SECTION 2

DESIGN OF STEEL AND CAST-IRON MEMBERS

STEEL SHAPES AND PROPERTIES OF SECTIONS

1. Steel Shapes.—The steel used in structures is in the form of single pieces, or combinations of two or more pieces, to which the general term shapes is applied. The procedure in the manufacture of these shapes consists of the following operations: (1) smelting iron ore and producing pig iron; (2) converting the pig iron into rectangular prisms of steel, called ingots; and (3) rolling the ingots to the desired shapes. The shapes used in building construction are: Square and round rods or bars, flat bars or flats, plates, angles, channels, I-beams, H-sections, zees and tees. Flat members 6 to 7 in. wide and less are usually designated as bars or flats; over 6 to 7 in. wide are designated as plates. Zees and tees are not now used to any great extent. Zees have been used extensively for columns but are rapidly becoming obsolete. H-sections are designed for use as columns.

The process of rolling I-beams, channels, angles, etc. is in general as follows: The ingots are brought to a uniform temperature in the soaking pit, and then are taken out and passed several times through a set of rolls, called blooming rolls. These rolls give to a piece only the general shape (rectangular, flat, or square) of the finished product. The next step is to pass the steel through the roughing rolls, and then the piece is passed to the finishing rolls where the final shaping takes place. The pieces, still very hot, are then passed on by movable tables to circular saws where they are cut into required lengths.

The method of increasing sectional area of standard shapes is shown in Fig. 1. For example, suppose it is desired to roll channels or I-beams having the same depth, but different thicknesses of web. These sections are always rolled horizontally and the increase in thickness of web is accomplished by changing the distance between the rolls, the effect being to change the width of flange as well. Thus, two beams with the same height but different weights differ simply by a rectangle as shown. It will be seen, also, that for an angle with certain size of legs the effect of increasing weight is to change slightly the length of legs, and to increase the thickness.

Bethlehem beam, girder and II-sections are shaped by four rolls instead of the two grooved rolls used for manufacturers' standard shapes. The use of so many rolls makes possible a variation of height as well as width, and both are increased with additional weight in II-sections.

Plates when rolled to exact width, the width being controlled by a pair of vertical rolls, are known as universal mill or edged plates. Plates rolled without the width being controlled have uneven edges and must be sheared to the correct width. Such plates are known as sheared plates.
The properties of the standard shapes manufactured by the different steel companies are the same. The standard shapes of the Assoc. of Am. Steel Mfrs., are rolled by all mills, but each company also has its own list of special shapes. These special shapes, which are different for the different mills, are not as likely to be in stock as the standard shapes.

Standard I-beams are rolled in depths from 3 to 24 in. and standard channels from 3 to 15 in. The different depths of standard I-beams are: 3 to 10 in. consecutively, then 12 in., 15 in., 18 in., 20 in., and 24 in. For channels, 3 to 10 in., consecutively, then 12 in. and 15 in. For each depth of I-beam and channel, there are several standard weights.

Minimum sizes of steel shapes are more likely to be found in stock and are the most efficient for resisting bending considering the weight of material used. The rolls are made especially for these sections and the heavier sections for a given depth of beam are obtained by spreading the rolls as explained above.

I-beams and channels, 15 in. and under, and angles 6 in. and under, take the base price. Heavier sections are charged for at a higher rate, usually 10 c. per 100 lb., above base price.

2. Properties of Steel Sections.—The fundamental properties of sections may be said to be: Sectional dimensions, location of the center of gravity, and the moments of inertia about the various axes. The distance from the center of gravity to the most stressed fiber \( c \); the section modulus \( s \); and the radius of gyration \( r \) follow from these. The methods of finding the properties of sections are given in Section 1 and Appendix B.

To facilitate the work of the designer, properties of steel sections are published. The facility with which a designer can find and use these properties, which are given in manufacturers' handbooks and elsewhere, has much to do with the amount of work which he can accomplish.

Beams.—The steel manufacturers' handbooks give very complete tables of properties of steel beam section. Uniformly loading I-beams, channels, and angles should be selected from the tables of safe or allowable uniform loads. These tables can also be adapted for other loadings, such as for a load concentrated at the center, in which case a beam should be selected which will carry twice the load, uniformly distributed. For a number of load concentrations, approximately equal in amount and spacing, the load may be considered as uniform.

For irregular loadings on I-beams and channels the moment and shear should be computed and the tables used which give the allowable resisting moment and shear of the various shapes. If desired, however, the beams may be designed by computing the section modulus and selecting the proper size of beam from the tables of properties. Angles, tees and other miscellaneous shapes used as beams must usually be designed by use of the section modulus, as few tables of safe loads or resisting moments and shears are given for these shapes.

Bethlehem beams and girders differ from the manufacturers' standard sections rolled by other manufacturers. The beams have heavier flanges, and, where moment is the consideration, they are lighter for the same strength than other sections. Their webs are lighter than in standard sections. Bethlehem girder sections are, for their depths, the strongest sections rolled. They have nearly twice the carrying capacity of the manufacturers' standard section for the,
same depth, but they are uneconomical where there is room for a deeper section. Tables of uniform loads for Bethlehem sections are given in Bethlehem Handbook. The common properties are also given.

Built-up steel beam properties usually have to be computed with the properties of the component parts as a basis. Some properties of the more common plate-girder sections are given in the principal steel handbooks.

Columns.—I-beams are occasionally used as columns. Their properties will be found as noted under beams. The only rolled steel column section in common use is the H-section. The Carnegie Co. rolls 4-, 5-, and 6-in. H-sections; and the Bethlehem Co. rolls 8-, 10-, 12- and 14-in. H-sections in a large range of weights. The properties of various built-up columns of pairs of channels, both latticed and with cover plates, and of plate and angle sections are given in the steel handbooks.

There are also patent columns such as Lally columns\(^1\) and cast-iron columns for second-class construction or light loads, whose properties are given in books issued by the manufacturers.

Struts and Ties.—In the design of struts and ties, it is found convenient to have tables giving the values of the radius of gyration \(r\), and also tables giving net areas deducting rivet holes. The principal steel handbooks give values of \(r\) for pairs of different angles back to back, and also the net areas for angles. It should be noted that the minimum \(r\) for a single angle is not about an axis parallel to either leg. This minimum \(r\) is given in the tables of the properties of angles.

STEEL BEAMS

BY C. R. YOUNG

3. Stress Conditions to be Met.—In order that a steel beam may be able to perform the service required of it satisfactorily, it must be secure against failure by bending, flange buckling, vertical or horizontal shear and web crippling. At the same time it must not deflect to such an extent as to endanger or damage dependent construction. By carefully selecting the type of section to be employed, large economies may often be effected in meeting each of the above tendencies to failure. Where bending moment and tendency to excessive deflection are of paramount importance, the selection of beams possessing heavy flanges and large depth in relation to area will be found in the interests of economy. If the beam be without lateral support for a long distance in relation to its width, the selection of a broad-flanged type is desirable. Where shear and the accompanying tendency to web crippling are large, it may be prudent to select relatively shallow beams with heavy webs which do not require stiffening or reinforcing. To meet any stress condition, it is most desirable to utilize, where possible, single rolled sections, of standard rather than special type, and so obviate the relatively expensive procedure of building up a section from several shapes and plates.

