter; an ammeter reading up to at least full load intake amperes, and a source of supply SS at the normal voltage and frequency for which the motor is built and capable of reduction to about \( \frac{1}{4} \) full normal voltage at full normal frequency, as before, together with—

For motors with mesh-connected stator—2 similar wattmeters having a capacity of about \( \frac{1}{2} \) of the full-load output of the motor.

For motors with well-balanced star-connected stator—1 wattmeter having a capacity of about \( \frac{1}{3} \) of the full-load output of the motor.

The ohmic resistance of a complete stator phase (and of a rotor phase for reference later, if needed) will be required, and can be obtained from a separate measurement with a high-reading ammeter and low-reading voltmeter, by the full of potential method (vide p. 84).

Observations.—(1) Connect up as in Fig. 101 if the motor stator is mesh connected or if the stator is star connected but not well balanced, and adjust the pointers of these instruments which require it, to zero. The terminals of the stator of the motor \( M \) are 1, 2 and 3, whether it is star or mesh connected.

(2) Close the switch \( S_1, S_2, S_5 \) and start up \( M \), finally short-
circuiting the starter. Then with the supply at normal voltage and frequency note the speed of \( N \) on the tachometer and the readings of all the instruments, the motor running quite light and at "no load."

N.B.—If the power factor of the system is low, one of the wattmeters will read negatively. Reverse the connections to its shunt coil and take the reading which must be considered — and subtracted from the reading of the other wattmeter to get the total true power.

(3) Open \( S_y \), \( S_z \), \( S_b \) and when the motor comes to rest, clamp the shaft in any convenient way to prevent it rotating, and place the starter at short-circuit. Now apply any convenient lower voltage, say, \( \frac{1}{3} \) to \( \frac{1}{2} \) of the normal value at full normal frequency, and again read all the instruments as in Test 2 above and switch off and open the starter.

Note.—A lower voltage has to be used in this test for the larger sized motors, because the normal voltage would cause dangerously large currents to flow which would probably damage the stator winding in even their brief application. The true static current will now be the observed current \( I \) by the ratio of the two voltages, while the corresponding static watts will be those observed \( x \) by the square of this ratio (see table). The reason for this is that the static watts are nearly all copper loss and hence \( x \) to (current)².

(4) Measure in a convenient manner (pp. 84 and 203) the resistance \( R_s \) between any two terminals of the stator and rotor, preferably while warm. Then the resistance of each complete stator phase (star connections) \( r_s = \frac{R_s}{3} \), and for (mesh connections) \( r_s = \frac{2}{3} R_s \). Also in the case of a slip ring rotor obtain the ratio of transformation given by the ratio of any stator voltage to the corresponding rotor voltage with rings open-circuited and rotor in the position giving max. voltage.

Record your results as follows—
Motor: No.
Rated Full Load: H.P. = . . . Line Volts (V) = . . . Line Amps = . . . Speed = . . .
Rmp at = . . . Normal frequency = . . .
Blade Phases connected: . . .
Resistance—complete Blade Phase, \( r_x = . . . \) Rotor Phase, \( r_y = . . . \)

<table>
<thead>
<tr>
<th>Blade Phase</th>
<th>Volts Across Blade Phase at</th>
<th>Line A.C.</th>
<th>Each Blade Phase A.C.</th>
<th>Total Blade Volts</th>
<th>Watts per Phase</th>
<th>Power Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Blade of Motor supplied with line voltage than normal, but at normal speed.</th>
<th>Watts per Phase</th>
<th>Power Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ap. Plated Watts per Phase</td>
<td>Watts per Phase at Voltage ( V )</td>
<td>Watts per Phase at Voltage ( V ) = ( V )</td>
</tr>
<tr>
<td></td>
<td>( P )</td>
<td>( P )</td>
</tr>
</tbody>
</table>

Note.—With star-connected stators: Volts per phase = line volts and amps. per phase = \( \sqrt{3} \) line amps.; with star-connected stators: Volts per phase = \( \sqrt{3} \) line volts and amps. per phase = line amps.

It will be seen that the Power Factor for the "no load" and for the "short-circuit" tests is found from the relation:

\[
\text{Cos.} \, \phi = \frac{\text{True Watts Absorbed per phase}}{\text{Amps. per phase} \times \text{Volts per phase}}
\]

but it is perhaps more convenient to calculate from the experimental readings by means of the fraction given in the above table.

From the data contained in the above table, the Hoyer Diagram can be constructed, giving the performance of the motor.
Construction of Heyland Diagram.—This will be understood more easily by working it out from tests recently made by the author on a 360 H.P., 500 volt three-phase induction motor, running at a speed of 300 r.p.m. with a normal frequency = 60 ~ per sec. The stator windings were star connected, the rotor windings being also star connected, and led out to three slip rings which were connected to a starting resistance. The method of procedure with this motor was as follows—

With an ammeter in one line and a wattmeter connected between the neutral point and one terminal of the motor so as to measure the true watts absorbed by one phase (see Fig. 107, p. 386), the following measurements were made—

Motor running light at normal speed, frequency, and voltage with rotor short circuited.—Wattmeter reading = 3600 watts per phase = \(\frac{n}{3}\)

Line current \(I_1\) = 142 amperes.

Resistance of each phase of stator (coil): \(r_s = 0,0122 \text{ ohm}\), or by calculation about 0.013 ohm, (not) on the assumption of a maximum temperature rise of 70° F., above that of the air.

Resistance of each phase of rotor (coils): \(r_r = 0.0051 \text{ ohm}\).

Ratio of transformation in rotor 500 : 325.

Bearing in mind always that the diagram is constructed with reference to one phase of the motor, and not the motor as a whole. Further that in testing motors with mesh-connected stators it would be the total power absorbed that would be measured by the two line wattmeter method (Fig. 101, p. 278) instead of that per phase.

Hence the power factor of any phase, whether in star or mesh-connected stators, can be calculated best from the general relation

\[
\cos \theta = \sqrt{\frac{3I_1P}{P}}
\]

where \(P\) = total power in watts absorbed by motor running light with a line current \(I_1\) and line voltage \(V\). The numeral 3 reduces \(P\) to watts per phase, and \(\sqrt{3}\) reduces the line voltage \(V\) or line amperes \(I_1\) to the corresponding quantities per phase in the case of star- or mesh-connected stators respectively. Thus in the present case
ELECTRICAL ENGINEERING TESTING

\[
\cos \theta_1 = \frac{\sqrt{3} E_y}{3A_y V} = \frac{\sqrt{3}}{3} \times \frac{10000}{127 \times 500} = 0.9878
\]

or \( \theta_1 = 89^\circ \).

Motor at standstill with rotor short circuited and stator supplied at normal frequency.—With these conditions we require the line current that would flow, and also the true watts absorbed, at normal line voltage. As, however, a line voltage as great as the normal value would, in most motors, produce an abnormal current that would be inconvenient to measure and liable to damage the windings, a smaller voltage sufficient to give a convenient line current is applied. Thus in the present case the line current \( A' = 104 \) amps,

\[
P_p = 127 \text{ volts.}
\]

Wattmeter reading \( \left( \frac{V_p^2}{3} \right) \) = 8000 watts per phase.

