(132) Measurement of the Electrostatic Capacity of a Concentric Cable Ballistically.
(Standard Magneto Inductor Method.)

Introduction.—When some standard form of magneto inductor is available, the form devised by Dr. W. Hibbert being a very convenient and easily manipulated one, the capacity of a concentric or other electric light cable can be readily determined, provided a few other additional pieces of apparatus are available. The reader should note the general introductory remarks on p. 369 concerning the capacity of cables in general, and also those of the alternating current method of measuring the capacity of cables.

The present test can be employed for finding the capacity of cables in tanks and of ordinary and concentric mazes. As, however, the former are best tested by the "method of mixtures" (p. 371), we shall here only consider the test of a concentric cable by this inductor method.

Apparatus. — Standard inductor to be tested \( \beta \) (Fig. 138); sensitive ballistic galvanometer \( G \); concentric cable to be tested \( C \), of which \( \beta \) is the free and well-insulated end, \( \beta \) the inner conductor, and \( C \) the other; box of known resistances \( r \); battery \( B \), of known M.E., or, if this is unknown, a standard voltmeter to measure the P.D.; two-way spring
tapping-key $K$ (p. 585); ordinary spring tapping-key $K_1$; damping-coil with its cell and key.

Observations.—(1) Connect up as in Fig. 138, and adjust the galvanometer needle to zero, carefully prepare the free (far) end of the cable, viz. $K_2$ in the manner described on p. 375, and also the (near) end as well; by so doing the variation of the results from leakage across the cable ends will be avoided.

(2) $K_1$ and $K_2$ being open, adjust $r$ to a low value, such that pressing $K_2$ nearly a full-scale throw $d_r$ is obtained on slipping down $L$. Note this value of $d_r$, and the box resistance $r$ ohms.

(3) $K_1$ being open, adjust the voltage of the battery $B$ to such a value that on closing $K_2$ for two or three seconds, then opening it, and immediately closing $K_1$, a first throw $d_1$ is obtained on discharging $C$, as nearly as possible equal to the former.

N.B.—Two or three throws should be taken in both 2 and 3, and the means noted as being more accurate.

(4) Obtain the mean throw on the charge in a similar way by first closing $K_1$ for a few seconds so as to completely discharge $C$, and then opening it and closing $K_2$ afterwards.

Note.—Care must be taken that $C$ is each time discharged before taking the charge throw.

(5) If possible employ three or four different voltages and repeat 2 and 3 with each of them, keeping the deflections $d_r$ and $d_1$ about equal to one another, preferably by varying $r$ to suit.

(6) Calculate the capacity of the cable tested from the relation

$$C = \frac{P}{100} \frac{d_0}{V_0},$$

where $P$ = total magnetic flux in the air-gap of the inductor and $E$ = total resistance in ohms of the inductor circuit.

Tabulate as follows—

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement</td>
<td>$B = \ldots$</td>
</tr>
<tr>
<td>Sub-circuit resistance $r$ = \ldots</td>
<td>ohms</td>
</tr>
<tr>
<td>Cable tested</td>
<td>Type \ldots</td>
</tr>
<tr>
<td>Length of Cable $L = \ldots$</td>
<td>miles</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mass first throw</th>
<th>Resistance in Ohms</th>
<th>P.D. of voltage variable</th>
<th>Capacity in Microfarads</th>
<th>Mean in</th>
<th>Capacity of Cable in Mica. per mile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_r$</td>
<td>$d_0$</td>
<td>$E = r_0 + r_2 + L$</td>
<td>$C$</td>
<td>$C$</td>
<td>$C$</td>
</tr>
</tbody>
</table>


Measurement of the Electrical Power absorbed in Alternating Current Inductive Circuits. (Three-voltmeter Method.)

Introduction.—The measurement of alternating current power depends on the nature of the external circuit. Thus, if this circuit is non-inductive, then the true power \( W = \text{amps.} \times \text{volts} \times \cos \theta \), where \( \theta \) = angle of phase difference of the current \( I \) behind the voltage \( V \) and the product \( I \times V \) is called the apparent power absorbed.

The measurement, therefore, of this electrical power accurately is more difficult than that in the case of a direct current circuit, owing to the effects of self and mutual induction and capacity which appear in alternating-current (A.C.) working. In such a case a Wattmeter may be used, but it must be practically non-inductive to give accurate results. Another method to employ, which will give accurate results even though most of the circuit is highly inductive, is that known as the "three-voltmeter method," and it has the advantage that only one A.C. voltmeter is required, though three similar ones may be used if available. By it the true power absorbed by the circuit may be obtained with any degree of accuracy desired by using an accurately graduated voltmeter, and by carefully repeating the readings two or three times and noting the mean in each case.

The three-voltmeter method, which was simultaneously suggested by Prof. Ayrton, Dr. Sampson, and Mr. Swinburne, gives a true measure of the power given by any current, whether harmonic or otherwise, to any circuit, inductive or not. It has the disadvantage that, as the differences of squares of quantities is being taken, a small error in the quantities themselves may make a considerable error in the final result, especially if the angle of lag \( \theta \) is large.
The voltmeter used must be such as will not alter the voltage across the points to which it is applied. In other words, it must have a high resistance compared with that between these points.

An electrostatic voltmeter most accurately fulfils this condition, but if a lead-wire voltmeter is used (of relatively low resistance), the main current must be large compared with its own current, or an error will thus be introduced.

With this apparatus we are in a position to investigate the following important characteristics of an inductive circuit formed by, say, a exciting coil or the primary of a static transformer, etc., namely—

(1) The true power absorbed in the whole and each part of the circuit.

(2) The angle of phase difference between the current and both the supply and choke voltages.

(3) The impedances, ohmic, and inductive resistances, and self-induction of the choke.

The vector diagram for the circuit \( PR \) is that shown in Fig. 139, and is constructed as follows: set off a vector on equal to the total voltage \( V \) across \( PR \) to any convenient scale. With nulls \( ab = V_1 \) and \( ba = V_2 \) and centres \( o \) and \( a \) respectively, draw axes intersecting at \( b \), join \( b \) to \( o \) and \( a \) and produce \( ab \) to meet \( a \) perpendicular from \( a \) in the point \( c \). Then \( abc \) is the triangle of E.M.F.s for \( PR \), and see that for \( PQ \) where \( v_1 \) and \( L \) are the ohmic resistance and self-induction of the inductive portion \( PQ \). Since \( QR \) is non-inductive, the current \( A \) and voltage \( V_3 \) are always in phase, and hence by Ohm's law \( V_3 = Ar \) where \( r \) is the ohmic resistance of \( QR \), and \( ac \) will be coincident with the current vector. Thus \( \theta \) will be the angle of phase difference.
between the current (A) and total voltage \( V \), while \( \theta \) will be that between \( A \) and the voltage \( V_1 \).

**Note.**—Errors in \( V \), \( V_1 \) or \( V_2 \) or in the graduation of the voltmeter scale, will have least effect on the result when \( V_2 = V_2 \), which is the condition for maximum accuracy, and the resistance \( r \) of \( QR \) should first be adjusted if possible to obtain this condition.

Should the non-inductive resistance \( QR \) not be accurately known, or be likely to alter in value through heating due to the passage of the current \( A \), then its equivalent in terms of \( V_2 \) and \( A \) can be substituted in this formula. Hence, if \( A \) is the \( \sqrt{\text{mean square}} \) current in amps as given by a Siemens dynamometer or other direct reading alternating current ammeter, we shall have

\[
W = \frac{A}{2V_2} \left( v^2 - v_1^2 + v_2^2 \right) \text{ Watts},
\]

for the true mean power given to the whole circuit \( PR \). \( QR \) may consist of a bank of electric glow-lamps, as the resistance \( r \) of \( QR \) can vary if it likes with the different mean currents.

It can easily be shown, in like manner, that the true mean power given to the inductive portion \( PQ \) of the circuit is

\[
W = \frac{1}{2P} \left( v^2 - v_1^2 - v_2^2 \right) = \frac{A}{2V_2} \left( v^2 - v_1^2 - v_2^2 \right)
\]

The method is not based on any assumptions as to the nature of the current (whether periodic or otherwise) or of the circuit, which may contain either self or mutual induction, and capacity, or all three. It is based solely on the differences in phase between the current and voltage.

If \( \theta \) = angle of phase difference or lag of the current behind the voltage, then if both are sine functions

\[
\cos \theta = \frac{v^2 - v_1^2 - v_2^2}{2V_1 V_2}
\]

**Apparatus.**—Alternator \( D \) and its exciting circuit; inductive portion \( PQ \) of the circuit in series with a strictly non-inductive portion \( QR \); two 2-way keys \( K_1 \) and \( K_2 \) (p. 387); an A.C. voltmeter \( V \); main switch \( S \); A.C. ammeter \( A \). For comparison of methods, \( A \) may be used, and also a non-inductive Wattmeter \( W \) for measuring directly the power used up in \( PR \). Frequency meter \( f \) connected across the supply \( D \).
In the electrical circuit of the motor $M$ may be used a voltmeter $V_2$, ammeter $A_2$, switch $S_2$, rheostat $R_2$, source of continuous current $S_1$.

**Experiments**—(1) Connect up as shown in Fig. 140. Adjust the pointer of all the instruments to zero, levelling such as need it.

(2) See that all lubricators in use feed properly, then start $D_1$ running slowly.

(3) Adjust its speed so as to get $\frac{1}{4}$ of the max. $\cdots$ per sec., at the same time varying the excitation of $D$ to alter its voltage ($V$), so

![Diagram](image)

as to send a convenient current ($A$) through $PQ$. Note $A$, and in quick succession (the speed being constant) the voltages $V_1, V_2$ and $V_3$ across $PQ, PQ$, and $QR$. (See Note above.)

(4) Repeat 3 for about 5 frequencies between the max. and min. values possible, using the same current $A$ in each case by suitably altering the excitation.

(5) Repeat 3 at a constant frequency of about normal for five different current values, rising by equal increments up to the maximum allowed, by varying the excitation.

Tabulate your results as follows—
ELECTRICAL ENGINEERING TESTING

Four. of Inductive Circuit PQ test

- Non-Iml. resistance PQ used...

<table>
<thead>
<tr>
<th>Voltages</th>
<th>Power in Watts absorbed</th>
</tr>
</thead>
<tbody>
<tr>
<td>V, v, V′</td>
<td>PR, PQ, QR</td>
</tr>
<tr>
<td>A, B</td>
<td>PR, By CV exp.</td>
</tr>
<tr>
<td>C</td>
<td>PR, PQ, QR</td>
</tr>
<tr>
<td>D</td>
<td>PR, PQ, QR</td>
</tr>
<tr>
<td>F</td>
<td>PQ, QR</td>
</tr>
<tr>
<td>G</td>
<td>PQ, QR</td>
</tr>
</tbody>
</table>

(G) Plot curves having values of A as abscissas with values of $r_A \cos \theta_A$ and $w_A$ respectively, for the inductive portion PQ as ordinates.

(7) Draw the vector diagram (Fig. 139) for the maximum current used.

(8) Compare $V$ with the algebraical sum $(\sqrt{\sum} V_2)$, also $W$ with $w$.

Inferences.—Prove the formula in column 7, and state any assumptions made in deducing it. What can be inferred from the results of the test and from the curves?

(134) Measurement of the Electrical Power absorbed in Alternating Current Inductive Circuits. (Three-Ammeter Method.)