4. Proportioning for Moment.—In selecting a rolled section, such as an I-beam, channel, angle or tee for the resistance of a given bending moment, it is convenient to employ the flexure formula \(f/c = M/I\) in the form \(S = M/f\),

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\(^1\) The Lally Column Co., New York and Chicago.
where \( S \), known as the section modulus, = \( I/c \). Having found the required section modulus \( S \) by dividing the computed bending moment by the permissible fiber stress, all that is necessary in designing for moment is to find from a book of tables a section having a section modulus at least as great as that required. It should be remembered, however, that adequacy for moment is by no means a guarantee of entire safety.

**Illustrative Problem.**—If the permissible bending stress on a steel beam which is subjected to a bending moment of 30,000 ft.-lb. is 16,000 lb. per sq. in., suggest a suitable section.

Care must be taken to express the bending moment in the formula in inch-pounds, since the unit of length involved in the permissible fiber stress and in the tabulated section moduli is the inch.

\[
S = \left( \frac{30,000 \times 12}{16,000} \right) = 22.5
\]

An economical section would be the Carnegie supplementary 10-in., 22.4-lb. I-beam for which the section modulus = 22.7, or a Bethlehem 10-in., 23.5-lb. I-beam with a section modulus = 24.6. The standard 10-in., 25.4-lb. I-beam with a section modulus of 24.4 would also suffice. Relative pound prices and availability would govern the choice, provision being made of course for shear, web crippling and flange buckling (see Arts. 16, 17 and 15 respectively).

**Illustrative Problem.**—If a girder of 21-ft. span, consisting of one 18-in., 54.7 lb. I-beam, is to be loaded by two equal concentrated loads at the third points (including the weight of the girder), and the permissible flexural stress = 16,000 lb. per sq. in., find how great each of the concentrated loads may safely be, so far as bending is concerned.

If \( P \) be one of the concentrated loads, the maximum moment = \( (P)(7)(12) = 84P \) in.-lb.

Moment of resistance of section, or its capacity to resist moment

\[
M = S_f = (88.4)(16,000) = 1,415,000 \text{ in.-lb.}
\]

Hence

\[
P = \frac{1,115,000}{84} = 16,850 \text{ lb.}
\]

Great saving in time in proportioning beams for moment may be effected by using the tables of flexural capacity for rolled sections in the handbooks. By using these tables it is possible to find very easily the necessary size of rolled shape to carry a given uniform load over a given span, or to find the carrying capacity of a given beam over a given span.

Tables of safe loads per foot of span given in the handbooks have the merit that the safe bending capacity for any practicable span may be found by dividing the safe load per foot, or the coefficient of strength as it is sometimes called, by the span. The correctness of this is evident from the formula for the total safe load on a uniformly loaded beam,

\[
W = \frac{8SF}{l}
\]

where \( l \) = the span in inches and \( S \) and \( f \) are as previously defined. The capacity of spans of a fractional number of feet may be found in this way without interpolation.

**Illustrative Problem.**—What is the total safe load in pounds of a 7-in., 15.3-lb. I-beam of 9.37-ft. span at a permissible bending stress of 16,000 lb. per sq. in.?

From Cambria, the coefficient of strength, or allowable load for a span of 1 ft. = 110,410 lb.

Safe load for stipulated span,

\[
W = \frac{110,410}{9.37} = 11,780 \text{ lb.}
\]
In tables of capacity, the weight of the beam is included, so that if the superimposed, or net load is desired, the weight of the beam must be deducted from the quantity taken from the tables.

5. Economic Section for Flexure.—In most cases it is more economical for the resistance of moment, so far as the beam itself is concerned, to select a section of the minimum weight available for a given depth than to use an intermediate or a maximum weight. The addition of area to a section that comes from thickening the web and widening the flanges an equal amount, as is done in widening the rolls to produce the heavier weights of I-beams and channels, is much less effective in increasing the bending capacity than would be the addition of area by increasing the depth. For a rectangular section the section modulus increases as the square of the depth and as the first power of the width. A somewhat similar rule applies to variations in width and depth of rolled sections. From Fig. 2 it is seen for Carnegie I-beams: (1) that the section modulus per square inch of sectional area is in every case greater for the minimum weight than for the maximum weight beam of the same depth, and (2) that this quantity gradually increases with the depth of beams. It is, therefore, economical of steel, so far as moment is concerned, to utilize the deepest practicable sections available.

**Illustrative Problem.**—Compare the bending capacities per square inch of area of the following standard I-beams: 15-in., 45-lb.; 15-in., 55-lb.; 18-in., 48.2-lb.; and 18-in., 54.7-lb.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Section modulus (in.²)</th>
<th>Area (in.²)</th>
<th>Section modulus per sq. in. of area</th>
<th>Relative efficiencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-in., 45-lb.</td>
<td>60.5</td>
<td>13.12</td>
<td>4.61</td>
<td>1.09</td>
</tr>
<tr>
<td>15-in., 55-lb.</td>
<td>67.8</td>
<td>16.06</td>
<td>4.22</td>
<td>1.00</td>
</tr>
<tr>
<td>18-in., 48.2-lb.</td>
<td>81.9</td>
<td>14.09</td>
<td>5.82</td>
<td>1.38</td>
</tr>
<tr>
<td>18-in., 54.7-lb.</td>
<td>88.4</td>
<td>15.94</td>
<td>5.55</td>
<td>1.32</td>
</tr>
</tbody>
</table>

It is thus evident that by increasing the depth of beams from 15 to 18 in. with little or no change in weight, the bending capacity per unit of area or volume is increased at least 32 per cent and that the efficiency of an 18-in., 48.2-lb. I-beam although 12.4 per cent lighter than a 15-in., 55-lb. is 38 per cent greater.

Formerly, only the so-called "standard" sections of I-beams were to be had, but in 1902 in Germany and in 1908 in America the rolling of broad-flanged beams
on the Grey mill began. The Bethlehem series of beams brought out at the latter date was designed to have the same bending strength as standard sections then existing but with generally 10 per cent less weight. This result was attained by thinning the web and widening the flange, thus dispensing with what was generally unnecessary shear material and adding it in the flanges where it was useful for the major requirement, the bending moment. In recent years the rolling of Carnegie "supplementary" beams has made available a highly economical beam so far as flexure is concerned. Care must be taken in using these beams, as also in using Bethlehem beams, to see that the safe stresses in shear and web crippling are not exceeded (see Arts. 16 and 17).

6. Relative Efficiencies of Beams and Channels.—In selecting a rolled section for a given duty, the respective advantages of the I-beam and the channel should be studied. The fact that I-beams have a larger percentage of their area in the flanges than have channels and a smaller percentage of their area in the web, would indicate the superiority of the I-beam for flexure and the channel for shear and web crippling. If then, the governing stress is a flexural one, the I-beam is best, but if it is shear or web crippling, the channel is best.