From which we calculate the following static standstill values—

Line current

\[
A_w = \frac{\text{normal volts} \times 401}{\text{applied volts}} = \frac{500}{127} \times 401 = 1591 \text{ amps.}
\]

Total watts absorbed

\[
e_1 = \left( \frac{500}{127} \right)^2 \times (3 \times 6000) = 372,000 \text{ watts,}
\]

whence \( \cos \theta_1 = \frac{\sqrt{3} E_y}{3A_y V} = \frac{\sqrt{3}}{3} \times \frac{372000}{127 \times 500} = 0.9871 \)

or \( \theta_1 = 89.3^\circ \).

Current Circle.—Referring to Fig. 102, draw two lines \( OB \) giving the phase of the supply volts and \( OA \) perpendicular to one another, and from the point \( O \), as origin, set off a straight line \( OC \) (= current which would be taken by the stator if directly across normal voltage), making an angle \( \theta_1 \) of 74.3° with \( OB \) and another line \( OD \) (= no-load current) making an angle \( \theta_1 \) of 89° with \( OB \). Now choose a convenient linear scale of current, e.g. in the present case, 100 amps = 1 cm., and thus make

\[
OC = 1591 \text{ amps.} = \frac{1591}{100} = 15.91 \text{ cm. long.}
\]
and make $OD = 142'' = \frac{142}{100} = 1.42$ cm. long.

Then with a centre $L$ (on $OD$) draw a semi-circle $ACZD$ through the points $C$ and $D$, and cutting $OD$ in $A$ and $F$, thus determining the point $A$. From $D$ draw $DF \perp EF$, the energy component of no-load current perpendicular to $OA$ and cutting it in $F$, then $BFO$ is the no-load triangle of currents $OF = \phi_0$, the no-load magnetizing current; $OF = \phi_0 = \phi_0$, the no-load power factor. Noor current $= FFB \times \text{rati of transformation stator intake} = BV$ and $AXF$ is the current circle for the motor.

Output Circle.—Join $CA$, and from $A$ draw $AH$ perpendicular to $AC$, cutting the perpendicular $HX$, to $OA$ through $I$, in the point $H$. With centre $H$ draw the output semi-circle $AXF$ through the points $A$ and $F$. Since the angle $CAH = 90^\circ$, the line $CA$ is a tangent to the output circle, the ordinate of which at $A$ is zero, thus satisfying the condition of no output corresponding to this point and the point $O_1$, output of motor $= 5^\circ$ which has a max. value $= XJ$.

Torque circle.—The torque of an induction motor is proportional to the rotor flux $R_F \times$ the rotor current $I_F$ and in fig. 102 the right-angled triangle $ACO$ is a triangle of fluxes in which $AC \propto R_F; AO \propto$ stator flux $S_0$ which is also $\pi$ applied voltage per phase and $OC \propto$ leakage flux $F_p$. Consequently the actual rotor flux will be $\propto AC$—copper drop in stator resistance, and we proceed by first finding the voltage represented by $AC$ when the volts per phase of stator are represented by $AO$. Thus—

$$\text{volts per stator phase} = \text{length of } AO = 16.7 \text{ cm.,}$$

$$\text{volts represented by } AC = \frac{\text{length of } AC}{\text{length of } AO} = \frac{4.55}{16.7} = 0.27 \text{ volts.}$$

whence—volts represented by

$$AC = \frac{AC}{AO} \times \text{volts per phase} = \frac{4.55(500)}{16.7(\sqrt{3})} = 78.65 \text{ volts.}$$

Now the copper drop for stator phase (for the short circuits current $J_s = 1591 \text{ amperes.)}$

$$= A_{s}F = 1591 \times 0.013 = 20.68 \text{ volts.}$$

Therefore from the point $C$ mark off along $CA$ the length

$$CN = 20.68 \text{ volts, which is proportional to}$$
Now find a centre \( K \) in \( RH \), such that a semi-circle \( ANER \)
described from it passes through the points \( ANF \). This is the
required torque circle. Since the rotor flux is represented by
\( AN \), the total torque is \( \alpha \cdot AN \times CF \) which is \( \alpha \) in the area of
the right-angled \( \triangle ANF \) (the line \( HF \) not being shown in Fig. 103).
But the base \( AB \) is constant. Hence the total torque is \( \alpha \) to
the altitude of the triangle. For example the total torque is
\( \approx RW \) for the stator current \( OB \). Again, in the \( \triangle \) load triangle
of currents \( ODF \), the no load wattless magnetising current \( = OP \)
lagging 90° in phase behind the R.M.F. \( OE \), and having a load
current component in phase with \( OE = PD \). The resultant
no load stator current as read off on the ammeter = \( OD \). Hence,
since useful torque = total torque - torque spent in overcoming
internal frictions for the same stator current \( OB \), we have
useful torque \( \approx RW - UW \approx R \). Starting torque would be
\( NM \) at the starting current \( OF \) and a power factor \( \cos \theta \).

Slip of the Motor.—Since we know that the slip is directly
\( \alpha \) to the rotor current \( NA \), and inversely \( \alpha \) to the rotor flux
\( AN \), we see that it will have its maximum value at \( C \) and
minimum value at \( B \) for \( \frac{NP}{NA} \) (which is \( \alpha \) to slip) has its
maximum value unity at \( C \) when the motor is at standstill.

From \( C \) therefore draw \( CP \) perpendicular to \( AK \) cutting \( AD \)
in \( P \). Then the slip \( \alpha \approx \frac{NP}{NA} \approx \frac{QP}{AP} \), since \( AP \) is constant
and the triangles \( ANP \) and \( APQ \) are similar.

Now maximum slip corresponding to the point \( C \) and the
motor at standstill is \( \alpha \approx CP \), which scales 455 cm, and
represents 100 % slip.

The slip at the output lead \( ST \) when taking a stator current
\( OB \) is length \( PQ \times 100 \% = 0.3 \times 100 = 0.9 \% \).

Stator copper loss = \( (BV - R) \) watts and rotor copper loss = \( (RU - ST) \) watts each to a scale of watts suitable for
input \( BV \) and output \( ST, RU \) and \( RS \) = ohm drop of volts in
stator and rotor respectively to same scale as \( OA \) gives stator
volts per phase.
Application of Diagram.—The diagram can now be employed for determining the performance of the motor at any load corresponding to any point such as $B$ taken anywhere on the current curve $AZF$. Suppose that we choose the point $B$ as being the point of contact of the tangent $OB$ with the current circle $AZF$. Join $AF$, cutting $CP$ in $Q$, the circle $AXF$ in $S$, and $AYF$ in $R$, and draw the perpendiculars $ST$, $RUV$, and $HY$.