Introduction.—This method, though inferior to that of the Wattmeter, is nevertheless instructive, and therefore a brief remit of it will be given here. As will be seen, it is very similar to the 3-voltmeter method of measuring power, the formulae in the two cases being strikingly similar. There is, however, one chief difference between the methods, namely, that practically three ammeters are necessary for a satisfactory test, as large errors...
may occur if only one ammeter is employed and interchanged between the circuits, while in the case of the allied method one voltmeter can easily be made to do and no appreciable error need be introduced. The actual arrangement is shown in Fig. 141, in which PQ represents the circuit in which it is desired to measure the power taken up, $A_1$, $A_2$, and $A_3$ are three non-inductive ammeters, at least $A_1$ should be of this nature, while $r$ is a non-inductive resistance connected as shown, and which is large compared with that of $A_1$.

Fig. 141.

Greater accuracy will be obtained when $A_1 = A_2$ and under these conditions it will be seen that $r$ consumes as much power as $Q$. Hence twice as much power has to be available at the source for operating this method as is taken up in $PQ$, but practically no excess voltage is needed in this case as it was in the 3-voltmeter method. If $u_3$ be the power in Watts absorbed by $PQ$, then

$$u_3 = \frac{1}{2}r[A_1^2 - A_2^2 - A_3^2]$$

and

$$\cos \theta_1 = \frac{A_3^2 - A_1^2 - A_2^2}{2A_1A_2}$$

where $\theta_1$ = angle of lag of the current $A_2$ in the inductive circuit $PQ$ behind the terminal voltage.

It will thus be seen that the method is based on the difference of phase of the various currents, and, as in the 3-voltmeter method, a small error in observing the currents introduces large errors in the answer. The possibility of such occurring can be minimized by using accurately calibrated non-inductive ammeters and taking the mean of three or four similar readings at each value of, say, $A$. If the value of non-inductive resistance $r$ is not accurately known, or if it is liable to alter through heating due to the passage of the current $A$, its equivalent value $A_2$ may be used instead in the formula, which will therefore become:

$$W = \frac{V}{2A_1} \left( A_1^2 - A_2^2 - A_3^2 \right) \text{Watts}$$
where \( V \) = the voltage across the extremities of \( PQ \). There is no objection to using a bank of incandescent lamps for \((r)\), since the resistance may vary if it likes with the different mean current strengths. It will be observed that if the resistance of \( A_2 \) is appreciable, an amount of power may be absorbed in it which is comparable with that in \( PQ \). In such cases the former must be deducted from the result as given by the above relation in order to obtain the true power absorbed in \( PQ \) alone.

If \((r)\) is accurately known, we may dispense with \( A_1 \) and put \((r)\) directly across the mains, then on placing a voltmeter (preferably an electrostatic one) across the mains as in the last instance, we may substitute the value of \( A_1 \) in the first formula, when we shall have

\[
W = \frac{1}{2} r \left( A_2^2 - \left( \frac{V}{r} \right)^2 - A_2^2 \right)
\]

The preceding remarks will be understood more clearly from the vector diagram, Fig. 142, for the circuit of Fig. 141, constructed as follows: set off a vector \( \Delta A \) equal to the total current \( A \) in the main line to any convenient scale with radii \( ab = A_1 \) and \( bc = A_2 \) and centres \( a \) and \( c \) respectively, draw arcs intersecting at \( b \). Join \( b \) to \( c \) and \( c \) and produce \( bc \) to meet a perpendicular from \( a \) in the point \( c \). Then \( abc \) is the vector triangle of currents for the main and both branches altogether, while \( bca \) is that for the inductive branch only. Since \( c \) is non-inductive, its current \( A_2 \) and voltage are always in phase, and hence by Ohm's law \( A_2 = \frac{V}{r} \). If \( R \) equals the ohmic resistance of \( PQ \) and \( L \) the self-induction, then the energy or magnetizing component of the current \( A_2 \) in \( PQ \) which is in phase with the voltage \( V \) across it, is \( bca = \frac{V}{R} \), while the idle or wattless component of the current \( A_2 \) in quadrature with the
voltage $V$ is on. The angle of phase difference between the main current $A$ and $V$ will be $\theta$, and that between $A^2$ and the same voltage $V$ across $PQ$ will be $\theta_2$.

From the geometry of Fig. 142 we see that

$$ A_2^2 = A^2 + A^2 - 2AA_2 \cos \theta $$

but $A_2 = \frac{V}{r}$ by Ohm's law, and

$$ A_2^2 = A^2 + A_2^2 - 2A_2 V \cos \theta $$

and $A_2 V \cos \theta = \text{total power given to the whole parallel circuit}.$

The total power absorbed in the whole circuit is

$$ w = V \times I = \frac{V}{r}(A^2 + A_2^2 - A_4^2) \text{ watts,} $$

the power absorbed in the non-inductive branch

$$ w_1 = A_1^2 r = A_1 V \text{ watts,} $$

and the power absorbed in the inductive branch

$$ w_2 = V \times I_2 = \frac{V}{r}(A_2^2 - A_4^2) \text{ watts,} $$

where the power factor for the whole circuit is

$$ \cos \theta = \frac{A^2 + A_4^2 - A_2^2}{2AA_1} = \frac{r(A^2 + A_2^2 - A_4^2)}{2AA_1}, $$

and the power factor for the inductive circuit $PV =

$$ \cos \theta_2 = \frac{A_2^2 - A_4^2}{2A_1 A_2} = \frac{r(A_2^2 - A_4^2)}{2AA_2}. $$

In this three-ammeter method the non-inductive parallel branch is equivalent to an added current, while in the three-voltmeter method the non-inductive series resistance means an added voltage. Both methods, therefore, require the supply of practically twice as much power as that needed for the circuit under test. Further, the losses in the ammeters, voltmeter, and wattmeter may cause serious errors in the results if the currents are small. For the above reasons, no one would use either method for measuring power if a wattmeter was available, except from a purely scientific interest. While the power absorbed and phase difference may be calculated in each method from the vector diagram, constructed for each set of readings, it would usually be obtained from the respective formulas.

Apparatus.—That indicated in Fig. 141, where $D$ is an adjustable source of alternating current, preferably a motor-driven alternator, the frequency, current and voltage of which can be
varied independently. A voltmeter $V$ is connected across the parallel combination, and a wattmeter inserted so as to measure the total watts absorbed in the parallel combination merely for the comparison of the three ammeter and wattmeter methods.

**Observations**—(1) Connect up as in Fig. 141, levelling and adjusting to zero each of the instruments as need it.

(2) See that all lubricating arrangements are in operation on starting up.

(3) Adjust the speed to get maximum frequency, and also the voltage $V$ of the alternator (by varying its excitation) so as to send the maximum safe current $A_3$ through $PQ_6$ and note in rapid succession the readings of $A_1$, $A_2$, $A_3$, $W$ and $V$.

**Note.**—If possible adjust the non-inductive resistance $r$ so as to obtain the conditions for maximum security (other things being the same) of $A_1 = A_2$.

(4) Repeat (3) for about six different frequencies between the maximum and minimum values possible, using the same current $A_3$ in each case by suitably adjusting the speed and excitation of the alternator.

(5) Repeat (3) at constant maximum frequency for about six different values of current $A_3$ between the maximum and minimum values possible by varying the excitation, and tabulate your results as follows—

![Table](image-url)
(6) Plot curves having values of $A_4$ as abscissae with values of $V_4 \cos \theta_4 \frac{P}{A_4}$ and $w_4$, respectively, for the inductive portion $JQ$ as ordinates.

(7) Draw the vector diagram (Fig. 143), for the maximum current used.

(8) Compare the value of $A$ with the algebraical sum $A_2 + A_3$; also $W$ with $w$.

Inferences.—What can you infer from the results of the test and from the curves?


Introduction.—The measurement of the electrical power absorbed in a three-phase alternating current circuit might at first sight appear somewhat complicated. In reality, however, it is very little more so than in the case of single-phase circuits and the actual extent to which it is, depends mainly on the nature of the circuit in which the measurement is being made. It has already been seen that the non-inductive Wattmeter forms the best means of obtaining the true power absorbed in a single-phase circuit, but with multiphase circuits usually, though not always, two such instruments are necessary.

The object consequently of the present investigation is not only to state the methods of measuring, but also to prove the truth of them under the several distinctive conditions met with in practice.

The circuit in which the power has to be measured may be of the type shown at (a) Fig. 143, which is known as the star or open form, or of the type shown at (b) Fig. 143, known as the mesh or closed form. (c) represents the circuit containing the measuring instruments, which may be connected to either (a) or (b) arrangements at will, $E$ being the source of polyphase supply.

Now let $A_1, A_2, A_3$ and $a_1, a_2, a_3$ be the mean square values of the currents flowing in the mains and branches respectively for Fig. 143 (a and c), and $V_1, V_2, V_3$ and $v_1, v_2, v_3$ the mean values of voltages across the mains and branches respectively for Fig. 143 (a and c); then if the mains are equally loaded we have:

$$A_1 = A_2 = A_3 \text{ and } a_1 = a_2 = a_3 \text{ and } V_1 = V_2 = V_3$$
whence \( A_1 = 2a_1 \sin 60^\circ = \sqrt{3}a_1 \), and \( A = \sqrt{3}a \), since the mains are equally loaded and the load non-inductive.

For Fig. 143 (a and c) we have, if \( A_1 = A_2 = A_3 \) and \( E_1 = E_2 = E_3 \), that \( A_1 = a_1 \), \( A_2 = a_2 \) and \( A_3 = a_3 \), and since \( V \) will now lag 30° in phase behind \( V \), in each main and the corresponding branch circuit, we have \( V = 2V \sin 60^\circ \sqrt{3}/2 \), providing the load is non-inductive.

Circuits equally loaded and non-inductive—Here if each main carries the same current \( A \), and if the pressure between each pair of mains is \( V \), then the True Power absorbed in a non-inductive load, Fig. (b)

\[
W = 3aV = 3V \frac{A}{\sqrt{3}} = \sqrt{3}AV \text{ Watts,}
\]

True Power absorbed in a non-inductive load, Fig. (a)

\[
V = 3aV = 3A \sqrt{3} = \sqrt{3}AV \text{ Watts.}
\]

If, however, the load is inductive, then if \( \theta \) = angle of phase difference between voltage and current, we have, as in the case of single-phase work, that for equal load the True Power absorbed in the inductive load, Figs. (a) or (b)

\[
W = \sqrt{3}AV \cos \theta \text{ Watts.}
\]

This latter can best be obtained by means of the non-inductive Wattmeter for each of the two following conditions met with in practice.

Circuits equally loaded and inductive—One Wattmeter only needed to obtain the true power. Assuming this to be \( W_1 \), Fig. 143 (c, d) or (e, f), then with the thick coil in any main \( A_3 \) say, as shown note the Wattmeter reading \( (w) \) with its fine coil on
to main \( A_3 \) and the reading \( (\omega_2) \) with it on to main \( A_2 \) immediately after, then the True Power absorbed in the *equally loaded* inductive circuit \( W = \omega_5 \times \omega_2 \), where both \( \omega_4 \) and \( \omega_5 \) will vary with load and power factor.