The truth of this will be evident from the comparative figures for representative beams and channels listed in Table 1. In the second column the amount of section modulus per square inch of each I-beam is seen to be greater than that for the channel with which it is most naturally compared. On the other hand, in the third column it is seen that the percentage of shear area is higher for channels than for the corresponding I-beams. As is explained in Art. 16, it is assumed that the entire shear is resisted by the web, taking its area as the depth of the beam d multiplied by the web thickness.

Table 1.—Relative Flexural and Shearing Efficiencies of Typical I-Beams and Channels

<table>
<thead>
<tr>
<th>Section</th>
<th>Section modulus per square inch of total area S/A</th>
<th>Shearing area per square inch of total area d/t/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>8-in. 11.5-lb. channel</td>
<td>2.41</td>
<td>0.523</td>
</tr>
<tr>
<td>8-in. 18.4-lb. 1-beam</td>
<td>2.86</td>
<td>0.405</td>
</tr>
<tr>
<td>10-in. 15.3-lb. channel</td>
<td>3.00</td>
<td>0.538</td>
</tr>
<tr>
<td>10-in. 25.4-lb. 1-beam</td>
<td>3.30</td>
<td>0.420</td>
</tr>
<tr>
<td>10-in. 35.0-lb. channel</td>
<td>2.24</td>
<td>0.798</td>
</tr>
<tr>
<td>10-in. 35.0-lb. 1-beam</td>
<td>2.86</td>
<td>0.581</td>
</tr>
<tr>
<td>12-in. 20.7-lb. channel</td>
<td>3.55</td>
<td>0.557</td>
</tr>
<tr>
<td>12-in. 31.8-lb. 1-beam</td>
<td>3.89</td>
<td>0.453</td>
</tr>
<tr>
<td>15-in. 33.9-lb. channel</td>
<td>4.22</td>
<td>0.607</td>
</tr>
<tr>
<td>15-in. 42.9-lb. 1-beam</td>
<td>4.72</td>
<td>0.492</td>
</tr>
<tr>
<td>15-in. 55.0-lb. channel</td>
<td>3.55</td>
<td>0.757</td>
</tr>
<tr>
<td>15-in. 55.0-lb. 1-beam</td>
<td>4.22</td>
<td>0.606</td>
</tr>
</tbody>
</table>

7. Location of Neutral Axis of Punched Beams.—When holes are punched or drilled through the flanges or web of a beam on the tension side of the neutral axis, some diminution of bending capacity is thereby brought about. It is
customary to assume that if holes on the compression side are filled with rivets, no reduction of strength is occasioned. Such could not be assumed, however, for holes filled with loosely fitting bolts.

Before the effect of holes on the tension side can be calculated accurately, it is necessary to determine the position of the neutral axis of the punched beam. If it be assumed that this axis is at the center of gravity of the right section passing through the rivet holes in question, the moment of inertia and section modulus of the net sect on must then be computed about that axis—a time consuming operation if many cases have to be considered.

There are reasons, however, for believing that the position of the neutral axis does not change greatly by reason of the insertion of a few holes on the tension side even though they be spaced at the minimum practicable distances apart. As the deformation of the beam, and hence the stress variation in it, depends upon

![Diagram](image)

Fig. 3.—Effect of holes on tension side of beams on position of neutral axis.

the gross rather than on the net section, the position of the neutral axis is likely to be affected more by the gross section between the rivet holes than by the net section through them. In order to show the small movement of the neutral axis that results if the predominance of the gross section be given due weight, consider the two extreme cases shown in Fig. 3: One a 24-in., 79.9-lb. I-beam with three horizontal rows of 1-in. holes in the lower half of the web and two rows of holes of the same size in the tension flange, the web and flange holes being opposite; the other a 6-in., 12.5-lb. I-beam with a line of 3/4-in. holes along the center line of the web and two lines of holes in the tension flange, the holes being opposite. Assume a longitudinal spacing of 3 in. in the first case, Fig. 3a, and of 2 in. in the second case, Fig. 3b.

That no important change in the position of the neutral axis is brought about, even in the case of such extreme punchings as those shown, seems probable when one considers the small diminution of volume brought by the insertion of the holes. The volume of the holes in a 3-in. longitudinal section of the 24-in. I is 2.56 cu. in. or 3.7 per cent of the gross volume of the section. If the moment of the volume
of the holes be taken about the neutral axis of the gross section and be divided by the net volume of the 3-in. section, the upward shift of the neutral axis is found to be 0.32 in. For the 6-in., 12.5-lb. I, the loss of volume of a 2-in. length of beam brought about by the punching shown in Fig. 36 is 6.1 per cent of the gross volume, and the movement of the neutral axis would be 0.14 in. If the neutral axis is assumed as located at the center of gravity of the net sectional area through a transverse line of holes, this eccentricity would be respectively 0.90 and 0.55 in. for the two beams of Fig. 3.

Just where the neutral axis actually lies at a reduced section it is impossible to say, but no doubt it is somewhere between the two extreme positions found, and rather nearer the neutral axis of the gross section than a consideration of net sectional area alone would indicate. Since in the examples the arrangement of rivets is an extreme one and the beams selected are of minimum-weight sections, in which the effect of punching is relatively important, the change in position of the neutral axis brought about by punching cannot be great in the average case.

In any accurate investigation of stress conditions at a cross section a knowledge of the extreme possible position of the neutral axis is of assistance. To show the alteration in position of the neutral axis brought about by an arrangement of rivet holes as severe as is likely to be commonly encountered in average practice, Table 2 has been computed. The figures therein contained are based on the extreme assumption that the neutral axis depends only on the extent and disposition of the net sectional area at the transverse section in question. Two opposite holes of a diameter equal to the size of the maximum permissible rivet or bolt plus \( \frac{1}{8} \) in. are assumed to be located in one flange only. The sections are, in general, the minimum and maximum weights of Carnegie and Bethlehem I-beams and Bethlehem girder beams. From the right-hand column it is seen that the alteration in position of the neutral axis is, in relation to the depth of the beam, a maximum for the shallowest and lightest beams and a minimum for the deepest and heaviest ones. It ranges from 10.5 per cent for a 3-in., 5.7-lb. I to 2.2 per cent for a 30-in., 200-lb. Bethlehem girder beam. The shift is, in general, greater for the lighter weights of any depth of beam than for the heavier weights of the same depth. With the aid of this table it is easy to determine the shift of the neutral axis for any rolled beam with two maximum holes in one flange. If there be but one hole in one flange, the shift in position of the neutral axis ranges from about 45 to 48 per cent of that for two holes in one flange, the former figure applying to very shallow, light beams and the latter to deep and heavy ones. An average of 47 per cent may be safely taken.

8. Proportioning of Punched Beams.—If it be assumed that the neutral axis of a punched beam passes through the center of gravity of the area of the reduced section, the capacity of the beam, even considering rivet holes in the tension side only, may be calculated in the same manner as the capacity of an unpunched beam from the formula \( M = \Sigma f \). The section modulus is to be taken for the net section, being equal to the net moment of inertia divided by the distance of the extreme fiber from the neutral axis. For an assigned working stress, the capacity of the beam is consequently reduced in the same ratio as the reduction of section modulus.