Power Factor.—For any given point on $AZF$ the power factor is given by the cosine of the angle between the join of this point with $O$ and the line $OB$. At $B$ it has the maximum value possible, since $OB$ is a tangent to the circle; namely \[ \cos \theta = \cos 32^\circ = 0.819. \]

Stator Current per phase = $OB$, which is also the line current in the present case since the stator is star connected.

This scales 4.95 cm. corresponding to 405 amperes (since 100 amps. = 1 cm.).

Total apparent Watts absorbed = $\sqrt{3} \cdot V \cdot \text{line amperes} = \sqrt{3} \times 600 \times 405 = 428,500$ watts.

Total True Watts absorbed = $\sqrt{3} \cdot V \cdot (\text{component of } OB \text{ in phase with and parallel to } OE). = \sqrt{3} \cdot V \cdot BV \text{ with star connection} = \sqrt{3} \times 500 \times 416 = 380,000$ watts.

The same result is given by $3(R1') \times \text{volts per phase}.$

Stator copper loss for this load = $3 \times \text{component } F2 \text{ of stator current} = 3 \times 0.013 \times 435^2 = 7381$ watts.

This loss is proportional to $BR$ which = 0.43 cm., and the Rotor copper loss is $\propto RS$ which = 1.0 cm., and this loss therefore = $\frac{1}{0.43} \times 7381 = 17,195$ watts.

The total loss in the motor at the load corresponding to the intake stator current $OB = 10,800 + 7381 + 17,195 = 35,386$ watts.

The output of the motor therefore = $350,200 - 35,386 = 324,814$ watts = 436 B.H.P.

The efficiency of the motor therefore = $\frac{324,814}{350,200} = 92.9\%$

when giving 436 B.H.P. or 21% overload with a power factor already found of 84%.
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The ordinate ST thus represents 436 R.H.P. or the scale of the ordinates of the output circle is \( \frac{436}{4} = 110.5 \) R.H.P. per 1 cm., consequently the rated full load of the motor, namely 360 R.H.P., will be given by an ordinate such as SE, but only 301.4 cm. long.

The efficiency, power factor, and slip, etc., can now be worked out for this full load point which gives another point, such as F on AB nearer to F and a line such as AB, at a smaller angle to OA. Hence the performance at any load can be determined.

Further, if \( T \) = the torque in pound feet, and \( \omega = \) the angular velocity of the rotor = \( 2\pi n \),

where \( n = \) the revs. per min. of the rotor at a given R.H.P.,

then \( \text{K.W.} = \frac{\omega T}{2\pi} = \frac{33000}{33000} = \frac{33000}{33000} \) or \( T = \frac{33000 \text{ B.H.P.}}{2\pi} \).

Now since at F the B.H.P. = 436 and the slip = 5-9% ; \( n = \frac{300 \times 33}{100} = 33 \) r.p.m. \( \therefore \quad T = \frac{33000 \times 436}{2\pi \times 33} = 8210 \) lb. ft.

Hence the scale of the ordinates, such as UW, of the torque circle is known from all other ordinates from the above, and

\[ \frac{8210}{340} = 8210 = \frac{8210}{340} = 2400 \text{ lb. ft. per 1 cm.} \]

The maximum torque which the motor can exert before pulling up is represented by JY, and the maximum B.H.P. by JX.

The starting torque corresponding to the current CO is NH.

The Heyland diagram becomes more accurate the smaller the no load losses as compared with the copper losses, i.e. the larger the motor tested. The performance of the motor is slightly better than given by the diagram, while for motors smaller than for 4 or 5 H.P., the line AB should be drawn downwards from D at an arbitrary angle of about 25° to OA for greater accuracy, in correcting the small error (slightly affecting the accuracy of the diagram) due to the greater proportion of no-load to load losses in small motors.

---

1 For higher accuracy see Theory of Induction Motors by Diagram, G. Georgius. Zeitschr. Electrotechn. Wien, 17, pp. 221-242 (1889), and Circle Diagram, by J. L. In Court (same journal), 21, pp. 613-645 (Nov. 1908).
Note.—If $OG$ be drawn perpendicular to $BF$ produced, then $AB$ and $OG$ will always be parallel at all loads and perpendicular to $BG$.

The sides $OB$, $BG$, and $OG$ of the triangle $OBC$, represent the stator current, rotor current, and magnetising current respectively. Now, in any electromagnetic circuit the total flux $\Phi$ is the sum of the useful flux $\Phi_u$ and the leakage flux $\Phi_l$, while the ratio $\Phi_u/\Phi_l$ is called the leakage factor $\nu$, which is always greater than unity, and the ratio $\Phi_u/\Phi$ is called the coefficient of magnetic dispersion $\sigma$ which should be always much less than unity.

In the Heyland diagram, Fig. 102, the leakage factor

$$\nu = \frac{OA}{FA} = \frac{OF + FA}{FA} = \frac{OF}{FA} + 1$$

and the dispersion coefficient

$$\sigma = \frac{OF}{OA} = \frac{OF}{FA} - 1 = \frac{1}{\nu} - 1.$$

(102) Complete Test for Efficiency, Slip, Power Factor, and Temperature Rise, of Three-Phase Induction Motors.

(Sumpner and Weekes Method.)

Introduction.—This method, due to Dr. W. R. Sumpner and R. W. Weekes, is an application of the principle of the ordinary Hopkins test of a pair of d.c. dynamos (p. 230) to the test of a pair of three-phase induction motors. The arrangement, which entirely avoids the use of a brake, is extremely convenient for obtaining the temperature rise due to a long run at any particular load, and is shown in Fig. 103. $M$ and $G$ are the two machines to be tested, of which $M$ is made the motor and $G$ the generator. Their motor terminals $T$ are connected to the main supply $SS$, which is at the normal voltage and frequency required by $G$ and $M$. A belt $B$ drives the rotor pulleys, which must be of different diameters in order to enable the generator $G$ to run at a higher speed, and the motor $M$ at a lower speed than that of synchronism.

1 Communicated by the authors to "Section G" of the British Association, August 22, 1904, at Cambridge.
If \( D_M \) and \( D_N \) are the diameters of these pulleys; then assuming the rotors are to remain short-circuited during the test, the ratio \( \frac{D_M}{D_N} \) must be such as to cause the right slip for the load required. For example,—if the slip of each machine \( M \) and \( G \) is 3\% at full load, and if the probable efficiency of each is 85\%, then 35\% of the load is lost in each, and the overload of the motor \( M = 30\% \) roughly, corresponding with \( 25 + \frac{1}{2} \times 2.5 \) or 3.25\% roughly. The other machine \( G \) working as a generator develops full load and has a negative slip of 2.5\%. \( D_M \) and \( D_N \) must therefore differ by \( 25 + 3.25 \) or 0.75\%, assuming no mechanical slip of the belt \( B \) on the pulleys. If, however, 1.25\% be allowed for this, the pulleys must differ by \( 0.75 + 1.25 \) or 7\% in diameter, and the machines under test will, when switched into circuit, take a perfectly definite load which can be maintained for any length of time.