The reason why \( \omega_1 \) or \( \omega_0 \) alone does not give the power of the circuit is because \( A_3 \) and \( V_3 \) are not in phase, even in a non-inductive circuit, but differ in phase by an angle \( = 30^\circ \pm \phi \), for both star and mesh connections. Therefore \( W_1 \) will read the product \( A_3 V_3 \cos 30^\circ = \frac{\sqrt{3}}{2} A_2 V_2 \) for unity power factor. Thus we see that \( \omega_3 = A_3 V_3 \cos (30^\circ + \phi) \), and \( \omega_2 = A_2 V_2 \cos (30^\circ - \phi) \), and the sum of these after expansion \( = 2.14 \cos 30^\circ \cos \phi = \omega_3 + \omega_2 \), which is the true power in the circuit. If now the load is so highly inductive that \( \phi \) exceeds 50°, i.e., the power factor \( \cos \phi \) is less than 0.5, then \( \cos (30 + \phi) \) becomes \(-1\), and the wattmeter will reverse for one of its readings \( \omega_3 \) or \( \omega_2 \), which must therefore be considered as \(-1\), since the volt-coil connections must be reversed to get a scale reading.

The total power \( W = \omega_1 \times \omega_0 = 2A_3 V_3 \cos 30^\circ \sin \phi = 2A_3 V_3 \times \frac{1}{2} \sin \phi = V_3 \times A_3 \sin \phi \), and \( V_3 \times A_3 \sin \phi \) = \( A_3 \sin \phi \) = the standstill or idle true current.

The above results will be readily understood by a reference to Fig. 144 (corresponding to Fig. 143, a and c), in which \( OA, OB, OC \), represents the voltages across the respective star stator phase windings in magnitude and phase difference (= 120°). \( AC, CB, BA \) = voltages \( V_D, V_2, V_3 \) between mains in relative magnitude and phase (= 120°).

Then obviously \( V_1 = AC = \sqrt{3} OA = \sqrt{3} OC \), also \( V_2 = CB = \sqrt{3} OC = \sqrt{3} OB \), and \( V_0 = BA = \sqrt{3} OA = \sqrt{3} OB \).

Now, since the three phase windings are inductive we can draw three equal lines, \( Oa, Ob, Oc \), to represent the currents in them lagging in phase by equal angles \( \theta \) behind their respective voltages \( OA, OB, OC \).

Then the current \( Oa \) in its phase winding differs in phase from the voltage \( AB (= V_3) \) by an angle \( \alpha DA = \theta - 30^\circ \), and the
current \( I_B \) in its phase winding differs in phase from the voltage \( AB \) \( (= V_2) \) by an angle \( \theta = 30^\circ \).

Now, if the current coil of wattmeter \( w_1 \) is in the circuit of \( OC_3 \) and hence of main 3, with its volt coil across \( AB \) (main 3 and 1), it will carry the current \( I_B \) at a voltage \( V_2 \). Similarly, the current coil of wattmeter \( w_2 \) in the circuit of \( OA_1 \), and hence of main 2, will carry the current \( I_B \) with its volt coil across \( AB \) (main 2 and 1) at a voltage \( V_3 \).

Then 
\[
w_1 = A_3 V_2 \cos (\theta - 30^\circ),
\]
\[
= A_3 V_2 \cos \theta \cos 30^\circ - \sin \theta \sin 30^\circ,
\]
and 
\[
w_2 = A_4 V_3 \cos (\theta - 30^\circ)
\]
\[
= A_4 V_3 \cos \theta \cos 30^\circ + \sin \theta \sin 30^\circ.
\]

The total power of the circuit \( W = w_1 + w_2 = 2AV \) \( \cos \theta \cos 30^\circ \) \( = \sqrt{3}AV \cos \theta \), on the assumption that \( A_1 = A_2 = A_3 = A_4 \), and \( V_1 = V_2 = V_3 \), which should be the case.

By adding and subtracting the values of \( w_1 \) and \( w_2 \) first, given we have
\[
w_1 - w_2 = \frac{1}{3} \tan \theta, \quad \text{and putting} \quad \frac{w_3}{w_1} = a
\]
we have the power factor
\[
\cos \theta = \frac{a + 1}{2 \sqrt{a^2 - a + 1}} = \frac{1}{\sqrt{1 + 3 \left( \frac{w_1}{w_1 + w_2} \right)^2}} \quad \text{(see p. 330)}
\]
when \( \theta = 0 \) the values of \( w_1 \) and \( w_2 \) are equal, and each
\[
eq \frac{1}{\sqrt{3}}AV \cos \theta.
\]
As \( \theta \) increases, \( w_1 \) decreases and \( w_2 \) increases, when \( \theta = 30^\circ \) the value of \( w_3 = \frac{1}{2} A_3 V_2 \) or \( \frac{1}{4} AV \), and of \( w_2 = A_4 V_3 \) or \( AV \); when \( \theta = 60^\circ \) the value of \( w_1 = 0 \), and of
\[
w_2 = \frac{\sqrt{3}}{2} A_4 V_3 \text{ or } \sqrt{3} AV \);
Since in this case the current in the series coil of $w_1$ differs 90° in phase from that in its pressure coil, any further increase in $l$ will make $w_1$ negative and reverse its deflection, so that the connections of one of its coils must be interchanged in order to bring the deflection on to the scale again.

Hence, in measuring the power of any inductive 3-phase circuit by either 1 or 2 wattmeters the total power = $w_1 + w_2$, i.e. if one of the readings reverses, subtract the smaller reading from the larger one to obtain the total power.

Circuits equally loaded and inductive—Two Wattmeters only needed to obtain the true power. Assuming the Wattmeters to have their thick coils in any two mains, as shown in Fig. 113 (a or b), then True Power absorbed $P = W_1 + W_2$.

Hence, when merely the true power in Watts only is required in a three-phase circuit, whether of the star or mesh type, one or two Wattmeters are required according to whether the circuits are equally or unequally loaded respectively. Also when such a three-phase circuit is both equally loaded and non-inductive the true power in Watts is given by the product $\sqrt{3}\times V\times I$, in one main x volts across any pair of mains. (See p. 365 et seq.).

Apparatus.—Sources of three-phase alternating current (A) and circuit of variable nature to experiment upon (a and b, Fig. 143).

Two Wattmeters $W_1$ and $W_2$; two Siemens dynamometers or Parr direct reading dynamometer ammeters $A_1, A_2, A_3$; three electrostatic or hot-wire voltimeters $V_1, V_2, V_3$.

Note.—It must be remembered that for any specific measurement, the foregoing rules, and the instruments they entail, can be at once used without reference to the following test, which is devised solely in order to prove these rules.

Observations.—(1) Connect up as in Fig. 113 (a and c), and adjust the instruments to zero, levelling them if necessary.

(2) With the load non-inductive and the circuits equally loaded, take the readings of all the instruments for five or six different loads, noting the Wattmeter reading when placing the fine coil of, say, $W_1$, successively on to $A_1$ and $A_2$ mains at each load.

(3) With an inductive load and circuits equally loaded, take the readings of all the instruments for five or six loads, placing the fine coil of, say, $W_1$, successively on to $A_1$ and $A_2$ mains at each load and noting its reading at each. Tabulate your results as follows—
Inferences.—State very clearly all that can be inferred from your experimental results.


Introduction.—Two distinct forms of circuits are met with in the distribution of electrical energy by means of two-phase alternating currents of electricity.

The first entails the use of four wires, forming two circuits completely independent of one another, one to each phase. Since this requires four wires it is usually employed in short distance transmissions.

The second entails the use of only three main wires, and is therefore more economical in first outlay of copper than the above. It will therefore be at once obvious that the measurement of power in two-phase alternating current circuits will be made in more than one way, depending on the form and nature of the circuit in question. We will now deal with such measurements in the case of each possible condition.

Two-Phase Circuits of the 4-Wire Form.

Here two cases are possible according to whether the circuits are carrying non-inductive loads, such as incandescent lamps, or inductive loads, such as two-phase motors or transformers, etc.

Non-inductive load.—The product of the amperes and volts in each circuit, obtained in the usual way, when added together gives the true power delivered from the generator; and if the two circuits are equally loaded, twice the product for one circuit gives the Total True Power.

Inductive load.—Owing to the lag in phase between the current and voltage in each circuit, two non-inductive Wattmeters are necessary, one in each circuit, connected up in the ordinary way
as in single-phase circuits. Then the Total True Power delivered by the generator = sum of the two Wattmeter readings.

If the two circuits are equally loaded, as should be the case when supplying such as two-phase motors, then twice the reading of one Wattmeter gives the Total True Power, and only one such instrument is then necessary.

**Two-Phase Circuits of the 3-Wire Form.**

Here also there are two or three cases depending on whether the circuits are inductive or otherwise.

- **Equally loaded non-inductive sections.**—Total True Power absorbed = twice the product of the current in one outer main and the voltage across the section.

- **Equally loaded inductive sections.**—Total True Power absorbed = twice the reading of a Wattmeter connected with its thick coil in series with either outer main, and its thin coil connected to the centre or larger main which is common to both outer.

- **Unequally loaded inductive sections.**—Total True Power absorbed = sum of the two readings of the Wattmeters connected with their thick coils in the outer respectively, and their thin coils connected to the common centre wire AB as shown in Fig. 145. This last case would be the one met with when the circuit was partly a lighting and partly a power one, running two-phase motors.

Where the reader may not be quite conversant with the preceding methods of measuring power in two-phase alternating current circuits, a most useful experiment will be to prove the above statements in much the same manner as was set forth in the preceding test on three phase measurements of power, only three or four ammeters and voltmeters with the two Wattmeters $W_2'$ and $W_4'$ and the variable two-phase rheostat being required.

As a summary, with some additions, to the methods as given on pages 388-394, the principal arrangements of wattimeters employed for measuring the true power in different kinds of alternating current circuits commonly met with in practice, are given here in diagrammatic form.

**SINGLE PHASE**

\[ W = \frac{E_1 I_1 \cos \phi}{W = W_1} \]

Fig. 146.

**TWO PHASE (UNBALANCED)**

\[ W = \frac{E_1 I_1 \cos \phi}{W = W_1} \]

Fig. 147.

**TWO PHASE (BALANCED)**

\[ W = B C \cos \phi \]

\[ W = 2W_1 \]

Fig. 148.

**THREE PHASE (UNBALANCED)**

\[ W = D C \cos \phi \]

\[ W = 3W_1 \]

Fig. 149.

\[ W = \text{the total watts in the system.} \]

\[ W_1, W_2, W_3 = \text{readings of the various wattimeters.} \]

\[ E_1, E_2, E_3 = \text{voltages in various sections.} \]

\[ C, C_1, C_2, C_3 = \text{currents in various sections.} \]
The diagrams and deductions accompanying each explain the principle clearly enough.

In Fig. 150, if the power factor of the system is less than 0.8, one of the wattmeters will read negatively and the connections of its fine wire circuit will have to be interchanged in order to obtain deflections on the scale. In this case the difference of the two wattmeter readings gives the total power.

In Fig. 152, the resistances \( r_1 = r_2 = (r + \text{fine wire coil}) \), but an artificial neutral point can be formed by lamps without the expense of the resistances \( r_1, r_2 \).

In Fig. 156, unless the resistance of the fixed current coil is
small compared with the resistance of the phase in series with which it is connected, its insertion will throw out the balance of a three-phase system and $3N_L$ will not be the total true power. The

**THREE PHASE (BALANCED)**

![Diagram](image)

**THREE PHASE (UNBALANCED)**

![Diagram](image)

Fig. 154.