This reduction is brought about by two causes. First, while the depth of the tensile portion of the section is increased, its area is reduced, since the diminution
<table>
<thead>
<tr>
<th>Section, Standard and Carnegie I-beams</th>
<th>Dia. of holes = diam. of maximum rivet + ( \frac{1}{3} ) in.</th>
<th>Shift of neutral axis</th>
<th>Ratio of shift to depth of beam</th>
<th>Section, Bethlehem I-beams</th>
<th>Dia. of holes = diam. of maximum rivet + ( \frac{1}{3} ) in.</th>
<th>Shift of neutral axis</th>
<th>Ratio of shift to depth of beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>(in.) (lb.)</td>
<td>(in.)</td>
<td></td>
<td>(in.)</td>
<td>(in.) (lb.)</td>
<td>(in.)</td>
<td></td>
<td>(in.)</td>
</tr>
<tr>
<td>3 × 5.7</td>
<td>0.50</td>
<td>0.32</td>
<td>0.105</td>
<td>10 × 23.5</td>
<td>0.875</td>
<td>0.56</td>
<td>0.056</td>
</tr>
<tr>
<td>3 × 7.5</td>
<td>0.50</td>
<td>0.23</td>
<td>0.075</td>
<td>10 × 28.5</td>
<td>0.875</td>
<td>0.46</td>
<td>0.046</td>
</tr>
<tr>
<td>4 × 7.7</td>
<td>0.625</td>
<td>0.39</td>
<td>0.099</td>
<td>12 × 28.5</td>
<td>0.875</td>
<td>0.58</td>
<td>0.048</td>
</tr>
<tr>
<td>4 × 10.5</td>
<td>0.625</td>
<td>0.27</td>
<td>0.068</td>
<td>12 × 36.0</td>
<td>0.875</td>
<td>0.59</td>
<td>0.049</td>
</tr>
<tr>
<td>5 × 10.0</td>
<td>0.625</td>
<td>0.45</td>
<td>0.090</td>
<td>15 × 38.0</td>
<td>1.0</td>
<td>0.75</td>
<td>0.050</td>
</tr>
<tr>
<td>5 × 14.75</td>
<td>0.625</td>
<td>0.28</td>
<td>0.057</td>
<td>15 × 71.0</td>
<td>1.0</td>
<td>0.69</td>
<td>0.046</td>
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<tr>
<td>6 × 12.5</td>
<td>0.75</td>
<td>0.52</td>
<td>0.086</td>
<td>18 × 48.5</td>
<td>1.0</td>
<td>0.75</td>
<td>0.042</td>
</tr>
<tr>
<td>6 × 17.25</td>
<td>0.75</td>
<td>0.35</td>
<td>0.059</td>
<td>18 × 59.0</td>
<td>1.0</td>
<td>0.60</td>
<td>0.031</td>
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<tr>
<td>7 × 15.3</td>
<td>0.75</td>
<td>0.48</td>
<td>0.069</td>
<td>20 × 59.0</td>
<td>1.0</td>
<td>0.75</td>
<td>0.038</td>
</tr>
<tr>
<td>7 × 20.0</td>
<td>0.75</td>
<td>0.35</td>
<td>0.050</td>
<td>20 × 82.0</td>
<td>1.0</td>
<td>0.61</td>
<td>0.032</td>
</tr>
<tr>
<td>8 × 17.5</td>
<td>0.875</td>
<td>0.56</td>
<td>0.070</td>
<td>24 × 73.0</td>
<td>1.0</td>
<td>0.80</td>
<td>0.033</td>
</tr>
<tr>
<td>8 × 18.4</td>
<td>0.875</td>
<td>0.63</td>
<td>0.078</td>
<td>24 × 84.0</td>
<td>1.0</td>
<td>0.75</td>
<td>0.031</td>
</tr>
<tr>
<td>8 × 25.5</td>
<td>0.875</td>
<td>0.50</td>
<td>0.063</td>
<td>26 × 90.0</td>
<td>1.125</td>
<td>0.90</td>
<td>0.034</td>
</tr>
<tr>
<td>9 × 21.8</td>
<td>0.875</td>
<td>0.69</td>
<td>0.067</td>
<td>28 × 105.0</td>
<td>1.125</td>
<td>0.90</td>
<td>0.032</td>
</tr>
<tr>
<td>9 × 35.0</td>
<td>0.875</td>
<td>0.40</td>
<td>0.045</td>
<td>30 × 120.0</td>
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<td>0.89</td>
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of section caused by the rivet holes is greater than the increase of section due to the upward shifting of the neutral axis. In the second place, the compression area is lessened, as will be seen from Fig. 4, and the maximum stress developed on the extreme compressive fiber, if the maintenance of a plane section be assumed (as usual), will be only

\[ f_r = f_t \left( \frac{c - e}{c + e} \right) \]

where \( f_t \) = extreme fiber stress on tensile side; \( c \) = distance from center of gravity of gross section to extreme tensile fibers, and \( e \) = shift of neutral axis towards compression side due to insertion of holes. In the case of, say, a 24-in., 79.9-lb. I, with two holes in the tension flange,

\[ f_r = f_t \left( \frac{12 - 0.94}{12 + 0.94} \right) = 0.855 f_t \]

The calculation of the moment of resistance of a typical section on the assumption that the neutral axis passes through the gravity axis of the net section will illustrate the procedure necessary for exact analysis.

**Illustrative Problem.**—Compute the safe moment of resistance of a 15-in., 73-lb. Bethlehem girder beam with two 1-in. rivets through one flange. \( f = 16,000 \) lb. per sq. in. Assume the neutral axis to pass through the center of gravity of the net section.

Statically moment of two 1\( \frac{1}{8} \)-in. holes through 1\( \frac{3}{16} \)-in. metal of flange taken about axis through center of section = (2)(1.125)(0.688)(7.16) = 11.10 in. \(^2\)

Shift of neutral axis = statical moment of holes \div net area of section =

\[ 21.49 - \frac{(2)(1.125)(0.688)}{11.10} = 0.56 \text{ in. above center of web} \]

Moment of inertia of net section about center of gravity of net section is

\[ I + Ae^2 - a \left( \frac{d - g + e}{2} \right)^2 \]

where \( I \) = moment of inertia of gross section about its own gravity axis, \( A \) = gross area of section, \( e \) = shift of neutral axis; \( a \) = area of two holes; \( d \) = depth of beam; \( g \) = thickness of beam flanges at rivet lines, or grip. This is

\[ 883.4 + (21.49)(0.56)^2 - 1.55 \left( \frac{15.0 - 0.688}{2} + 0.56 \right)^2 = 797.7 \]

Section modulus of reduced section = 797.7/(7.5 + 0.56) = 99.0, as compared with 117.8, the section modulus of the gross section, a reduction of 16 per cent.

Safe moment of resistance = \( SF = 99.0(16,000) = 1,585,000 \) in.-lb.