An interesting feature of the test lies in the fact that \( G \), the machine used as the generator, gives current of about the same power factor as that of the current supplied to the motor \( M \), while the current from the supply mains \( S' \) is the difference of the power components of the machine currents, together with the sum of the inductive components of these currents. Consequently, the power factor of the main current from \( S' \) is very small, and may be only \( \frac{1}{2} \) of that of the machine current, the main current from \( S' \) may be equal to, or greater than, that through the machines, while the actual power taken from \( S' \) may be less than \( \frac{1}{2} \) of that circulating round the machines \( G \) and \( M \).

Alteration of load with two given pulleys can be obtained by alteration of resistance in the rotor circuits between starter and slip rings, or by using a low resistance starter which can stand the full load rotor currents. The CVR losses in these rotor resistances, if appreciable, must be deducted from the power taken from \( S' \) in order to get the total loss in the two machines \( G \) and \( M \) and in the belt drive. The machines can be run at various loads, with resistance variation such as above, if the pulleys are chosen with sufficient difference to obtain the maximum slip required.

The magnitude of the mechanical slip at the pulleys is determined by the ratio of their circumferential speeds, a quantity difficult of determination with any accuracy in ordinary belt drive, but most easily found in the present method. Thus—
let \( S_p \) = frequency of the supply current,
\[ G \]
\( G_r \) = generator rotor current,
\( M_s \) = motor rotor current;
then ratio of the rotor speeds \( R = \frac{S_p + G_r}{S_p - M_s} \) and ratio of the circumferential speeds of the pulleys \( R = \frac{H}{H_s} \).

\( G \) and \( M_s \) can be very accurately determined, and are each small compared with \( S_p \), so that although \( S_p \) cannot be so accurately found, the value of \( R \) is not much affected by small errors in \( S_p \).

The belt losses are easily determined, as shown in the table, and are caused by (a) extra bearing friction and in bending and driving the belt, (b) heating of the pulleys due to belt slip.

Apparatus.—The two similar three-phase induction motors to be tested: suitable pulleys and belt; four alternating current ammeters \( a_1, a_2, A_1, A_2 \); an alternating current voltmeter \( \mathcal{V} \); four wattmeters \( w_1, w_2, W_1, W_2 \); source of three-phase supply \( SS \) of normal voltage and frequency for which the machines under test have been built; three-throw switch \( s_1, s_2, s_3 \).

N.B.—Switches must be used with the motor connections if the rotors are of the "short circuited" type, but are not wanted if the rotors are supplied with slip rings and starting resistors. Only two wattmeters will be needed if one is connected between neutral point and a terminal in the case of each machine, since in this case total power = 3 \times \text{power of one coil}, which is sufficiently accurate for commercial work. Two wattmeters to each machine, as shown in Fig. 103, is, however, the best and most accurate arrangement, and has the additional advantage that the ratio of the readings of a pair of wattmeters gives the power factor independent of the usual method of getting the power factor from true watts + apparent watts. With two wattmeters the reading of one will be -\( \sqrt{3} \) (owing to the low power factor of the supply current.

Observations.—(1) Connect up as shown in Fig. 103, and adjust these instruments to zero which requires it.
(2) With the supply \( SS \) at the normal voltage and frequency needed for \( H \) and \( G \), and the belt \( R \) off, close \( s_1, s_2, s_3 \) and start up \( H \) and \( G \), which must run in the same direction. If they do not stop them, change the connections at \( T \), and start up again. Note the readings of all the instruments, and denote those of \( w_1 \) and \( w_2 \) by \( w_{11} \) and \( w_{22} \) respectively.
(3) Stop $M$ and $G$, place the belt on, and start up again, with only one of the machines in circuit, and acting as a motor, but running the other by belt with its stator excited and rotor open circuited. Note the readings of all the instruments, and denote those of $w_1$, $w_2$, by $w_{m1}$ and $w_{m2}$ respectively.

Thus the belt loss $W_2 = (w_{m1} + w_{m2}) - (w_{m1} + w_{m2})$.

(4) In Tests 2 and 3 above, and in all future load tests, the slip of each machine can most conveniently and accurately be determined by measuring the frequency of the rotor currents by suitably shunting any ordinary low reading d.c. voltmeter to the slip rings of the rotor, and noting the number of periods made by the pointer in, say, 20 seconds, these being slow enough to be easily counted. If $F_p$ = number of periods per second, in, say, the case of the motor, rotor currents, and $s_p$ = frequency of the supply current in periods per second, then the slip of the motor = $s_p \times 100 \%$.

(3) Take readings at different loads, obtained by altering the resistance in the rotor circuit and tabulate as shown on page 293.

The values of the losses given in columns 31 and 32 are accurate enough for commercial purposes if the two machines are of the same make and of about the same rated full-load output and if subjected to the same voltage. A greater degree of accuracy may be obtained, however, by subdividing these
losses. In addition to the diameters of the pulleys indicating which machine is the motor and which the generator, the latter is the one which must always run at the higher speed. If the belt is removed and the machines run light, the wattmeters \( W_1 \) and \( W_2 \) will read negatively if \( G \) is the generator.

Column 44 is obtained from the curve Fig. 101, which is drawn in the following manner.

The ratio of the two readings of wattmeters \( W_1 \) and \( W_2 \) connected as shown in Fig. 103, varies from +1 to −1. Hence the power factor \( \cos \phi \) can be calculated from the readings of \( W_1 \) and \( W_2 \) by substituting different values of \( \phi \) (the angle of lag) between current and voltage in the formula shown.

\[
\frac{W_1}{W_2} = \frac{\text{current}}{\text{voltage}}
\]

The value of \( \frac{W_1}{W_2} \) so found plotted against the value of \( \cos \phi \) gives the curve, Fig. 104. As an example of the application of this curve, let \( W_1 = 6500 \) watts, \( W_2 = 15000 \), then \( \frac{W_1}{W_2} = 0.43 \) and \( \cos \phi = 0.77 \). Again, let \( W_1 = (−) 2000 \) watts, \( W_2 = 4000 \), then \( \frac{W_1}{W_2} = (−) 0.50 \), and \( \cos \phi = 0.18 \).
<table>
<thead>
<tr>
<th>Time of Reading</th>
<th>Time in Hours from Start</th>
<th>Volts</th>
<th>Amps.</th>
<th>Watermeter Reading</th>
<th>True Watts</th>
<th>Power Taken from Mains</th>
<th>Amps.</th>
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<td>4</td>
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<table>
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<tr>
<th>Amps. d.p.</th>
<th>Watermeter Reading</th>
<th>True Watts</th>
<th>Output of Generator in Watts</th>
<th>No. of Periods of Motor Currents in 30 sec.</th>
<th>Frequency of Motor Currents</th>
<th>Frequency of Variation of B.H. in %</th>
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<thead>
<tr>
<th>Mechanical Slip of Motor</th>
<th>Motor Eff. %</th>
<th>Motor Output in fraction of the Rated Full Load</th>
<th>Generator Eff. %</th>
<th>Power Factor of Motor</th>
<th>E.M.F. of Generator</th>
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<thead>
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<th>Machine No.</th>
<th>Maker</th>
<th>Full Load—Volts</th>
<th>Amps.</th>
<th>Speed</th>
<th>Speed used as Generator</th>
<th>No.</th>
<th>Resistance of each Stator Coil</th>
<th>Motor Coil</th>
<th>Trans. of Motor Polar 8e =</th>
<th>Trans. of Stator Polar 8e =</th>
<th>In Test No. 3 W =</th>
<th>In Test No. 4 W =</th>
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<th>Amps.</th>
<th>Speed</th>
<th>Speed used as Motor</th>
<th>No.</th>
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<th>Motor Coil</th>
<th>Trans. of Motor Polar 8e =</th>
<th>Trans. of Stator Polar 8e =</th>
<th>In Test No. 3 W =</th>
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(103) Relation between Efficiency, Slip, Torque, Load, etc., in an Induction Motor with Variable Rotor Circuit Resistance.