Fig. 155.

arrangement in Fig. 170, or if the system is balanced one wattmeter with a two-way key for connecting one end of the fine

**THREE PHASE (BALANCED)**

![Diagram](image)

Fig. 156.

wire coil in quick succession to the remaining two mains, is much to be preferred.

Fig. 155 shows a method of connecting two wattmeters in three-phase high tension mains $ABC$ using current and pressure.
transformers. \( P \) and \( K \) are two 20 to 1 series transformers, giving a secondary current of 3 amperes at full load; while \( II \) is a 10 to 1 series transformer, giving a secondary current of 1 ampere at full load. \( T \) and \( S \) are each 100 to 1 pressure transformers, with 10,000 volts on the primaries. It can be shown that each wattmeter indicates 868 kw. at full load, the total power of the circuit being 1732 kw.

**Note.** The above methods are equally applicable to measuring the output of a generator or input into a motor or rheostate.

The measurement of power factor in alternating current circuits can be made by means of power factor indicating...
instruments, or by the method described on page 388, in the case of three-phase circuits. Another method is shown in Fig. 155 for three-phase circuits in which a wattmeter \( W_p \) connected as shown indicates the wattless power in a phase, or the power factor, if the scale be suitably graduated. The instrument in this case has a central zero and deflects to one side or the other according to whether the current lags or leads with respect to voltage.

As with such an arrangement, a considerable P.D. will exist between the fixed and moving coils, it can only be recommended for the lower voltages.

Another and safer method can be employed with balanced three-phase circuits using one wattmeter in one main and a two-way key for connecting one end of its fine wire circuit in quick succession to the remaining two mains.

If \( d_1 \) and \( d_2 \) are the two deflections so obtained, then the power factor of the circuit is:

\[
\text{Power factor of the circuit} = \frac{1}{\sqrt{1 + 3 \left( \frac{d_1}{d_2} \right)^2}}
\]

Corracting Factor for Wattmeters.—Considering the most common form, namely the electro-dynamometer type, used in practice, it is well known that the current through the moving coil should be exactly in phase with the P.D. at its terminals for the instrument to read true values correctly. It is therefore both interesting and important to know the magnitude of the error introduced into the reading of the wattmeter and the correcting factor to be applied to obtain true values when the current and pressure in the fine wire coil are not in phase due to the coil possessing inductance, which it must necessarily have to some small extent. The following considerations are quite general, and assume that the current and voltage are sine functions—

Let
- \( C \) = the maximum current in the fixed coil,
- \( I \) = the current (at time \( t \)) in the fixed coil,
- \( E \sin \omega t \) = potential difference between the mains (at time \( t \)),
- \( \phi \) = angle of lag of the current in the mains,
- \( i \) = the current (at time \( t \)) in the moving coil of self-induction \( L \) and total ohmic resistance \( R \), including any resistance in series with it.
Then the relations given in Fig. 107 can be shown to hold good.

The relations apply to all types of wattmeters if \( \phi \), \( L \) and \( \omega \), the only quantities which vary with the nature of the load and type of wattmeter, are known.

When \( \phi = 90^\circ \) the multiplying factor becomes zero, and the reading of the wattmeter is zero; since there is no force between the coils carrying currents which differ in phase by 90°. The curves (Fig. 107) can be used as follows—Suppose we know that \( L = 0.02 \) henry, \( R = 628 \) ohms, frequency = 50 ~ per sec., and the power factor = 0.6. Then \( \frac{L}{R} = 0.01 \).

Hence curve C is to be used. Now the horizontal line through 0.5 on the power factor scale cuts the power factor curve \( \phi \) at a point, the vertical line through which passes through \( \phi = 60^\circ \) and cuts curve \( \phi \) at 0.98, which is the correcting factor of the wattmeter.

**Fundamental Considerations Relating to Alternating Current Static Transformers.**

**General Remarks.**—Before considering actual methods of testing static transformers, the importance of which, in alternating current systems of distribution of electrical energy, arises from the ease with which a small current at high pressure can be converted to a large current at low pressure or vice versa by such an appliance and with very little loss, some introductory remarks are considered desirable.

There are a great many different forms and ways of building the kind of transformer in question, but they all come under one or other of two main heads, namely—

(a) Those with closed magnetic circuits in which the magnetic induction or lines of force are continued solely, or nearly so, in iron.

(b) Those with open magnetic circuits in which the lines of force run partly in the iron cores of the transformer, and partly in the air through which they complete their path. This type,

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however, has now become practically obsolete. In either case (a and b) the iron core is surrounded by or wound with two distinct and separate coils of insulated copper wire termed the primary and secondary. In all cases the former is the coil connected to the source of supply, while the latter has induced in it an E.M.F. which supplies current to some separate circuit, usually at quite a different E.M.F. to that acting on the primary.

The primary may be either the high tension (pressure) coil or the low, according as to whether the transformer is used as a step-down or step-up appliance respectively. Hence to avoid confusion, the primary will always be that coil which is connected to the source of supply, whether this be high or low tension.

It may now be well to consider certain phrases and quantities met with in static transformers, and which appear in testing work on them. Transformers with "closed" magnetic circuits only need be considered, the "open" magnetic circuit type not having been made for many years. The induced secondary voltage is evaluated as follows—

Let \( N \) = total magnetic flux threading the secondary winding of \( T_s \) turns,
\[ f = \text{periodicity of the primary supply-current, and hence of this flux,} \]
\[ E_p \] and \( E_s \) = maximum values of E.M.F.s at the terminals of primary and secondary.

Now since in one period of the current wave, the current and hence the flux varies from 0—max., max.—0, then reverses and again varies from 0—max. and then max.—0, the average rate of change of the flux = \( 4N \) lines per cycle or period, and the average change = \( 4f \) lines per sec. Therefore the average E.M.F. induced per turn = \( \frac{4Nf}{10^6} \) volts, and therefore the average E.M.F. induced in the secondary winding of \( T_s \) turns = \( E_s = \frac{4NfT_s}{10^6} \) volts. Since the virtual E.M.F. = average E.M.F. × form factor of the voltage wave, the virtual or R.M.S. E.M.F.,

\[ E_s = \frac{4 \times 1.11 N f T_s}{10^6} = \frac{4.44 N f T_s}{10^6} \text{ volts, where } 1.11 \text{ is the value of the form factor of a sinusoidal wave. There will also be an} \]
induced E.M.F. due to self-induction in the primary winding of \( T_1 \) turns, and since the same flux threads this also, this back E.M.F. of self-induction must be \( \frac{4.44 N T_1}{10^8} \) volts. On open secondary circuit the primary supply pressure only exceeds this back E.M.F. by a very small amount, namely, that sufficient to force the energy current through the resistance of the primary winding and provide the necessary magnetizing current for producing the flux in the core. We therefore have the following important relation, namely—

\[
\frac{E_s}{E_t} = \frac{4.44 N T_1}{4.44 N T_2} \times 10^8 = \frac{T_1}{T_2} 10^8
\]

very approximately, which is called the voltage ratio of conversion or ratio of transformation.

If \( I_p \) and \( I_s \) are the currents flowing in the primary and secondary having resistance \( R_p \) and \( R_s \), then the ohmic drop of voltage in each is \( I_p R_p \) and \( I_s R_s \) respectively, and the core flux is produced by an effective voltage \( E_p - I_p R_p \), where the bar over the expression indicates that it is a vectorial—and not an algebraical—difference, the primary supply E.M.F. \( E_p \) and energy voltage, \( A_p R_p \), not being in phase as indicated in Fig. 118.

The no-load secondary induced voltage will therefore

\[
\frac{E_s}{E_t} = \frac{(N_p - I_p R_p) T_1}{T_2} \text{ volts,}
\]

and the secondary voltage on load

\[
\frac{E_s}{E_t} = \frac{(N_p - I_p R_p) T_1}{T_2} + A_s R_s \text{ volts.}
\]

When the secondary circuit is open, the total loss occurring in the transformer is called the open-circuit loss, and the current flowing in the primary is called the no load primary current.

The open-circuit loss is made up of the copper loss due to the no-load current flowing in the primary winding, and which is usually very small compared with the remaining loss due to eddy currents and magnetic hysteresis which are termed the iron core losses.
The no-load primary current, such as would be indicated by an ammeter, consists of two components in quadrature, namely,
(c) the true magnetizing component, which being an idler or wattless current lags 90° behind the supply voltage, and (b) the energy or load component in phase with the supply voltage, and overcoming the above open-circuit losses due to eddy currents, hysteresis, and copper loss.

These three currents can therefore be represented by a right-angled triangle such as Fig. 158, in which BB would be the no-load current, BA the energy component, and CD the magnetizing component. Thus, since $BB = \sqrt{BA^2 + CD^2}$, we see that the no-load current $= \sqrt{\text{energy current}^2 + \text{(magnetizing current)}^2} = \sqrt{I_a^2 + I_m^2}$, and this no-load current would be in quadrature with the supply volt, except for the energy current, which makes the phase difference slightly less than 90°.

The magnetization or core flux, being directly proportional to the supply voltage at constant frequency, is constant at all secondary loads with a constant voltage supply, and hence the iron losses are constant at all loads. Further, since we have seen that $\frac{R_e}{R_s} = \frac{P_r}{P_s}$, it follows that $\frac{A_e}{A_s} = \frac{P_r}{P_s} = \frac{K_e}{K_s}$, i.e., the primary and secondary currents are inversely as the voltages.

The measurements of current, voltage, and power in both connected, not only with transformers, but also with alternating currents generally, should be made with instruments possessing practically no self-induction and little or no iron. The best results will be obtained when employing electrostatic, hot-wire, and d ynamometer instruments, for such measure the \textit{mean} square values of pressure and current and are independent of the variations of frequency. If a circuit supplied with alternating current is \textit{non-inductive}, as for example a bank of electric incandescent lamps run off the secondary of a transformer, then the \textit{mean} square values of the amperes x that of the volts $= \text{true or mean power}$ in Watts taken up by that circuit or bank of lamps.

If, however, the circuit is inductive this product (amps. x volts) gives what is called the \textit{apparent power} in Watts absorbed, which is in all cases greater than the \textit{true power}. This would be the case if we tried to measure the power given to the primary
of a transformer, which is always very inductive. Resource must
in such cases be had to the so-called non-inductive Wattmeter, the
fine wire coil of which must have as few a number of fine wire
turns as will give the requisite sensibility. Such an instrument
will measure the actual or true mean power given to any circuit,
however inductive it is, and no difficulty presents itself in the
use of the Wattmeter on a low tension circuit. If, however, the
power absorbed in a high tension circuit is required, then a special
arrangement of Wattmeter is needed (see p. 42). It is much
better, however, to have all measuring instruments on the low
tension circuit, and this can be accomplished by employing one
of the double conversion methods given in the following pages,
which are almost always possible in works and central stations in
which two similar transformers are required, size and output can
generally be obtained.

Another method of measuring the power given to or developed
by a transformer is the 3-voltmeter one, and in the case of the
primary circuit, a non-inductive resistance of such a known value
is placed in series with this coil, that the P.D. across its termi-
nals = across the primary coil, or preferably as nearly so
as possible, as this gives maximum accuracy. The method con-
sequently has the somewhat serious disadvantage that the E.M.F.
of the supply has to be double that required for the primary
alone, which would in the majority of cases preclude its use.
Then again a small error in observation may cause a large error
in the results.