**9. Net Section Modulus.**—Obviously the accurate computation of the section modulus and moment of resistance of a punched beam on the supposition that the neutral axis is at the center of gravity of the net section is too laborious a task to be often undertaken. To obviate this labor, Table 3 has been computed. This gives generally for the minimum and maximum weights of Carnegie and Bethlehem I-beams, Bethlehem girder beams and standard channels, the percentage of gross section modulus developed with holes in the flanges. The holes are \( \frac{1}{8} \) in. larger than the maximum rivet or bolt specified for the section considered in the handbooks. In the case of beams, the punchings assumed are one hole in one
flange or in each flange, and two opposite holes in one flange or in each flange. For channels, one hole is assumed in one or in each flange.

In order to appreciate the relation of the reduction of section modulus on the above hypothesis to that obtained on the assumption that the neutral axis does not move, the percentage reductions for unsymmetrical punchings on the latter basis have been listed parallel with the others. The grip, or thickness of the flanges at the rivet lines has been taken at the fractional values given in the handbooks, and the moment of inertia of the holes about their own gravity axis has been neglected. Since the error involved in these assumptions is unimportant, and may be offset by the selection of somewhat different gauges from those assumed, this procedure is justifiable, the more so because of the uncertainty as to just where the neutral axis actually does lie. The diameters of the holes have been taken as the diameter of the maximum rivet or bolt plus \( \frac{3}{8} \) in.

From a study of Table 3 it is evident that the percentage reduction of section modulus brought about by punching of the flanges is greatest for shallow, light beams and channels and least for deep, heavy ones, ranging, in the case of two holes in one flange, from 41.6 to 10.2 per cent. For beams of a given depth, the reduction is, in general, greatest for the minimum weight beam and least for the maximum weight. Where the punching consists of but one hole in one flange, the percentage reduction is very nearly one-half of that occasioned by two holes in one flange only.

Where the punching is symmetrical—that is, the same number of holes are taken out of each flange—the uncertainty respecting the position of the neutral axis disappears and the computation of net section modulus is thereby greatly simplified. From the table, the fact appears that the percentage loss in section modulus is only slightly more with one hole out of each flange than with one hole out of one flange only. Thus, the percentage of gross section modulus developed in a 9-in., 21.8-lb. I with one \( \frac{3}{8} \)-in. hole out of one flange is 84, assuming the neutral axis at the gravity axis of the net section, whereas with one hole out of each flange it is 81.4, thus showing losses of 16 and 18.6 per cent respectively. Although the loss of area in the second case is double that in the first case, the loss in section modulus is only as 1.16 to 1. The same general principles apply if two holes are taken out of one flange and two out of each flange. The greater weakening accompanying unsymmetrical punching, despite the fact that there are only half as many holes, is due to the important effect of the shifting of the neutral axis.

The importance of the alteration in position of the neutral axis brought about by punching is further exhibited in the relation of the relative percentages of gross section modulus developed on the assumption (1) that the axis moves, (2) that it is stationary. In the latter case the reduction is everywhere less. Thus, for a Bethlehem 18-in., 48.5-lb. I, with two holes out of one flange, the relative percentages are 81.7 and 89.2. If the neutral axis were to take up a position based wholly on the net area through the punched section in question, the common assumption of stationary axis would be seriously in error. It leads to reductions in section modulus that are only from 55 to 65 per cent of what they should be if the assumption of a shifting axis holds. The first figure applies to the shallower and lighter sections and the second to deep and heavy ones. A close approximation to the net section modulus on the basis of the shift theory can be
## Table 3.—Uncorrected Percentage of Gross Section Modulus Developed by Beams and Channels with Holes in Flanges

<table>
<thead>
<tr>
<th>Section</th>
<th>Diameter of holes = diameter of maximum rivet + ( \frac{3}{4} ) in.</th>
<th>Percentage of gross section modulus developed</th>
<th></th>
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<tr>
<td></td>
<td>(in.) (lb.)</td>
<td>Neutral axis at center of gravity of net section</td>
<td>Neutral axis at center of gravity of gross section</td>
</tr>
<tr>
<td></td>
<td></td>
<td>One hole in one flange</td>
<td>Two holes in one flange</td>
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<td>Standard and Carnegie I-beams</td>
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<td>78.8</td>
<td>58.4</td>
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<tr>
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<td>80.0</td>
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<td>0.625</td>
<td>82.8</td>
<td>64.4</td>
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<tr>
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<td>0.625</td>
<td>86.6</td>
<td>73.0</td>
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<tr>
<td>5 × 11.75</td>
<td>0.625</td>
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<td>64.4</td>
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<tr>
<td>6 × 12.5</td>
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<td>86.1</td>
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</tr>
<tr>
<td>6 × 17.25</td>
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<td>81.0</td>
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<td>85.8</td>
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</table>

Bethlehem I-beams

<p>| 10 × 23.5        | 0.875                                           | 88.7 | 77.5 | 93.8 | 87.6 | 87.6 | 75.2 |
| 10 × 28.5        | 0.875                                           | 90.5 | 80.4 | 94.4 | 88.8 | 88.8 | 77.6 |
| 12 × 28.5        | 0.875                                           | 89.8 | 79.5 | 94.1 | 88.2 | 88.2 | 76.4 |
| 12 × 36.0        | 0.875                                           | 89.5 | 79.0 | 94.0 | 88.0 | 88.0 | 76.0 |
| 15 × 38.0        | 1.0                                             | 89.2 | 78.4 | 93.7 | 87.4 | 87.4 | 74.8 |
| 15 × 71.0        | 1.0                                             | 90.0 | 80.0 | 94.2 | 88.4 | 88.4 | 76.8 |
| 18 × 48.5        | 1.0                                             | 90.8 | 81.7 | 91.6 | 89.2 | 89.2 | 78.4 |
| 18 × 59.0        | 1.0                                             | 92.0 | 84.1 | 95.2 | 90.4 | 90.4 | 80.8 |
| 20 × 59.0        | 1.0                                             | 91.6 | 83.1 | 95.0 | 90.0 | 90.0 | 80.0 |
| 20 × 82.6        | 1.0                                             | 92.5 | 85.2 | 95.6 | 91.2 | 91.2 | 82.4 |</p>
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<td>96.6</td>
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<td>84.5</td>
<td>89.7</td>
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<td>89.7</td>
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<td>87.5</td>
<td>91.8</td>
<td>83.6</td>
<td></td>
</tr>
</tbody>
</table>
made by computing the percentage of gross section modulus developed on the basis of the stationary axis theory and then reducing it by multiplying it by a factor less than one. An examination of Table 3 shows that this factor should range from about 0.75 to 0.98 depending upon the size of the beam and whether there are one or two holes in each flange.

Where the diameter of the holes is less than the maximum permissible diameter, or the diameter assumed, the percentage loss of section modulus may, with sufficient accuracy be assumed as bearing the same ratio to those tabulated in Table 3 as the new diameter of hole does to the diameter assumed in the table.

Illustrative Problem.—Find the percentage of gross section modulus developed in a Bethlehem 18-in., 59-lb. I-beam with holes for two 3/4-in. rivets opposite each other in the tension flange. Assume that the neutral axis is at the center of gravity of the net section.

Effective percentage of gross section modulus with two 1-in. holes (from Table 3) = 84.1, and hence loss of gross section modulus = 15.9 per cent.