Introduction.—The present test is obviously just an extension of, and similar in almost every way to, test No. 100, which is therefore repeated here but with different amounts of the starting resistance (r) in circuit instead of it being all cut out to short-circuit as in that test. Consequently there will be one set of curves, such as was obtained in test No. 100, for each different value of rotor circuit resistance used in the present investigation.

Further, if, say, five different rotor circuit resistances were used, giving five complete sets of curves as in test No. 100, then any of the variables plotted, say, against load, for the different constant rotor resistances can be transferred and replotted against rotor resistance, e.g., there would be five efficiency-load curves, then if a straight line was drawn through, say, full-load point, parallel to the axis of efficiency, and cutting the five efficiency curves; the five different efficiencies obtained by the five intersection points can be plotted against the five values of rotor resistance of the efficiency-load curves to give a curve of five points between efficiency and rotor resistance only.

If \( E_2 \) = E.M.F. induced in each stator circuit due to the rotating field,

\[ N_a N_b = \text{number of turns per phase in each stator and rotor winding respectively,} \]

\[ \omega_a \omega_b = \text{angular velocities of rotating stator field and rotor respectively,} \]

\[ r_a = \text{resistance of each rotor circuit,} \]

\[ L_a = \text{self induction of each rotor circuit,} \]

\[ m = \text{number of pairs of poles in the rotating stator field,} \]

\[ f = \text{frequency of the stator supply voltage and currents,} \]

\[ K = \text{the slip.} \]

Then we have \( K = \frac{\omega_2 - \omega_1}{\omega_1} \) and the frequency of the induced E.M.F. and currents in the rotor circuits will be \( \frac{\omega_2 - \omega_1}{\omega_1} \times f = \)
The running torque \( T \) is given by the relation

\[
T = \frac{N_s^2 B_r^2 r_b K}{N_s^2 L_s (K^2 (2\pi f L_s)^2 + r_b^2)} = \frac{N_s^2 B_r^2 r_b K}{N_s^2 L_s (K^2 (2\pi f L_s)^2 + r_b^2)}
\]

where

- \( f \) is speed of the stator rotating field in revs. per sec.,
- \( 2\pi f/\rho = \omega_1 \) is its angular velocity,
- \( K^2 r_b^2 L_s \) is resistance per rotor circuit when in motion and which is \( \propto \) to slip,
- \( K^2 (3\pi f L_s)^2 + r_b^2 \) is (impedance)\(^2\) per rotor circuit when in motion.

Further, if \( I_s \) is current per phase of the rotor, and \( B \) is leakage flux, then the coefficient of self-induction per phase of the rotor is \( \frac{2}{10^6 L_s} \) henries, which can also be calculated from the shape of slots and winding in them.

The above expression for \( T \) shows us that the running torque \( T \) of the induction motor is \( \propto \) to the square of the stator voltage, i.e., to the square of the stator flux, and increases as both the supply frequency and resistance per phase of the rotor decreases, becoming a maximum (at \( b \)) when

\[
K = \frac{\tau_s}{2\pi f L_s} \quad \text{and} \quad T = \frac{N_s^2 B_r^2}{\frac{N_s^2}{f^2} (2\pi f L_s)^2} = \frac{N_s^2 B_r^2}{f^2} \frac{r_b^2}{L_s}
\]

Thus, since the last expression for the maximum value of the running torque does not contain \( r_b \), we see that it is constant and independent of the rotor circuit resistance, but for different values of rotor resistance will attain the same maximum value \( b \) at a different slip, as indicated in Fig. 105. The motor starts.
at 0 with 100% slip and reaches the same maximum running torque $\omega = \omega_0$ for slips of $\omega_0%$ and $\omega = 100%$ (i.e. full speed) respectively, with rotor circuit resistances $r_a$ and $r_s$; since the stator current depends on the constant no-load current and rotor currents, and the latter will always be the same for a given torque, it follows that each value of torque will have a definite stator current which is independent of the rotor resistance.

Apparatus.—Precisely that for the preceding efficiency-load test No. 100), the starting resistance in the rotor circuit being of sufficient current-carrying capacity to enable it to carry, without overheating, the full-load rotor currents.

Observations.—Connect up as in Fig. 100, and carry out the tests precisely as directed in that test, for as many different values of starter resistance as possible.

Tabulate as shown on p. 276, adding two extra columns for values of starter resistance ($r$) and total rotor circuit resistance $r_{a_s} = (r_a + r)$ respectively.

Plot the following curves having torque $T$ in lb. ft., and rotor current $A_s$ as ordinates with percentage of full speed (or slip), and stator current $A_p$ respectively as abscissa for each value of rotor circuit resistance ($r_a$).

Also curves having efficiency, power factor, slip and true stator watts absorbed as ordinates with B.H.P. as abscissa for two widely different values of $r_s$.

Inferences.—From a careful study of the numerical results and curves state clearly what can be deduced.

(Rotor at Standstill.)

Introduction.—Under these conditions the slip will be 100%, since the speed is zero. The power absorbed will include both iron and copper losses, and the motor will approximate to a static transformer with a non-inductive secondary load.

Now the frequency $f$ of the stator supply will also be that
of the motor circuits when at rest, and if we put \( \omega = 1 \) in the
expression for the running torque (p. 295), since the rotor is
now stationary, it will be seen that the starting torque \( T_0 \) is
given by the relation

\[
T_0 = \frac{N e^2 P B_r}{N e^2 2\pi f (2\pi f L_m)^2 + r_a^2},
\]

where \( \omega \) = speed of the rotating field in revs. per sec, and \( \frac{2\pi f}{p} \)
its angular velocity, \( (2\pi f L_m)^2 + r_a^2 \) = the square of the
impedance, and \( 2\pi f L_m \) = the reactance per phase of the rotor
when at rest. From the above relation we see that the starting
torque \( T_0 \) is \( \propto \) to the square of the stator voltage, i.e. to the
square of the stator flux, and increases as both the supply
frequency and reactance per phase of the rotor decreases,
becoming a maximum when \( 2\pi f L_m = r_a \).