(136) The Effect on the No-Load Voltage
Ratio, Current, and Watts of a Trans-
former, of Change of Primary Supply
Voltage and Frequency. (Magnetization
Curve or Open Circuit Characteristic.)

Introduction.—The present investigation is a very important
one, in that, amongst other results, it gives the relation between
primary terminal voltage (to core flux at constant frequency)
and magnetizing current, and which is termed the “open-circuit
characteristic” or “magnetization curve” of the transformer.
The voltage used for the relation should, strictly speaking, be that of the back E.M.F. of self-induction, and therefore the vectorial difference $V_p - ABp$; but as both $A$ and $Bp$ are small, their product is negligibly small compared with $V_p$, and can be neglected. Even with the special low-loss iron now used in transformer cores, these are seldom worked at magnetic induction densities outside the limits, 3500 to 7500 lines per sq. cm., in order to minimize the power (due to the iron-loss or energy current component) absorbed in magnetization, which is converted into heat in the core. For this reason the "knee" of the curve, which corresponds to about 15,000 to 17,000 lines per sq. cm., and is all-important in the design of D.C. apparatus, is never reached in the magnetization curve of a transformer.

Further, since the core loss is obtained in this test and is well known to be practically constant at all loads, it follows that, knowing the resistance of the windings, and hence copper losses $(O+P)$ in $P$ and $S$ at any load current, the efficiency can be predetermined at all loads.

The test also shows that both the no-load current and watts decrease as the frequency increases, and hence that higher frequencies reduce the size of core and cost of manufacture for a given output.

**Apparatus.**—Transformer under test, of which $P$ is the primary and $S$ the secondary; low-reading wattmeter $W$; voltmeters $V_p, V_s$; switch $K$; frequency meter $F$; low-reading ammeter $A$; source of supply $E$, preferably a motor-driven alternator, the speed and excitation of which is variable over a wide range.

**Observations.**—With Variable Voltage Supply and Constant Frequency.

(1) Connect up as in Fig. 159, levelling and adjusting such instruments to zero as need it, the terminals $H$ of the high
tension winding used as the secondary being open-circuited as shown.

(2) With the frequency adjusted to the normal value for the transformer, and the field regulator of the alternator full in, close K and take simultaneous readings of $V_p$, $I_p$, $P$, $W$ and $A$ at each of some ten different values of $V_p$, rising by about equal increments from the lowest readable values to not exceeding $20\%$ above normal, by adjustment of field regulation or otherwise, and at constant normal frequency.

(3) With Variable Frequency Supply at Constant Voltage.

With the voltage adjusted to the normal value for the transformer, take simultaneous readings of $V_n$, $I_n$, $P$, $W$ and $A$ at each of some ten different values of $P$, rising by about equal increments between the lowest and highest values convenient, at constant voltage $V_p$.

(4) Measure the ohmic resistances of the primary and secondary windings $P$ and $S$; that of $P$ by either (a) the ammeter voltmeter method (p. 86), using Ohm's law and a direct current supply for $I$, taking care to connect a suitable main current variable rheostat in circuit between $I$ and $P$; or (b) the comparative deflection method (p. 84). The resistance of $S$ may be obtained by either (c) method (a) above mentioned, or (d) a Wheatstone's bridge. In the case of the ammeter-voltmeter method, the voltmeter must be connected to the actual terminals $TP$ or $ST$ of the windings.

Tabulate all your results as follows:

<table>
<thead>
<tr>
<th>Transformer</th>
<th>Type</th>
<th>Make</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal output</td>
<td>$V_p$</td>
<td>$I_p$</td>
</tr>
<tr>
<td>Secondary: volts</td>
<td>$V_n$</td>
<td>$I_n$</td>
</tr>
<tr>
<td>Resistance $R_n$</td>
<td>$\omega$</td>
<td></td>
</tr>
<tr>
<td>Transformer $P_n$</td>
<td>$\omega$</td>
<td></td>
</tr>
<tr>
<td>Primary: volts</td>
<td>$V_p$</td>
<td>$I_p$</td>
</tr>
<tr>
<td>Resistance $R_p$</td>
<td>$\omega$</td>
<td></td>
</tr>
<tr>
<td>Transformer $P_p$</td>
<td>$\omega$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Primary $V_p$</th>
<th>Secondary $V_n$</th>
<th>Amperes A</th>
<th>Apparent Power $W$</th>
<th>True Power $P$</th>
<th>Angle of Lag $\phi$</th>
<th>Power Factor $\cos \phi$</th>
<th>Power Factor Correction $\Delta \cos \phi$</th>
</tr>
</thead>
</table>


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(3) From obs. 2 plot the "open-circuit characteristic" (otherwise known as the "magnetization curve") of the transformer having values of \( V_p \) as ordinates, with magnetizing current \( A_m \) as abscissa.

Also curves having the same scale values of \( V_p \) as ordinates, with (a) total no-load current \( A \); (b) hysteresis component \( A_h \); (c) no-load watts \( W \) (practically all iron-core losses); (d) voltage ratio \( V_p/V_o \), as abscissa, in each case. From obs. 3 plot on another curve-sheet curves having values of frequency \( f \) as ordinates, with \( A \), \( A_h \), \( A_m \), \( W \) and \( V_p/V_o \), respectively, as abscissa.

Interferences.—From a study of the table of results and shape of curves state clearly all that can be deduced.

\( 137 \) Measurement of Copper Losses in a Transformer (by the Short Circuit Test).

Introduction.—The total internal loss \( W \) in any static transformer is made up of the iron loss \( W_i \) due to eddy currents and magnetic hysteresis in the iron core, together with the copper loss \( W_c \) due to the currents \( I_p \) and \( I_s \) in the primary and secondary windings of resistances \( R_p \) and \( R_s \).

Then \[ W_c = A_p R_p + A_s R_s \] and \[ W = W_i + W_c. \]

Knowing \( R_p \) and \( R_s \), the copper loss \( W_c \) can be calculated for any of a series of measured load currents, but the value so found may differ considerably from the actual working or effective value, owing to the eddy current and "skin effect" present with the larger sizes of conductor, when carrying alternating current, causing an apparent increase in the resistances \( R_p \) and \( R_s \). The present test, comprising the direct measurement of the total copper loss, would therefore appear to be a means of obtaining it under working conditions, and hence more accurately than by calculation.

Another source of error may, however, now creep in, for the wattmeter necessary for measuring the loss must obviously be a low-reading one, and have a current capacity equal to that of full load for the winding chosen as primary, while its pressure coil will be subject to a small fraction of what would probably be its normal pressure (a condition introducing an error in its
induction) unless, of course, the wattmeter is a specially designed one for low pressure. The small applied voltage necessary for keeping the short-circuit current within safe limits, will produce a very small induction, and therefore loss due to magnetization of the core. This last named may be negligibly small, when the wattmeter will indicate the copper loss only. If the iron loss is not so small, the wattmeter will give a reading at the applied voltage of short circuit when the secondary is open-circuited, and this reading must be subtracted from all of its indications on short-circuited secondary. Further, care must be taken that the wattmeter reading does not include any losses in connecting or short-circuiting cable. If it does, the loss in such must be separately calculated from their measured resistance and each current, and deducted from the reading.

If \( T_P \) and \( T_S \) = the number of primary and secondary turns respectively, and \( V_r \) = a small supply voltage applied to the primary in order to send full-load current \( I_P \) through it with secondary short-circuited, then the total resistance "drop"

\[
A_r \left( R_r + R_s \left( \frac{T_p}{T_s} \right) \right) \text{ volts.}
\]

From the values of this "drop" and \( V_r \) the characteristic triangle of the transformer can be drawn and the leakage drop determined.\(^1\)

Apparatus.—That for test No. 136, excepting that \( W \) and \( V_r \) must now both be low-reading instruments, while \( V_r \) is replaced by a low-resistance ammeter \( A_s \) for short-circuiting the terminals \( tt \) of the secondary winding \( S \), the range being large enough to indicate at least full-load current of that winding.

Observations.—(1) Connect up as in Fig. 139, with the ammeter \( A_s \) across \( dd \) and the pressure circuit of \( W \) across \( TT \), in order to eliminate errors due to including in the reading of \( W \) any copper loss in the primary connecting cables. Level and adjust to zero any instruments which need it.

(2) With an efficient short circuit of \( S \) the primary \( P \) will practically constitute a metallic resistance and require, by Ohm's law, probably only two or three volts or so to be applied to it in order to obtain full-load current through it.

This low voltage required at the normal frequency of the transformer, and from whatever source obtainable, must be adjusted by a suitable variable resistance in series with $P$, so as to give eight or ten currents through $P$, varying by about equal amounts from its full-load value to the lowest readable, the readings of $V_p$, $W$, $A$ and $A_x$ being noted at each.

3. Disconnect the secondary short circuit and note the iron-loss reading, $w_x$ on $W_x$ for the same value of $V_p$ as used in cls. 2 (if any is readable).

4. Measure the resistance $r_x$ of the short circuit (namely, $A_x$ and its two short connecting leads) and tabulate your results as follows:

<table>
<thead>
<tr>
<th>Transformer: 8A...</th>
<th>Made by... Type... Voltage Ratio...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full load: Output</td>
<td>$E_{P}$, Amps... Volts... Frequency...</td>
</tr>
<tr>
<td>Resistance: Primary ($R_p$)... ohms... Secondary ($R_x$)... ohms...</td>
<td></td>
</tr>
<tr>
<td>Secondary short-circuited ($r_x$)... ohms...</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Frequency (c.p.)</th>
<th>Total Volts ($V_t$)</th>
<th>Phase Factor</th>
<th>Phase Angle</th>
<th>Power Factor</th>
<th>Clock Drop</th>
<th>Core Loss</th>
<th>Actual Copper Loss</th>
<th>Actual Copper Loss</th>
</tr>
</thead>
</table>

5. Plot a curve having values of copper loss in $V_p$ as ordinates, with $A$ as abscissae.

**Note.**—The impedance voltage $V_p$ is entirely spent in overcoming the equivalent impedance of the windings with short-circuited secondary, being partly spent in overcoming resistance and partly in reactance.

The reactance voltage $= \sqrt{V_p^2 - (W_pA_p)^2}$. 
Deduction of the Regulation of a Transformer for any Load and Power Factor from the "Open" and "Short Circuit" Tests.

From the curves obtained in the preceding open and short-circuit tests, the drop in volts in a transformer on non-inductive or inductive secondary load can be predetermined. To obtain this drop is needed the "open-circuit" volts and the triangle of voltages relating impedance voltage, ohmic drop or resistance voltage, and the reactance voltage as obtained from the "short-circuit" test.

The voltage drop in the transformer for any load and power factor can thus be obtained from an exactly similar construction to that given on p. 182 for an alternator, and which will not, therefore, be repeated here.

(38) Determination of the Regulation of a Static Transformer. (Differential Method.)

Introduction.—The meaning of the term "regulation," as applied to a transformer, was explained and defined in text No. 139, p. 413, and its measurement in a single transformer there given. When, however, two similar transformers \( T_1, T_2 \) are available, the present method of measurement is both simple, convenient, and direct reading, whereas that of text No. 139 necessitates taking the difference between two voltages, and is less accurate. It is applicable to any pair of high-tension or low-tension transformers, but the secondary circuit should preferably be the L.T. side, on account of the greater safety in handling instruments at low tension.