Loss of section modulus with two 3/8-in. holes instead of two 1-in. holes = \[ \frac{0.875}{1.00} = 0.875 \] \( (15.9) \) = 13.9 per cent.

Percentage of section modulus effective = 86.1 per cent.

Since the moment of inertia of a portion of the section increases approximately as the square of the distance of its center of gravity from the neutral axis of the beam, and since at the same time holes through the flange remove a larger area of metal, flange holes have very much greater effect in reducing the bending capacity than web holes have. No material error is committed by neglecting the weakening effect of the latter so far as moment is concerned. An example will suffice to show this.

Illustrative Problem.—Determine the reduction of bending capacity of a 9-in., 21.8-lb. I-beam with two 3/8-in. holes opposite each other in the tension flange and one 3/8-in. hole in the web 2 in. out from the neutral axis with its center on the right section through the flange holes. Assume the neutral axis to be at the center of the web.

Gross \( S \) of 9-in., 21.8-lb. I = 18.9

Area of two flange holes, the grip of beam being 3/4 in. = \( (2)(0.875)(0.5) = 0.875 \) sq. in. in Distance of center of gravity of flange holes from neutral axis = 4.5 – 0.25 = 4.25 in. Moment of inertia of two holes about axis through center of web, neglecting their moment of inertia about their own gravity axes = \( (0.875)(4.25)^2 = 15.8 \).

Area of web hole = \( (0.875)(0.290) = 0.25 \) sq. in. Approximate moment of inertia of hole = \( (0.25)(2)^2 = 1.0 \).

Section moduli of 3 holes = \( (15.8 + 1.0)/4.5 = 3.7 \), hence net section modulus = 18.9 – 3.7 = 15.2, or 19.6 per cent less than the gross section modulus. This represents the reduction of bending capacity caused by the holes.

The relatively small weakening caused by the web hole indicates that but little error would be committed by neglecting web holes.

10. Correction of Approximate Section Modulus.—In making calculations respecting the bending strength of beams on the basis of the fixity of the neutral axis, it should be assumed that an originally plane cross-section does not remain a plane during flexure, but resolves itself into two planes intersecting at the neutral axis, as shown in Fig. 5a. This involves a greater extreme fiber stress at the tension edge than at the compression edge in order that the total tension, \( T \), on the punched half of the beam may equal the total compression, \( C \), on the compression half, as shown in Fig. 5c. Dividing the total moment of resistance into
two parts—that contributed by the tensile portion of the cross-section, \( M_t \), and that contributed by the compression portion, \( M_c \), we have

\[
M = M_t + M_c = f_t \cdot I_t + f_c \cdot I_c
\]  

(1)

where \( f_t \) and \( f_c \) are the extreme fiber stresses on the tensile and the compressive faces respectively, and \( I_t \) and \( I_c \) are the moments of inertia of the corresponding halves. In computing the total moment of resistance on the basis of this theory, therefore, it is incorrect to assume, as is commonly done, that for beams of the same depth—that is, with \( f/c \) constant—it varies directly with the net moment of inertia. Change in the number and arrangement of holes not only affects the value of \( I_n \), but also the value of the coefficient of \( I_c \)—that is, \( f_c \). A reduction of 20 per cent in the moment of inertia of the tension half, therefore, brings about a reduction of total moment of resistance of more than 20 per cent. If there were no holes, the moment of resistance would be

\[
M = M_t + M_c = f_t \cdot I_t + f_c \cdot I_c
\]  

(2)

but the punching not only reduces the first term of the right-hand member by 20 per cent, but the second term in the ratio of \( f_c/f_t \).

The reduction of flexural strength, assuming the neutral axis to remain fixed in position, may be computed for the general case of an I-beam with holes in one flange by substituting an equivalent rectangle for each flange and calculating the necessary increase in extreme fiber stress on the tension face as compared with that on the compression face in order that compensation may be made for the effect of the holes. Let \( f_c \) = extreme fiber compressive stress and \( f_t \) = extreme fiber tensile stress. If the equivalent section of the beam be as shown in Fig. 5b, \( h \) being the clear distance between flanges and \( d \) the depth of the beam, the total compression may be written

\[
A_f \left[ \frac{f_c + \frac{h}{d} f_c}{2} \right] + A_w \left[ \frac{h}{d} f_c \right] = \frac{1}{2} f_c \left[ A_f \left( 1 + \frac{h}{d} \right) + \frac{1}{2} A_w \frac{h}{d} \right]
\]

where \( A_f \) = gross area of one flange and \( A_w \) = gross area of entire web.
The total tension may in like manner be written

\[ A_f \left[ f_t + \frac{h}{d} f_i \right] + A_w \left[ \frac{h}{d} f_i \right] = \frac{\sqrt{2}}{2} f_t \left[ A_f' \left( 1 + \frac{h}{d} \right) + \frac{\sqrt{2}}{2} A_w \frac{h}{d} \right] \]

where \( A_f' \) = net area of one flange.

Equating the total compression and the total tension, there results the relation

\[ f_c = \frac{A_f' \left( 1 + \frac{h}{d} \right) + \frac{\sqrt{2}}{2} A_w \frac{h}{d}}{A_f' + \frac{\sqrt{2}}{2} A_w \frac{h}{d}} \quad (3) \]

By taking account of the reduced value of \( I_t \) compared with half the moment of inertia of the unpunched beam, or with \( I_c \), and also of the lesser value of the ratio \( f_c/c \) as compared with \( f_t/c \), as given by Formula (3), the net moment of resistance in rigid accordance with the arbitrary assumption of fixed neutral axis may be found.

**Illustrative Problem.**—Calculate the percentage reduction of the moment of resistance of an 18-in., 54.7-lb. I with two 1-in. holes in one flange, assuming the neutral axis as at the center of the web.

Moment of inertia of one-half the gross section = 397.8.

Moment of inertia of two 1-in. holes through \( \frac{3}{4} \)-in. flange material =

\[ (2)(1)(0.75)(8.625)^2 = 112.0. \]

Net moment of inertia of tension half of beam, \( I_t = 397.8 - 112.0 = 285.8 \), or 0.72 \( I_c \).

The gross area of one flange (taking its average thickness) = 4.15 sq. in.; the net area = 2.77 sq. in., the web area = 7.64 sq. in., the average flange thickness = 0.691 in., and the value of \( h/d = 0.924 \). Then, the ratio

\[ f_c = \frac{(2.77)(1.924) \times (0.5)(7.64)(0.924)}{(4.15)(1.924) + (0.5)(7.64)(0.924)} = 0.77 \]

Formula (1) becomes for the case in point

\[ M = \frac{f_t}{c} 0.72I_c + 0.77\frac{f_t}{c} I_t \]

or, since \( I_c = \frac{\sqrt{2}}{2} I \), or, half the gross moment of inertia of the entire beam,

\[ M = \frac{f_t}{c} \left( 0.36 + 0.385 \right) = 0.745\frac{f_t}{c} \]

or 74.5 per cent of what the moment of resistance of the gross section would be with an extreme fiber stress on both upper and lower faces equal to the maximum permissible stress \( f_c \).