For this last condition—

\[
T_0 = \frac{N e^2 P B_r}{2N e^2 2\pi f r_a} = \frac{N e^2 P}{4N e^2} \times \frac{B_r}{r_a}, \text{ i.e. inversely } \propto \text{ to } r_a
\]

we therefore have the following most important deduction:
amely, that for a given supply frequency, the starting torque is
a maximum when the resistance and reactance of the rotor
circuits are equal and each as small as possible. Since the
rotor currents are a maximum at starting, the present test enables the maximum value of the starting resistance to be
obtained under either of two conditions: namely, (1) for
maximum starting torque, or (2) for maximum safe starting
rotor current. In the former, by measuring the values of \( L_m \)
and \( r_a \) per phase winding of the rotor we know that for maximum
starting torque \( 2\pi f L_m \) must = \( r_a + r \), whence the external starter
resistance must have a maximum value \( r = 2\pi f L_m - r_a \) ohms
per phase.

In the latter, if \( V_n \) = the standstill slip-ring voltage at
normal stator volts and frequency, then \( V_n \sqrt{3} = \) the standstill
volts per phase winding, whence \( r = \frac{V_n}{\sqrt{3} L_m} \) ohms per phase for
a maximum safe starting rotor current \( I_a \). The gradation of \( r \)
between this maximum value and 0 depends on the number of switch contacts and sections chosen.

Apparatus.—That detailed for the no-load short-circuit test No. 98, using an induction motor having a slip-vane rotor connected to the usual form of three-phase equal variable starting resistance of a current-carrying capacity sufficient to allow the necessary time for taking readings without overheating. In addition, a block brake and lever, preferably similar to that shown in Fig. 95, will be needed to measure the torque exerted by the shaft.

Observations.—(1) Connect up as in Fig. 100, levelling and adjusting to zero such instruments as need it. On starting up see that all lubricating arrangements are feeding properly.

Starting Torque with Rotor Circuit Resistance for a Constant Supply Voltage and Frequency.—(2) Adjust the supply frequency \( f \) to the normal value for the motor and the supply voltage \( V_s \) to some convenient value, if necessary lower than the normal value for the motor in order to avoid excessive rotor currents and keep both constant. Then read the spring balance and all other instruments as quickly as possible, when \( f \) is moved one contact stud at a time from its “full in” position to such a position nearer that of short circuit at which the rotor current \( A_x \) reaches a safe overload value. Finally measuring the resistance of each motor circuit corresponding to each contact-stud position.

Starting Torque with Supply Frequency for Constant Rotor Circuit Resistance and Supply Voltage.—(3) With the supply voltage \( V_s \) and starter resistance \( r \) adjusted to convenient values for giving safe maximum rotor currents and kept constant, read the spring balance and all the other instruments as rapidly as possible at each of a series of supply frequencies \( f \) between the maximum and minimum values possible and convenient.

Starting Torque with Supply Voltage for Constant Rotor Circuit Resistance and Frequency.—(4) With supply frequency and starter resistance \( r \) adjusted to convenient constant values, read the spring balance and all instruments as rapidly as possible at each of a series of supply voltages \( V_s \) between maximum and minimum values giving safe maximum rotor currents, and tabulate all your results as follows—

Introduction.—The comparatively small starting torque of the induction motor to that necessary for electric traction work has led in recent years to the production, improvement, and utilization on an increasing scale of the so-called alternating-current commutator motor. It is well known that any ordinary direct-current series or shunt-wound electric motor will run in one and the same direction whichever way the supply current flows through it, for with every reversal of the supply current, the magnetization of both field and armature will also be simultaneously reversed, and the motor will continue to run as if nothing had been changed. From this it follows that any ordinary D.C. machine will run as a motor when supplied with
A.C., though insufficiently owing (1) to the large eddy-current loss due to heavy eddy currents which would be set up in the solid field system by reason of the rapid reversal of the magnetization, and (2) the demagnetizing effect of such currents on the field. If, however, the field system is well laminated, like the armature of any machine always is, the machine would run with reasonable efficiency on an A.C. supply, but will develop less power than when run with D.C., of the same mean voltage, owing to the smaller current and flux, and to the larger internal losses due to eddy currents and hysteresis resulting from an A.C. supply.

The field magnets of A.C. commutating motors are either bipolar or multi-polar, whether of the projecting pole form used in D.C. machines, or of the cylindrical form with uniform air-gap as used in induction motors, and with definite polarity produced by the windings but not otherwise so evident. The armature, however, presents the usual appearance of D.C. forms, although, along with its commutator, embodying features mentioned later and necessary for ensuring satisfactory operation.

These features will be appreciated after a brief consideration of the actions taking place in the machine, but at the outset it should be realized that a single-phase series-wound commutator motor, built on the best possible lines for a given voltage supply, will operate in every way as well, but even more efficiently when run from a D.C. supply of the same voltage. In fact, such motors have to run on A.C. in some parts and on D.C. in other sections of certain tramway undertakings.

Now, considering such a series-wound motor with (for simplicity) a two-pole field, and hence with one pair of brushes, with a D.C. supply, producing a unidirectional field in the poles and through the armature, there will be set up a unidirectional induced potential difference (P.D.) having its maximum value between the brushes, i.e. along the "diameter of commutation", which, with the motor running light, will be coincident with the "neutral axis" and perpendicular to the direction of the fixed field. This induced P.D. (or "back R.M.F." ) is set up solely by reason of the forced rotation of the armature conductors across the field, by the supply current flowing in them, and with a given field is entirely due and
directly to the speed (n) of rotation. On the other hand, with an A.C. supply producing an alternating field in the poles and through the armature, there will be set up two distinct alternating P.D.s: namely, (1) the induced P.D. having its maximum value between the brushes exactly as mentioned above, and with a given field entirely due and directly to the speed (n) of rotation; it is in phase with the field and also practically with the current, and consequently not in direct opposition of phase with the supply E.M.F., and (2) the self-induced P.D. having its maximum value between two points in the armature winding on a diameter perpendicular to the diameter of commutation. This self-induced P.D. is set up solely by reason of the transformer action due to the armature conductors cutting the alternating field, and will lag in phase behind the field flux by an angle of 90°. Its magnitude will depend only on the strength and rate of reversal (i.e. the frequency f) of the alternating field, and in no way on whether the armature rotates or is stationary. It has no effect on the action of the motor, nor on the supply; consequently, due to the main field, in the rotating armature of a single-phase commutator motor, there are induced two entirely distinct E.M.F.s—one caused only by and directly to the speed of rotation, the other caused only by transformer action and directly to the supply frequency.