Apparatus.—Source of A.C. supply \( E \), whether low or high tension; two transformers \( T_1, T_2 \) similar in all respects; voltmeter \( V_2 \); switches \( S_1, S_2 \); load resistance \( R \); ammeter \( A_1 \); and (if available—for interest, but not as a necessity) two ammeters \( A_{12} \) and \( A_{22} \).
Observations. — (1) Connect up as in Fig. 100, levelling and adjusting to zero such instruments as need it.
(2) First connect for $V_s$ a voltmeter capable of reading (or glow lamps capable of absorbing) the sum of the normal voltages $N_s N_s$. Then with $N_s$ and $S_s$ open, and $E$ giving the normal voltage and frequency of $T_2$ or $T_3$ close $S_2$. If $V_s$ shows a fairly large voltage, $N_s$ and $S_s$ are in helping series, and the connections of one of them must be interchanged to bring their voltages into opposing series, when $V_s$ will show very little.
(3) Now replace $V_s$ by a low-voltage voltmeter, and, with $N_s$ closed ($S_s$ still being open), note the readings of $A_{m}$, $A_{fr}$ and $V_s$ (if any). If $T_2 T_3$ are either exactly similar, or unloaded, or both, $V_s$ should now read 0.

(4) With the supply voltage constant, and $K$ non-inductive and full in close $S_2$, taking the readings of all the instruments for each of a series of six or eight load currents $I_m$ rising by about equal increments from 0 to the full-load current of $T_2$ or $T_3$.

Note. — $V_s$ gives the difference of the voltages between the terminals of the loaded ($T_2$) and unloaded ($T_3$) transformers, which is the required "drop."

The secondary output of transformers is usually expressed in kilo-volt-amperes (K.V.A.), irrespective of the power factor of the secondary circuits, and not in true K.W. at unity or some lower P.F. The value of $V_s$ may, however, be obtained if desired on inductive loads by repeating obs. 4 with a variable choker ($r$) (not shown), connected in series with the non-inductive resistance $R$ (preferably a bank of lamps) and a voltmeter with key to measure the volts $V_s$ across $R$ and $V_s$ across the choker at the same current, when the power factor of the circuit will be

$$\cos \theta = \frac{R}{\text{Impedance}} = \frac{V_s}{V_s'}.$$
Tabulate your results as shown—

<table>
<thead>
<tr>
<th>Currents in Winding</th>
<th>Secondary Load Current</th>
<th>Voltage Drop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{A_1}{A_2}$</td>
<td>$\frac{A_2}{A_2}$</td>
<td>$V_D$</td>
</tr>
</tbody>
</table>

(5) Plot curves having values of $V_D$ as ordinates, with values of $A_2$ as abscissae.

(139) Determination of the Efficiency and Regulation of Transformers. (Single Conversion Method.)

Introduction.—This method is one of the simplest, though not the most accurate, and entails using only one transformer to be tested. In all cases by the primary of the transformer is meant that winding connected to the supply mains whether these are at high or low tension.

The efficiency of any transformer, supplied at constant voltage and frequency, is the ratio of the secondary output to the primary input, or $\frac{W_2}{W_1}$.

The regulation of a transformer is the amount by which the secondary terminal voltage at any secondary load differs from that on open secondary circuit, i.e., it is the "drop" in voltage under load, and is due to both the resistance and reactance of the winding.

The regulation curve therefore relates secondary terminal voltage as ordinates and secondary load current as abscissae. The ordinate is the ratio between this curve and a horizontal straight line through the "open circuit" voltage point at any secondary load gives the "drop" of voltage at that load.

Caution.—In the case of transformers which have to be run off a high tension alternator and are tested by this method, the one operating the high tension instruments must not only wear a pair of carefully selected and good India-rubber gloves, but must also stand on an India-rubber mat, and to guard against the possibility of accidents even in the case of manipulating the high tension instruments, the one operating these must either wear the pair of India-rubber gloves provided or stand on an India-
rubber mat. Before switching on, the "danger boards" provided must be placed close to the high tension wires.

Note.—Great care must be taken that the india-rubber gloves are not scratched, cut, or pierced in any way, as this would tend to render them useless for the purposes of insulation.

Apparatus.—Alternating current ammeters $A_1 A_2$ (Fig. 251), and voltmeters $V_1 V_2$; non-inductive Wattmeters $W_1 W_2$ (with their separate anti-inductive resistances $r_1 r_2$, if any); load absorbing resistance $R$, preferably non-inductive (p. 698); switches $S_1 S_2$; source of alternating current supply and transformer $T$ to be tested.

Note.—$V_1$ and $V_2$ should be either hot-wire or electrostatic instruments, of which $V_2$ may preferably be of the latter type. If $R$ is strictly non-inductive, then $W_2$ could be dispensed with; it may, however, as well be used if available.

Tests.—(1) Measure the ohmic resistances $R_1$ of primary and $R_2$ of secondary coils in a suitable manner.

(2) Connect up the apparatus as indicated in Fig. 161, carefully levelling such instruments as need it, and seeing that their pointers are at $o$. Adjust the voltage and frequency (if possible) of the supply to the normal value required for the transformer.

(3) With $S_2$ open, close $S_1$ and note the readings of $A_1$, $V_1$, and $W_1$ simultaneously. The "open circuit losses" occurring in the transformer will thus be obtained.

(4) Make $R$ large, and close $S_2$ as well as $S_1$. Then note simultaneously the readings of all the instruments for about ten different secondary currents from 0 to full load or to 15% over full load, rising by about $x$ instruments.
In all cases the frequency and primary voltage must be kept constant.

(5) Repeat 4 with a higher and lower frequency than the normal.

(6) Repeat 4 and 5 on an inductive load, of constant power factor, or otherwise obtain the readings, as detailed on p. 182, necessary for plotting the regulation curves between secondary volts and current, each at different but constant power factors.

(7) Find, experimentally, the copper losses in primary and secondary by passing direct currents of various strengths, between 0 and full load, through the coils, and noting the losses by means of a Wattmeter.


<table>
<thead>
<tr>
<th>NAME ...</th>
<th>DATE ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer No. ...</td>
<td>Make by ...</td>
</tr>
<tr>
<td>Reactance ...</td>
<td>Kilovolts Phase ...</td>
</tr>
<tr>
<td>Frequency ...</td>
<td>Volts Sec. 750 ...</td>
</tr>
<tr>
<td>Primary Resistance R1 ...</td>
<td>Ohms at 75°C. Secondary Resistance R2 ...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Primary Watts</th>
<th>Secondary Watts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Volts</td>
<td>Secondary Volts</td>
</tr>
<tr>
<td>Primary Res. 1</td>
<td>Secondary Res. 2</td>
</tr>
</tbody>
</table>

Frequency used = ... Vols. per sec. Total Res. for high = ... Vols. |

(8) Plot the following curves having values of—(a) Total copper losses; (b) total iron losses; (c) secondary voltage; (d) primary power factor; (e) efficiency; (f) voltage ratio, respectively as ordinates and secondary load currents as abscissae in each case.

Inferences.—State concisely all the inferences which you can draw from the results of your experiments.

(140) Determination of the Efficiency of Transformers. (Double Conversion Method.)

Introduction.—This method can be used when two similar transformers are at hand, and particularly when only low tension measuring instruments are available.

Caution.—To guard against the possibility of accidents, even in the case of manipulating the low tension instruments, the pair
of India-rubber gloves provided must be worn by the one manipulating the tertiary circuit instruments, and the India-rubber mat must be used by the one reading those on the primary circuit. On no account must any part of the secondary (high tension) circuit be touched while "alive," and before switching on the primary current, the "danger boards" provided must be placed close to the high tension loads.

Note.—Great care must be taken that the India-rubber gloves are not scratched, cut, or pierced in any way, as this would tend to render them useless for the purposes of insulation.

Apparatus.—Alternating-current ammeters $A_1, A_2$ (Fig. 321), and voltmeters $V_1, V_2, V_3$, of which $V_2$ is not absolutely essential to the test; non-inductive Wattmeters $W_1$ and $W_2$ (with their separate anti-inductive resistances $r_1, r_2$, if any); switches $S_1, S_2, S_5$; load absorbing resistance $R$, preferably non-inductive (p. 598); source of alternating current supply and the two transformers $T_1, T_2$ to be tested.

Note.—Both $V_2$ and the high tension voltmeter $V_3$ should, if possible, be of the electrostatic type. If $R$ is strictly non-inductive, then $W_2$ can be dispensed with; it may, however, as well be used if available.

Tests.—(1) Measure the ohmic resistance of each of the coils of the transformers $T_1$ and $T_2$ in a suitable manner.

(2) Connect up the above apparatus as indicated, carefully leveling such instruments as need it, and seeing that their pointers are at zero. Adjust the voltage and frequency (if possible) of the supply to the normal value required for the transformers.
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(3) With \( S_4 \) and \( S_5 \) open, close \( S_3 \) and note simultaneously the readings of \( \Delta_2 \), \( \chi \), \( \phi \). The "open circuit losses" occurring in transformer \( T_1 \) will thus be obtained.

(4) Make \( R \) large and close all the switches. Then note simultaneously the readings of all the instruments for about ten different tertiary currents from 0 to full load, rising by about 1 increments.

(5) Interchange \( T_1 \) and \( T_2 \) so that the latter now becomes the "step-up," and repeat exp. 3 and 4, tabulating your results in two tables similar to that shown.

Note.—In all cases the frequency and secondary voltage must be kept constant.

(6a) Repeat 3—5 for a higher and lower frequency than the normal.

(6) Find, experimentally, the copper losses in each of the coils by passing direct currents of various strengths between 0 and full load through them, and noting the losses by means of a Wattmeter.

<table>
<thead>
<tr>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer ( T_1 ) No.</td>
</tr>
<tr>
<td>Type</td>
</tr>
<tr>
<td>Made by</td>
</tr>
<tr>
<td>Used as</td>
</tr>
<tr>
<td>( T_2 )</td>
</tr>
<tr>
<td>Normal Output</td>
</tr>
<tr>
<td>Kilowatt</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>Change ratio</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Primary</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
</tr>
<tr>
<td>( W_{1p} )</td>
</tr>
<tr>
<td>( k_2 )</td>
</tr>
<tr>
<td>( W_{2p} )</td>
</tr>
<tr>
<td>( k_n )</td>
</tr>
<tr>
<td>( W_{np} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tertiary</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_1 )</td>
</tr>
<tr>
<td>( \chi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \chi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Copper Losses in Watt</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
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<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Efficiency of</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
<tr>
<td>( \phi )</td>
</tr>
</tbody>
</table>

(7) Plot the following curves having values of—(a) Total copper losses; (b) total iron losses; (c) tertiary voltage; (d) power factor; (e) efficiency respectively as ordinates and tertiary load currents as abscissae in each case.

Inferences.—State clearly all the inferences which you can draw from your experimental results.
Efficiency of High Tension Transformers. (Sumpner's Differential Method.)

Introduction.—A neat and convenient method of measuring the efficiency of high tension transformers, and which is susceptible of greater accuracy than most methods, is that due to Dr. W. H. Sumpner, and detailed as follows:—A small auxiliary transformer (C), the output of which need not be greater than the waste of power occurring in the two larger transformers A and B, to be tested, at full load, is required for the purpose of furnishing a small extra voltage (say 3 to 12 volts) necessary for driving the full load or any other currents through A and B. Its efficiency, goodness, or badness is a matter of indifference, and all we need in connection with it, is the output (w) of its secondary in Watts as measured by the Wattmeter (W).