Comparing this with the efficiency of the same beam, when computed on the assumption that the neutral axis is at the center of gravity of the net section, it is found by consulting Table 3 that in the latter case it is 76.8 per cent, or somewhat higher than is obtained in accordance with the arbitrary assumption that the neutral axis is fixed at the center of gravity of the gross section. The latter must, therefore, be in error on the side of severity.

To compare the efficiencies developed in accordance with the two common assumptions upon which Table 3 is based with that found by the method illustrated in the last problem, efficiencies have been listed for a number of typical beams and girder beams in Table 4. These beams are assumed to have two maximum holes in one flange.

The efficiencies tabulated in column \( A \) are the same as those in column 4 of Table 3 for the same beams and are based on the neutral axis being at the center of gravity of the net section. Those in column \( B \) are based on the neutral axis
being at the center of gravity of the gross section, and take into account in accordance with the method illustrated in the last problem, the fact that $f_c$ must be less than $f_e$. The efficiencies in column $C$ are calculated on the assumption that the neutral axis is at the center of gravity of the gross section and that the fiber stress $f_i$ exists at both compressive and tensile extreme fibers. The figures correspond to those in column 6 of Table 3 for the same beams.

<table>
<thead>
<tr>
<th>Size of beam, (in.)</th>
<th>Diam. holes (lb.)</th>
<th>Neutral axis at center of gravity of net section</th>
<th>Neutral axis at center of gravity of gross section</th>
<th>Coefficient &quot;K&quot; by which method &quot;C&quot; results are to be multiplied</th>
<th>Method C results adjusted by coefficient $K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard 1-beams</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>6 $\times$ 12.5</td>
<td>0.75</td>
<td>64.4</td>
<td>61.4</td>
<td>79.6</td>
<td>0.90</td>
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<tr>
<td>6 $\times$ 17.25</td>
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<td>72.3</td>
<td>68.5</td>
<td>83.0</td>
<td>0.90</td>
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<td>12 $\times$ 31.8</td>
<td>0.875</td>
<td>74.7</td>
<td>71.9</td>
<td>85.0</td>
<td>0.95</td>
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<td>12 $\times$ 55.0</td>
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<td>79.3</td>
<td>77.0</td>
<td>87.0</td>
<td>0.95</td>
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<td>18 $\times$ 54.7</td>
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<td>74.4</td>
<td>86.0</td>
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<tr>
<td>24 $\times$ 73.0</td>
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<tr>
<td>Bethlehem girder beams</td>
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<td>18 $\times$ 92.0</td>
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<td>93.6</td>
<td>0.95</td>
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</tbody>
</table>

It will be seen from this table that if the resistance of the beam were developed in the manner assumed in calculations by method $B$, that the efficiency would be less than if the neutral axis were to shift to the center of gravity of the net section, which is in itself, according to the principle of least work, an indication that the axis does shift, and that the efficiency cannot be lower than given by method $A$. It probably does not shift to the center of gravity of the net section by reason of the influence of the gross section on either side of the holes, but it is reasonable to assume that it shifts to a mid-position, so that the actual efficiency developed
is neither so low as methods A or B would indicate nor so high as method C would indicate. Since the employment of the latter method greatly facilitates computation, it is recommended that in supplementing Table 3 it be followed and a correction applied to give results between those given by method A and by method C. If, for beams 6 in. deep and less, the efficiency found by method C be multiplied by the coefficient 0.90 and, for beams over 6 in. deep, including rolled girder beams, it be multiplied by 0.95, the efficiencies so obtained will probably be very close to those actually developed. In the last column of Table 4 the efficiencies found for certain typical beams by this procedure are given, affording a comparison with those obtained by methods A, B and C. For a greater refinement of design than is obtained by the usual arbitrary assumptions, it is recommended that for unsymmetrical punchings the quantities in the fifth and sixth columns of Table 3 be employed, multiplying them first by the appropriate coefficients \( K \) from Table 4.

The same correcting coefficient may be applied without material error to beams having only one hole in the tension flange or to beams having reinforcing plates riveted on the flanges.

Illustrative Problem.—A Bethlehem 28-in., 165-lb. girder beam has two 1\(\frac{1}{8}\)-in. holes opposite each other in its tension flange. Estimate the probable efficiency.

Gross moment of inertia of girder beam = 6,562.7.

Moment of inertia of two holes, assuming fixed axis (method C) = \((2)(1.188)(1.125)\) \((13.41)^2\) = 480. Net moment of inertia = 6,562.7 - 480 = 6,082.7.

If the correction factor be 0.95, as suggested in Table 4 for sections of this depth, the probable efficiency is:

\[
\frac{(0.95)(6,082.7)}{6,562.7} = 0.88 = 88 \text{ per cent.}
\]

This result may be obtained readily from Table 3, if available, by multiplying the efficiency given for this beam in column 6 by the factor 0.95.

11. Limiting Longitudinal Position of Flange Holes.—Where flange holes are located in unreinforced rolled beams at points remote from the center, no attention need be paid to the weakening effect produced by them. Within a certain distance from the center of a beam symmetrically loaded, and from the point of maximum moment of a beam unsymmetrically loaded, the reduction of section modulus should be computed in accordance with one of the methods given above. The exact position of this critical point depends on the nature of the distribution of the loading and on the end conditions. For the common cases of simple beams uniformly loaded, loaded at the third-points, and loaded at the center, the diagram of Fig. 6, similar to one suggested by Henry Kercher in *Engineering News-Record*, May 12, 1921, p. 800, will be found helpful. By means of the curves there given it is possible to determine directly the point in the span length, with respect to the center, at which a given fraction of the section modulus provided at the point of maximum moment would be permissible. If it be desired, for example, to place two holes in the tension flange of a symmetrically-loaded beam at a stated distance from the center, the ratio of the net section modulus at the holes to the section modulus at the center may be found by the methods already described or by the use of tables, and a reference to the appropriate curve will show whether this reduced section modulus would suffice at the proposed location of the holes.
In cases where plates are riveted to the flanges of rolled beams for purposes of reinforcement, it is very convenient to be able to discover readily if the net section of the beam at the point of cut-off of the plates is adequate.

**Illustrative Problem.**—An 18-in., 48.2-lb. I-beam of 21-ft. span carries a uniformly distributed load. It is desired to drill two 1-in. holes in the bottom flange at a point 3 ft. from the center. Is this permissible? Assume the neutral axis at the center of the web and apply the adjusting factor 0.95 recommended in Table 4.

From Table 3 the percentage of the gross section modulus developed on the assumption made is 89.6 per cent. This corrected = $(0.95)(0.896) = 0.852$.

![Graph showing proportion of maximum provided section modulus required at various points of span.](image)

**Fig. 6.—Proportion of maximum provided section modulus required at various points of span.**

On the curve for uniformly distributed loading in Fig. 6 find the point where the vertical through 0.852 on the horizontal axis intersects the curve, and directly opposite to the left is found 0.20. The holes may, therefore, not be located closer to the center than $(0.20)(21) = 4.2$ ft., and hence the proposed location is not permissible, without reinforcement of the section or use of a heavier beam.