Now, when a current flows through the armature, the latter becomes a powerful electro-magnet, the two halves of the winding in parallel between the brushes producing two similar semi-circular electro-magnets having a consequent north and a consequent south pole situated in the diameter of commutation, and at a distance apart equal to the diameter of the armature core. The flux of this armature magnetization will be in phase with the current, and have a direction therefore perpendicular to the main field flux, or in line with the diameter of commutation, giving rise to the phenomenon commonly known as armature reaction. It will react on the main flux in three ways: (1) by distorting and dragging it round in the direction of rotation, (2) by inducing in the armature conductors, as they rotate through it, an E.M.F. along an axis parallel to the main field, but which will not in any way affect the action of the motor,
(3) by inducing, through transformer action on the armature conductors, an E.M.F. of self-induction 90° in phase behind the current and acting along an axis joining the two brushes. The value of this self-induced or reactance voltage of the armature is \( L_2 2\pi f A \) where \( L_2 \) = coefficient of self-induction of the armature winding carrying a current \( A \), and \( f \) = the frequency of the current \( A \), which in this case is that of the supply to the motor. Since the motor is series wound, the same current \( A \) will flow in the field winding which will have a coefficient of self-induction \( L_f \). Consequently the series field coils will introduce into the circuit a self-induced, or back, or reactance voltage \( = L_2 2\pi f A \). Thus the total reactance voltage of the motor will \( = 3\pi f (L_2 + L_f) \). Now the reactance of the machine has the disadvantage of reducing the power factor of the circuit, and should therefore be minimized as far as possible.

That due to the field coils cannot be reduced, because the chief cause of its existence, viz. the flux, is also necessary for the operation of the machine as a motor.

The reactance of the armature can, and is, compensated for by an additional winding on the field system midway between the main field windings, and producing a flux equal and opposite to the reactance field of the armature, and which is connected either in series with the circuit or short-circuited on itself. In either case the effect is the neutralization of the armature reaction flux and reactance and an increase in power factor. Again, although the self-induced voltage in the armature coils, due to transformer action and main field has no effect on the action of the motor, it has an effect on the commutation. For example, an armature coil undergoing commutation is short-circuited by the brush while inactive, i.e. generating no E.M.F. by reason of its rotation across the field. Since, however, in an A.C. motor, the coil by transformer action has, during commutation, a self-induced E.M.F., this will produce in it, when short-circuited by the brush, a heavy current which when broken as the segments leave the brush will cause sparking and the deterioration of the commutator.

Now, the self-induced E.M.F. of a coil decreases with a decrease in the number of turns, and if the circuit of the coil is broken before the current has time to attain its full value the
spark will be decreased. Hence in commercial single-phase
commutator motors, sparking is minimized by (a) having as
few a number of turns per armature coil as possible, (b) an
increased number of coils and peripherally narrower commutator
segments and brushes, (c) as small a supply frequency as
possible, (d) brushes of special composition. The narrower seg-
ment in (c) reduces the time during which an armature coil is
short-circuited and reduces the short-circuit current in it.

The single-phase A.C. motor possesses much the same
characteristics as the D.C. form, using a variable-speed motor,
giving maximum torque on starting which decreases with increase
of speed, and is as to armature current, but independent of
power factor. It will tend to race in speed on suddenly re-
moving the load. Further, since the current is simultaneously

\[ \text{Fig. 103.} \]

reversed in armature and field, the torque will be unidirectional
though pulsating.

**Apparatus.**—An A.C. supply $E$, preferably a motor-driven
alternator having a speed and field control independently
variable between wide limits; ammeter $A$; variable non-inductive
rheostat $R$; frequency meter ($f$); voltmeter $V$ with
two two-way keys $K_1, K_2$; wattmeter $W$; switch $S$; and single-
phase commutator motor, to be tested, of which the series field
windings are $F$ and (c) the armature.

**Observations.**—(1) Connect up as in Fig. 103, levelling and
adjusting to zero such instruments as need it. N.B.—With a
turn's supply for $E$, the rheostat $R$ will be needed to start up
$M$ and for regulating the current afterwards, otherwise with a
motor alternator this may be done by field excitation.

(2) With $R$ or the alternator field rheostat full in, and the
armature shaft clamped to prevent it rotating, close $S$ and adjust
the frequency $f$ to the normal value for the motor. Now
gradually raise the voltage until \( A \) reads the full-load current of the motor, and note the readings of \( f, A, W \) and \( V \) (when switched by \( K_1, K_2 \) across \( q', p' \) and \( n + s' \), giving readings \( V_x, V_y, V_z \) and \( V \) respectively).

(3) For this same value of \( V \) and \( f \) unclamp the armature shaft so as to see what all the instruments, including tachometer \( V_x, V_y, V_z \) and \( V \), will read when the speed has risen to a constant value (which must not be excessive), the shaft running quite "light."

(4) With normal frequency, and the motor running perfectly "light," read all the instruments and speed at each of a series of voltages \( V \) between 0, 25%, 50% above normal value.

(5) Repeat obs. 4 with constant normal voltage and wide variation in frequency.

(6) With the normal frequency \( (f) \) and the motor running light, alter \( V \) so as to obtain normal speed on the tachometer, and note the readings of \( f, A, W, V_x, V_y \) and \( V \).

(7) With this same value of speed and \( f \) load up the motor to about 25% above full load in some eight or ten successive steps, noting the readings of all the instruments, the speed being kept constant by raising the voltage \( V \).

(8) With the motor running light at constant normal frequency, obtain the maximum safe speed allowable, note this and also the values of \( V, A \) and \( W \); next apply about ten different braking loads up to about 50% above normal, noting the values of the speed \( V, A \) and \( W \) at each—\( V \) and \( f \) being kept constant.

Tabulate all your results as follows—

<table>
<thead>
<tr>
<th>Motor Test No.</th>
<th>Type</th>
<th>Make</th>
<th>Weight</th>
<th>Full Load</th>
<th>Amps</th>
<th>Volts</th>
<th>Speed</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series Single</td>
<td>motor</td>
<td>1200</td>
<td>volts</td>
<td>800 r.p.m.</td>
<td>60 Hz</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Brake Force</th>
<th>Full Load</th>
<th>Volts</th>
<th>Armature Current</th>
<th>Water</th>
<th>Healthy State</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 lb.</td>
<td>100 lb.</td>
<td>100 lb.</td>
<td>100 lb.</td>
<td>100 lb.</td>
<td>100 lb.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Efficiency</th>
<th>100%</th>
<th>90%</th>
<th>80%</th>
<th>70%</th>
<th>60%</th>
<th>50%</th>
<th>40%</th>
<th>30%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(3) Plot the following curves—
From obs. 4 between voltage $V$ as abscissa with speed $A$ and $W$ as ordinates.
From obs. 5 between frequency $f$ as abscissa with speed $A$ and $W$ as ordinates.
From obs. 6 and 7 between loads $H_l$ as abscissa with values of $x$, cos $\phi$, $H_l$ and $A$ as ordinates.
From obs. 8 between speed as abscissa with values of $x$ and $H_l$ as ordinates.
From obs. 8 between torque as abscissa and values of speed and $A$ as ordinates.