The particulars as regards this output can be deduced as follows:—Suppose that two 2250 Watt transformers have to be tested each converting from 100 to 2000 volts or vice versa. Then their probable efficiency would be about 94% (say), and hence each would absorb 33.5 x 6 = 135 Watts at full load. Consequently the output of the auxiliary transformer C need not exceed 2 x 135 or 270 Watts.

Hence if used on low-pressure 100 volt mains the primary should take about 2.7 amps. at 100 volts, and the secondary give out 276 amperes at 12 volts. In the ordinary test of efficiency of high tension transformers, in which two similar ones are used—one as a step-up from the low-pressure primary mains, and the other as a step-down to the tertiary mains—the efficiency is deduced from measurements of primary input and tertiary output which are of nearly equal magnitude. Consequently the percentage error in measuring these two quantities reappears as the same percentage error in the efficiency so obtained.

The present method, which is much to be preferred of the two, consists in actually measuring the losses (w) occurring in the two transformers directly, and comparing these with the input to obtain the efficiency.

The method is economical in cost of energy used, especially when testing large transformers; with the methods used in tests
139 and 140 it would be a serious consideration, while the supply of full-load current would make a serious demand on a public supply, or necessitate a large loading alternator. The present method is accurate because the total loss (v) in the two transformers is obtained by adding together two quantities, and not by subtracting them, and is most convenient for finding the temperature rise after a run of a prescribed number of hours on full load.

The principle of the present method is strikingly analogous to Dr. Hopkinson's combined efficiency test of a pair of dynamos, the distinguishing feature of which is to couple two similar machines together both mechanically and electrically, one to run as a dynamo and the other as a motor. Energy is supplied to one by which it is transferred to the other, this latter returning it again to the source; the balance of energy supplied actually by the source is therefore equal to the waste which occurs in the double transformation and corresponds with the loss (v) above mentioned. This then is what takes place in the present case, for energy is taken from the mains by the "step-up" (A or B, whichever is used as such), then transferred to the "step-down" transformer, and finally back to the mains again.

Thus, while both transformers can be loaded to any extent by controlling the current circulating between them, the power taken from the supply is only some 4 to 20% of the full-load K.W. capacity of either, depending on their efficiency—being only that necessary to make up the total internal losses in the two transformers together. Whether the L.T. or H.T. windings are connected to the supply is merely a matter of convenience depending on which supply is available, but usually the L.T. sides are connected to an L.T. supply for safety in handling the more commonly available low-tension instruments, etc. Coiling whichever are connected to the supply the primaries the secondaries must be so connected in series that their E.M.F.s oppose each other. If the primary E.M.F.s are equal, so also will be the secondary E.M.F.s, and no current will flow in the secondary windings. By making the small auxiliary transformer in series with one of the primaries provide a +" or "+" boosting E.M.F., the out-of-balance primary E.M.F.s so produced will cause out-of-balance secondary E.M.F.s and a circulating secondary
current, the strength of this current depending on the difference between the E.M.F.s. Should the connections be such that the secondary E.M.F.s are in helping series instead of opposing series, as they should be, a short circuit will result. To avoid this and to ensure the connections of the secondaries being correct, close \( S \) and \( S' \) when \( B \) will induce a voltage in the L.T. winding of \( A \) equal to that of the supply, but opposite in phase if the connections are correct. Hence, if either a voltmeter or lamps, each having a voltage range equaling twice that of the supply, are connected across the open switch \( S' \) and neither show any voltage, the connections are correct for the two L.T. windings, and therefore also the two H.T. windings are then in opposition. If otherwise, the voltmeter or lamps will show twice the voltage of the supply. In this event the connections of one of the H.T. secondaries must be reversed. It should be noted that \( q \) will indicate the load current, while an ammeter \( q_1 \) in series with \( S' \) will give the magnetizing current.

If \( W = \) load in Watts supplied to the primary of the "step-up," and \( w_1, w_2 = \) the Watts at this load as measured by \( w_1 \) and \( w_2 \), and \( \lambda = \) loss of power in the connecting leads, current meter \( n \) and the current coil of \( W' \), then the total loss in the two transformers

\[
10 - w_1 + w_2 - \lambda.
\]

Hence the efficiency of double conversion \( = 1 - \frac{10}{W} \)

and the efficiency of either transformer \( = \sqrt{1 - \frac{10}{W}} \).

As the ratio of \( w \) to \( W \) is small—not greater than \( \frac{1}{4} \) with a transformer of 95\% efficiency, the efficiency of each transformer is given quite accurately enough by the relation

\[
X = 1 - \frac{10}{W} - \frac{1}{3} \frac{w}{W^2}.
\]

An error of 10\% in estimating \( W \) only affects the combined efficiency to 1\%, and that of either transformers to \( \frac{1}{3} \) only. Hence can be seen the superiority of the present method over the preceding one. The quantity \( W' \) can be obtained with quite
sufficient accuracy by the product of the current $A$ and the P.D. in volts supplied to the primary of the "step-up."

If $(A)$ is the "step-up," then $W = 100 \times$ current in low tension coil of $A$, whereas if $B$ is the "step-up," the power returned to the mains by $A$ is the above quantity, and hence the input of the primary of $B$ is

$$W = 100 \times \text{current of } A + (w_1 + w_2 - \lambda).$$

That transformer will be acting as "step-up" which has the higher P.D. of the two ($A$ and $B$) on its low tension coil. If, say, 12 volts are supplied by $C$ to the primary of $B$, the P.D. at the terminals of $B$ will be either 112 or 88 volts according as the 12 volts from the auxiliary and the 100 of the mains are in phase or opposite phases. If $R$ was very inductive, the above voltages would be out of phase, and $B$'s P.D. might be anything between 88 and 112 volts.

Apparatus.—The two high-tension transformers $A$ and $B$ to be tested are 3000/100 volts; an auxiliary Boosting one $C$, the primary of which is in series with a variable non-inductive resistance $R$ of sufficient range to produce only a few secondary volts; two non-inductive Wattmeters $W_1$, $W_2$; Siemens electro-dynamometer or direct reading alternating current ammeter $(a)$; switches $S_1$, $S_2$, $S_3$ and $S_4$; voltmeter $V$ for maintaining the mains at 100 volts; alternator $D$, or some other source of alternating current.

Caution.—On no account whatever is the high tension circuit of either $A$ or $B$ to be touched while "alive." The india-rubber gloves and mat must be used by the operators to ensure immunity from accidental shock, or break-down of the insulation between primary and secondary of $A$ and $B$.

Observations.—(2) Connect up as in Fig. 165, and adjust the instruments to zero where necessary. See that all humidifying arrangements are working properly, also that the gloves are in good order and the mat suitably placed.

(2) Measure the losses due to the resistance of the leads and instruments employing alternating currents, which can be done without altering any of the connections thus—with $S$ and $S_1$ open, short circuit the primaries of $A$ and $B$ and close $S_2$, $S_3$.

(3) Vary $R$ so as to obtain about six different currents through $(a)$ from 0 to the full load current of $A$ or $B$ by causing the secondary voltage of $C$ to vary suitably. Note the corresponding
readings on \( W_1 \) (the power given out by the secondary of \( C_1 \)), which therefore at once gives the required losses in Watts in leads and instruments for each particular current passing through them.

4. Measure the copper losses in the coils of the two transformers \( A \) and \( B \) by opening \( S_3 \) and closing \( S_1 \) and \( S_2 \) and observing the readings of \( W_2 \) for some six different currents from 0 to the maximum of \( A \) and \( B \), as read off on \( a \).

5. Measure the iron or core losses in the two transformers \( A \) and \( B \) by closing all the switches and observing the reading of \( W_2 \).

6. With \( S_3 \) and \( S_2 \) open, close \( S_1 \) and \( S \) and note the readings of \( a \) and \( W_2 \). Then \( a \) will indicate twice the no-load current of either transformer \( A \) or \( B \) and \( W_2 \) twice the no-load losses in either.

Tabulate your results as follows:

<table>
<thead>
<tr>
<th>Transformer and setting up: No. ... Type ... Model: ( a = \ldots a \ldots ) ... Others ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. ... Output: Volts ... Amps. ... Occasion of measurement: ( \ldots \ldots \ldots ) ...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Data ...</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Brand ...</th>
<th>Frequency ...</th>
<th>Current ...</th>
<th>Total Losses ...</th>
<th>Efficiency ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage ...</td>
<td>Power factor ...</td>
<td>Total Losses ...</td>
<td>Conductors' type ...</td>
<td>Total Losses ...</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
(142) Measurement of the Efficiency of ordinary Single-Phase Transformers by Blakesley's 3-dynamometer Method.

Introduction.—This method necessitates the use of two ordinary Siemens electro-dynamometers, in which of course the moving coil is in series with and carries the same current as the fixed coil, whereas the angle of torsion is proportional to the square mean value of the alternating current, and in addition the use of a third Siemens electro-dynamometer, arranged so that the moving coil has its own separate terminals, and is not in series with the fixed coil.

If then two alternating currents of equal period, from either the same or different sources, flow through the two independent coils, the periodic time of oscillation of the moving coil being very large compared with that of the current, the angle of torsion is proportional to the square mean product of the simultaneous instantaneous values of current throughout the period, and is called the "split dynamometer" reading.

If $A_0$ and $A_1$ are maximum values, and $A_1$, $A_1'$ the mean values of two simple periodic alternating currents, one of which lags behind the other by an angle $\phi$, then the ordinary dynamometer will give $A = \frac{1}{2} A_0 A_1$ and $A'^1 = \frac{1}{2} (A_1)^2$. On passing these currents through the split dynamometer its reading $\phi$ would be $\frac{1}{2} A_0 A_1 \cos \phi$, and hence $\cos \phi = \frac{A'}{\frac{1}{2} A_0 A_1}$.

The following method is quite general, and does not assume that the current is a simple sine function of the time, but does
assume that there is no magnetic leakage, i.e. that the number of lines cutting the primary and secondary are the same. This is not true in all types of transformers on full load, but is nearly so in closed magnetic circuit types.

Since the split dynamometer gives no reading on an open secondary circuit, this method is useless for determining the "open circuit" losses.

Apparatus. — Two ordinary Siemens electro-dynamometers $A_1, A_2$, and one split dynamometer ($A$); transformer $T$ to be tested; non-inductive resistance $L$ (such as a bank of lamps to take up the secondary load) (p. 158); alternator $D$; switches $S_1, S_2$; voltmeters $V_1, V_2$; non-inductive Wattmeter $W$ inserted merely for the purposes of comparison.

Observations. — (1) Connect up as in Fig. 164, and adjust the instruments to zero where necessary. See that all lubricating cups in use feed slowly and properly, then start $D$.

(2) $S_2$ being open, close $S_1$, and adjust the speed and excitation so that $V_1$ reads the normal voltage required for the primary at the normal frequency of the transformer. Note the readings of $A_1, V_1$ and $W$.

(3) Close $S_2$ and adjust $L$ so that $A_2$ reads about $1/2$ of the maximum secondary current, the voltage $V_1$ being kept normal by varying the excitation. Now note the readings of $A, A_1, A_2, V_1, V_2$ and $W$.