Inferences.—State clearly all that can be deduced from the tabular results and curves.

(106) Relation between the Field Excitation and Armature Current, or the "V" and other Curves, of a Synchronous Alternating-Current Motor Running Light or at Constant B.H.P.

Introduction.—All alternators, whether single or polyphase, are reversible machines, and will run as motors synchronously with the periodicity of the A.C. supply to their armatures, the field system being in all cases supplied with a separate source of direct current.

Synchronous motors are, however, not self-starting, for at every succeeding rapid reversal of the A.C. supply, the armature coils receive equal impulses but in opposite direction, and hence there is no resultant torque. If, however, the motor is first started up and run by some other driving source of power, at such a speed that any armature conductor passes through the distance between the centres of two poles (i.e. the pitch) in half the periodic time of the A.C. supply, then on switching it on to the supply it will continue to run as an efficient A.C. motor in dead synchronism with the supply frequency, irrespective of load, so long as this is not sufficient to pull it out of step with the supply current.

A single-phase synchronous motor therefore develops an alter-
nating armature polarity and torque which reverses with the rapidity of reversal of the A.C. supply, thus producing unidirectional rotation. On the other hand, the currents in the phase windings of a polyphase synchronous motor combine so as to form a constant polarity of fixed position relatively to that of the field, so causing a unidirectional torque and rotation.

Apparatus.—Sources of A.C. supply $E_1$, to synchronous motor $M_1$, and of D.C. supply $E_2$ to starting motor ($w$) and field of $M_1$ switch $S_1$; lamps $L_1 L_2$; A.C. ammeter $A_1$, voltmeter $V_1$; wattmeter $W_1$, D.C. ammeter $A_2$, voltmeter $V_2$. Field ammeter $A_3$, rheostats $r_6$ and $r_7$, switches $S_2$ and $S_3$, with starter or main variable resistance ($r$).

![Diagram](image)

**Fig. 107.**

**Note**—The lamps $L_1 L_2$ should be stamped for a voltage, each equal or even 10% higher than that of $M_1$, so as to avoid burning them out while synchronising.

**Observations.**—(1) Connect up as shown in Fig. 107, levelling and adjusting to zero such instruments as require it. On starting the machines, see that their lubricating arrangements are working properly.

(2) The synchronising or starting up of the A.C. motor under test can be effected as follows: with $S$ and $S_3$ open and ($r$) off, if a starter, or "full in," if a variable rheostat, close $S_3$, and operate ($r$) so as to start the machines up to about the normal speed of $M_1$; now close $S_2$ and adjust $r_6$ and $r_7$ until $V$ indicates the same voltage as that of the supply $E_1$, and the lamps $L_1 L_2$ cease to blink and go out definitely with a slow period. At this moment close $S$ and open $S_3$, when the A.C. machine $M$ will continue to run as a synchronous A.C. motor at a speed entirely governed by, and directly proportional to, the supply frequency.
Note.—At the above moment of closing $S$, the back E.M.F. ($V'$) of $M$ will be not only practically equal to, but also exactly opposite in phase with ($i.e.$ differ by $180^\circ$ from), that of the supply $E$.

The starting up may also be effected by one of the special forms of synchroscope now made for the purpose, e.g. the rotatory type or synchroscope of Messrs. Everett, Edgecombe & Co., the characteristics of which are as follows: with the supply and the motor connected to the respective pairs of terminals on the synchroscope, the speed and field of $M$ are adjusted until the frequency of $M$ is that of the supply (indicated by the rotating pointer coming to rest), and the voltage of $M$ is equal and opposite in phase to that of the supply (indicated by the pointer taking the vertical position); under these conditions, the dial will show a white light and $S$ can be closed. Briefly, therefore, close main switch when pointer stops vertically and white light shows. If $M$ is running too fast, the pointer rotates clockwise and a red light shows, whereas if $M$ is running too slow, the pointer rotates counter-clockwise and a green light shows.

Two- and three-phase machines are synchronised by the same single phase instrument with its 2 pairs of terminals connected across any one phase, either side of the main switch contacts of that particular phase. With the motor $M$ under test running synchronously with the A.C. supply, the following very interesting and important investigations can be made, namely—

(3) With $N_a$ open and $r$ and $s$ "fall in," $M$ will (unless coupled to and released from $M_a$ by an electro-magnetic clutch) simply be turning against the small windage, brush, and bearing friction, and will therefore practically be running light itself. For this no-load condition at normal supply frequency and voltage adjust $r$ to obtain minimum reading on $A$, and note simultaneously that of $V, W, a$, and the speed.

(4) Next vary $r_a$ and hence the exciting current ($a$), by a series of steps, above and below the value found in obs. 3, as will raise $A$ to a value not exceeding 25% over load, in each case noting $V, W, a, A$ and the speed at each excitation. The supply voltage $V$ and frequency being kept constant throughout at the value of obs. 3.

(5) Repeat obs. 3 and 4 for constant B.H.P. load outputs
from \( M \) of say \( 1, 2, 3 \) and full load respectively at the same constant supply volts and frequency, taking that value of \( a \), giving minimum main current \( A \) as the starting point of the "up and down series" of (a).

Note.—The brake load can most conveniently be taken up electrically in the coupled D.C. starting motor \( m \) by causing it to act as a D.C. generator and send current through a suitable current rheostat to be connected in series with a switch (neither shown in Fig. 107) across the points \( P \) and \( Q \). In this case \( v \) must be short-circuited and special precautions taken to keep \( S_m \) open.

The product \( v \cdot a \) = the power absorbed in the added rheostat, and, if the efficiency of \( M \) is known, the actual B.H.P. developed by \( M \) is at once obtainable, otherwise with constant excitation of (a), the power absorbed by it will be roughly \( v \cdot a \) to the currents (a) developed, and therefore to the B.H.P. given by \( M \).

Tapulate all results as follows——

<table>
<thead>
<tr>
<th>Synchronous Motor: No.</th>
<th>Full load current: B.H.P. = ... with Amps. at ... Volts and Speed = ... rpm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maker ...</td>
<td>Type ...</td>
</tr>
<tr>
<td>D.C. Starting Motor</td>
<td>Full load Amps. = ...Volts = ... Speed = ...</td>
</tr>
<tr>
<td>Efficiency ( E = \ldots ) = ... load.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supply</th>
<th>Ammeter A.</th>
<th>Ammeter B.</th>
<th>Motor A.</th>
<th>Motor B.</th>
<th>Speed in R.P.M.</th>
<th>Total Loaded Power cos ( \phi ) R.M.F.</th>
<th>Total Loaded Power cos ( \phi ) R.M.F.</th>
<th>D.C. Starting R.M.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

(6) Plot curves for running light and for each load on \( M \) having (1) amps \( A \), (2) power factor (cos \( \phi \)), and (3) watts \( W \) as ordinates with exciting current \( (a) \) as abscissae in each case.

Inferences.—From a careful study of the shapes of the curves and of the tabular results, state what can be deduced.