(4) Repeat 3 for about 10 secondary load currents, rising by
about equal increments to the maximum allowable, and tabulate as follows—

<table>
<thead>
<tr>
<th>NAME</th>
<th>TYPE</th>
<th>DATE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer tested: No.</td>
<td>Primary turns</td>
<td>Make</td>
</tr>
<tr>
<td>Secondary</td>
<td>Rectifying</td>
<td>&quot;C.</td>
</tr>
<tr>
<td>Normal</td>
<td>Voltage</td>
<td>&quot;C.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Speed, Horse Power</th>
<th>Transformer Rating</th>
<th>Amperage</th>
<th>Power Factor</th>
<th>Line Volts</th>
<th>Secondary Volts</th>
<th>Primary Volts</th>
<th>Current at full load</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>600</td>
<td>100</td>
<td>0.8</td>
<td>230</td>
<td>75</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>300</td>
<td>900</td>
<td>120</td>
<td>0.9</td>
<td>240</td>
<td>90</td>
<td>300</td>
<td>120</td>
</tr>
<tr>
<td>400</td>
<td>1200</td>
<td>150</td>
<td>0.85</td>
<td>250</td>
<td>120</td>
<td>400</td>
<td>150</td>
</tr>
</tbody>
</table>

(5) Plot curves having values of \( V' \) as abscissae, with efficiencies and \( V'' \) as ordinates.

(143) Measurement of the Efficiency of Multi-phase Alternating Current Transformers.

Introduction.—The determination of the efficiency of ordinary single-phase transformers has already been fully considered in the preceding pages.

The present test does not differ materially in principle from those in question, and practically the only difference is in the method of measuring the power absorbed and developed by the multiphase transformer, and which possesses some characteristic differences from that used in the case of the ordinary single-phase transformer.

Most of the preceding methods are equally applicable in the present case whether the transformer is of the two or the three phase type. The reader should refer to p. 388 for the method of measuring electrical power in two and three phase alternating current circuits, where a more detailed description of them will be found. If in the present instance, as in fact with any others, the rectifiers or circuits in which the load is to be absorbed are strictly non-inductive, i.e. are of the nature of incandescent lamps or water rheostats, then providing such load-absorbing devices operate equally on each of the sections of the circuit, they main-
taining a balanced system, the output can quite accurately 
be obtained from the ammeter and voltmeter readings in 
the manner set forth on pp. 383 et seq.

For the present test we will assume that the efficiency of a 
3-phase transformer is required by any, the single conversion 
method.

Apparatus.—The 3-phase transformer to be tested, of which P 
is the primary winding and S the secondary shown in Fig. 165, 
with the star or open winding; two non-inductive Wattmeters 
W_1 and W_2; three Darr's direct-reading ammeters 
A_1, A_2, and A_3 (p. 572); three voltmeters V_1, V_2, and V_3; 3-phase 
variable rheostat R (non-inductive) capable of operating equally 
on each line (p. 510); source of 3-phase current E; two 3-throw 
switches S, S, S and S, S, S,

Note.—If the 3-phase rheostat R is not non-inductive, then 
two additional Wattmeters will be necessary in the secondary 

![Circuit Diagram](image)

![Fig. 165](image)

circuits connected up in precisely the same way as those shown in 
the primary circuit, the secondary output being then given by the 
sum of their readings at any particular load.

Observations.—(1) Connect up as in Fig. 165, and adjust all 
the instruments to zero, labelling such as require it. See that 
all lubricating cups in use feed slowly and properly if the source 
of 3-phase current supply E is controllable.

(2) With S, S, S open, close S, S, S, and adjust the speed of the 
generator so as to give the proper periodicity for the transformer
and then the excitation, so as to have the desired voltage, shown by $V$ across the primary.

Note the respective Wattmeter readings $W_p$ and $W'_p$, and if possible that of $A$ in addition to $V$. Then $(W_p + W'_p)$ are the no-load primary losses + the magnetizing losses.

(3) With $R_a$ at its maximum, close $S, S', S''$, and note the readings of all the instruments for some ten or twelve secondary load currents from the smallest to the maximum permissible, rising by about equal increments at a time for constant secondary voltage.

(4) Calculate the secondary loads ($W_2$) from the relation

$$W_2 = \sqrt{3} A_1 V_1 = \sqrt{3} A_2 V_2$$

and tabulate as follows—

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer Test: $p_0$</td>
<td>$p'_0$</td>
</tr>
<tr>
<td>Rated: Volt</td>
<td>Amps</td>
</tr>
<tr>
<td>Resistor: Each Primary coil $q_1 = ...$ ohm; $q_2 = ...$ ohm.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Primary Circuits</th>
<th>Secondary Circuits</th>
<th>Total Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>$I_{1p}$</td>
<td>$P_{1p}$</td>
</tr>
<tr>
<td>$V_2$</td>
<td>$I_{2p}$</td>
<td>$P_{2p}$</td>
</tr>
</tbody>
</table>

(5) Measure the resistance of the transformer coils by means of either the Wheatstone bridge or Potential Difference method.

(6) Plot the following curves between—

- Efficiencies $\eta$ as ordinates and secondary loads ($\sqrt{3} A_2 V_2$) as abscissae.
- Power Factor as ordinates and secondary loads ($\sqrt{3} A_2 V_2$) as abscissae.

Inferences.—State clearly all that can be inferred from your experimental results.
(144) Efficiency of a Nodon Valve Electrolytic Rectifier.

Introduction.—The necessity of obtaining continuous current for certain purposes, such as electrolytic work and the charging of secondary cells, where, frequently, the only available public supply is alternating current, has led to the introduction of rectifiers for rectifying alternating into continuous or unidirectional current.

Of such appliances, there are now several commercially successful forms; that known as the nodon valve consists of as many pairs of cells grouped according to the Leo Gratz method (Fig. 166) as there are phases of current or distributing mains, in order to obtain a single rectified current. Each cell consists of plates formed of an alloy, mainly composed of aluminium, acting as cathode, immersed in a solution of borate or phosphate of ammonium or other salt formed from tartaric, acetic, oxalic or gallic acids. The solution is capable of rapidly altering the condition of the polarizing film formed by an alternating current on the aluminium. The containing cell is made of lead and constitutes the anode. The electrolytic action taking place is as follows—

In one half period of the alternating current, current tends to flow from aluminium to lead, but cannot, owing to an insulating film of very high resistances being formed over the aluminium (cathode) plate. In the next half (reversed) period, the current actually is able to flow from lead to aluminium owing to the instantaneous de-polarization or reduction of the film on the aluminium plate. The principle on which both semi-waves of the period of an a.c. supply are utilized, is that proposed by Leo Gratz, and shown in Fig. 166, for a single phase alternating—to direct—current transformation. A and B are the aluminium and lead plates respectively of the four cells I, I, and II, II. The continuous arrows represent the direction of current in the valve during one half of a period when the current of the alternating supply flows from P to R. The dotted arrows show the direction of current in the valve in the next half period when the supply current flows in the reverse direction from
Thus for the first half period it is blocked in cells \( R \), \( P \), and in the second half period it is blocked in cells \( I \), \( I \). A unidirectional current therefore always flows from \( D \) to \( C \) through any load \( r \) whether motor, secondary cells or resistance, etc. To obtain greater constancy or uniformity of d.c. voltage a suitable condenser can be connected across \( D \) and \( C \). A single valve will stand a.c. pressures up to 140 volts between \( Q \) and \( R \), that between \( D \) and \( C \) being about 90\% of this. For higher a.c. pressures two or more valves may be combined, or an "economy coil" type of transformer connected between valve and a.c. supply. The pressure between \( D \) and \( C \) may be varied to any extent by a corresponding variation of that of the supply between \( Q \) and \( R \).

The temperature of the electrolyte must not be allowed to rise much above 60\° C., and in large valves, forced air draught around the cells is resorted to in order to keep down the temperature. The valve may be used on any periodicity employed in practice up to 100 ~ per sec. or more. A starting resistance \( S \) must be employed with the valve when this has been out of use for a few hours, in order to reform the insulating pellicle on the aluminium plate. This only takes a few seconds and prevents a sudden heavy rush of current through the valve. The resistance or inductance of \( S \) is cut out entirely afterwards.

Evaporation of the solution is made up by adding distilled water and the solution need only be renewed at long intervals.

**Apparatus.**—The nodon valve complete; starting resistance \( S \); alternating current ammeter \( A \), voltmeter \( V \), wattmeter \( W \); direct current ammeter \( a \), voltmeter \( v \); load or variable resistance \( r \); thermometer; source of a.c. supply; economy coil or transformer if a.c. supply exceeds 140 volts; switches \( S_1, S_2 \).

**Observations.**—(1) Connect up as in Fig. 166, and adjust all the instruments to zero, levelling such as require it. \( Q \) and \( R \) are the terminals marked ALT on the valve, and are to be connected to the a.c. supply; \( D \) and \( C \) are the terminals marked + and _. If machinery is being run for supplying the valve, see that all oil cups feed very slowly and properly.

(2) With \( S_1 \) and \( S_2 \) open and \( B \) full in, adjust the a.c. supply so that \( V \) reads about 140 volts, the periodicity being kept constant at normal value. Now close \( S_3 \), and note the readings of all the instruments.
(3) With $S_1$ still open, gradually cut out $S$ to short circuit and again note all instrumental readings and the temperature of the electrolyte.

(4) Re-insert $S$ and with $(r)$ full in, close $S_2$ and gradually cut out $(S)$ to short circuit. Next adjust $(r)$ so that $(v)$ reads about $\frac{1}{4}$th full output current and note the readings of all the instruments.

(5) Readjust $(r)$ so as to obtain some ten different load currents on $(v)$ rising by about equal increments to the maximum for which the value is intended, and note the temperature and readings of all instruments at each.

(6) Repeat (5) for a widely different but constant periodicity (if available) above and below normal at the same voltage if possible.
(7) Repeat (6) for a constant supply voltage, say 50% less than before, at normal periodicity.

(8) Open \( S_1 \) and at constant normal periodicity, note the readings of all the instruments for ten different voltages between 0 and 140 volts.

(9) With a convenient constant supply voltage and \( S_2 \) open, take readings of all the instruments for ten different periodicities, ranging from the maximum obtainable downwards.

(10) Repeat both (8 and 9) for \( S_2 \) closed, constant full load being maintained on (a) by varying (c), and tabulate all results as follows—

<table>
<thead>
<tr>
<th>Name</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of coils</td>
<td>Area of coils</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value of ( \phi )</th>
<th>Temp of Heatsink</th>
<th>Current ( q )</th>
<th>Voltage (V)</th>
<th>Moving Coil B.</th>
<th>Test V.</th>
<th>App. Power ( PQ )</th>
<th>Power Factor</th>
<th>App. Volts ( V )</th>
<th>App. Watts ( W )</th>
<th>Voltage Ratio</th>
<th>Efficiency ( \eta )</th>
<th>Value of ( \phi )</th>
<th>Value of ( \psi )</th>
</tr>
</thead>
</table>

**Note.**—If the valve is cooled by forced air draught, the power absorbed in producing the draught must be added to the true watts \( (W) \), or watts \( (W) \), according to whether it is supplied by the primary or secondary circuit respectively.

(11) Plot curves on the same sheet, having values of—power factor; volts \( (V) \); efficiency; and voltage ratio as ordinates, with secondary load \( (\phi) \) as abscissae; also between efficiency as ordinates and temperature as abscissae at constant secondary load.

(12) Plot curves (for Exp. 8 and 10) with voltage as abscissae and the other quantities as ordinates; also (for Exp. 8 and 10) with periodicity as abscissae and the other quantities as ordinates.

**Inferences.**—State clearly all the inferences deducible from experimental results.