**12. Reinforcement of Beams for Bending.**—Occasionally through the desire to utilize a rolled section that chances to be on hand for a bending moment in excess of its normal capacity, or in order to keep the construction especially shallow, or to strengthen an overloaded beam already in position, a beam is reinforced for flexure by the addition of a plate or plates to each flange. Such procedure is only practicable where the shear requirement does not also demand reinforcement. The bending capacity of the beam is thereby increased in direct ratio to its increase in section modulus. To determine this increase, the moment of inertia of the plates on the compression flange, considered as unpunched, plus the net moment of inertia of the plates on the tension flange is added to the gross moment of inertia of the original section less the moment of inertia of the holes through the tension flange. The total net moment of inertia divided by the distance to the extreme fiber of the reinforced beam will then give the net section modulus. In making this computation it is most convenient to consider the
neutral axis as at the center of gravity of the gross area and then apply whatever correction is deemed reasonable for the section under consideration. Allowance for rivet holes on the tension side of the beam should be for the most unfavorable arrangement likely to be adopted. A saving in section may, however, be effected by staggering the rivets in the two gage lines of the flange, since for large spacing obtaining at the point of maximum moment, one flange hole only need then be deducted. While web holes on this critical section should also in strict accuracy be considered, it is generally unnecessary to do so unless there be several of them. If for any reason flange holes are opposite as they will be at the ends of the plate, allowance should be made for two holes in computing the section modulus.

Illustrative Problem.—Calculate the moment of resistance of a 12-in., 31.8-lb. I (Fig. 7) with one 6 × ¾-in. plate riveted to each flange by ¾-in. rivets, staggered in two lines. Permissible flexural stress = 16,000 lb. per sq. in.

Gross moment of inertia of I-beam = 215.8.

Approximate moment of inertia of two unpunched plates =

\[
(2)(6)(0.375)(6.19)^2 = 172.4.
\]

Total gross I = 388.2.

Moment of inertia of one hole through tension flange, including plate, the grip of the I-beam being ¾ in., is

\[
(1)(0.875)(0.94)(3.591) = 28.8
\]

Net moment of inertia of section = 388.2 − 28.8 = 359.4, and adjusted net section modulus (0.95)(359.4)/6.375 = 53.6, applying the correction factor 0.95 recommended in Art. 10 for sections over 6 in. in depth.

Moment of resistance = 53.6 (16,000) = 858,000 in.-lb.

It should be remembered when calculating the section modulus of parts of a compound section that this quantity is the moment of inertia of the part under consideration about the neutral axis of the compound section divided by the distance from the neutral axis to the extreme fiber of the total assemblage of parts existing at the section considered. If, for example, a 12-in., 31.8-lb. I with section modulus of 36.0 is used in a reinforced beam or box girder with one ¾-in. plate on each flange, the gross section modulus of the beam in the assemblage is

\[
\begin{align*}
6.00 \\
6.50 (36) = 33.2.
\end{align*}
\]

13. Length of Reinforcing Plates.—Because of the lessening moment near the ends, it is unnecessary to carry the reinforcing plates the full length of the beam. They should end, theoretically, at the points nearest the center where the net section modulus of the unreinforced beam is sufficient for the moment requirement. If the total loading is uniformly distributed, the position of these points may be found readily by either graphical or analytical means.

Since for a beam carrying a uniformly distributed load, the moment varies as the ordinates to a parabola with vertex at the center of the span and axis vertical, the required section modulus must vary in precisely the same manner, given a constant working stress in flexure. The truth of this is evident from the formula

\[
S = \frac{M}{f}
\]
It is possible, therefore, to plot, as has been done in Fig. 8, a curve of required section modulus at each point of the half span and, to superimpose on this, a diagram of provided section modulus. Such diagram may be prepared for any system of loading, although that of Fig. 8 is for uniform loading. The maximum ordinate $S$ represents the required net section modulus of the reinforced beam at the point of maximum moment, and the depths of the various strips required to cover the area beneath the curve of required section modulus represent the net section moduli of the punched beam and the successive flange plates with respect to the total cross-section existing at the point. The minimum length of these strips required to cover completely the area mentioned may be easily scaled from the diagram. The theoretical length, $x$, of any flange plate is twice the scaled distance from the center of the span to the point where the inner (lower) horizontal bounding line for the appropriate strip cuts the curve. To this length a certain addition is made, for reasons set forth below.

If it happens that by reason of staggered rivet spacing in the central portion of the reinforcing plates (or for any other cause), the net section modulus, $s_2'$ of the primary beam is less near the ends of the flange plates (where the rivets will be opposite each other) than the net section modulus, $s_2$, near the center, the upper horizontal bounding lines for the successive strips should be jogged down inside the end of the first (outside) flange plate, as shown in Fig. 8. For approximate work the net section modulus of the beam may be taken as the same throughout—that is, at the least value—and the jogging of the horizontal bounding lines thereby obviated. In general, the strip representing the net section modulus of the pair of outside flange plates will be wider than is actually required to cover the remaining area of the diagram, but this does not affect the graphical determination of length of the outside flange plates.

To determine the theoretical lengths by analytical means, let $x_1$, $x_2$, etc., Fig. 8, be the required half lengths of the flange plates numbered consecutively from the outside. Let the actual effective section modulus of the punched beam at the center be $s_2$ with respect to the maximum section and at the ends of the flange plates be $s_2'$ with respect to the beam section alone. Let $s_2$ be the actual effective section modulus at the center of the span for the two flange plates in immediate contact with the punched beam with respect to the maximum section, and $s_2'$ be the corresponding quantity at the ends of these plates with respect to an assemblage consisting of the beam and these two plates. Let $s_1$ be the required net section modulus of the two outer (first) flange plates with respect to the maximum section, and $s_1'$ be the required net section modulus of these plates at their
ends. Let $S$ be the total required net section modulus at the center of the reinforced beam. The relation of these quantities will be clear from Fig. 8. Then, since the curve $BD$ is a parabola

$$\left(\frac{x_1}{2}\right)^2 - \left(\frac{l}{2}\right)^2 = \frac{s_1'}{S}$$

or

$$x_1 = l \sqrt{\frac{s_1'}{S}}$$

In a similar manner the length of any plate may be derived; thus,

$$x_2 = l \sqrt{\frac{s_1' + s_2'}{S}}$$

$$x_n = l \sqrt{\frac{s_1' + s_2' + \ldots + s_n'}{S}}$$

To ensure that the reinforcing plates are able to take stress at the points where they are first needed, they should be carried past the points of theoretical ending a distance sufficient to accommodate at least two transverse rows of rivets. The addition of 9 in. at each end of the flange plates will make this possible, for the close spacing of rivets usually adopted at the ends of the reinforcing plates. Closer spacing at the ends of the plates than at the center is desirable since the increment of flange stress per lineal inch of girder is greater near the ends than near the center, and since it is well to transfer stress to the flange plates as near their ends as possible in order that they may be fully and uniformly stressed at the points where they are most needed.

**Fig. 9.—Details of flange reinforcement for I-beam.**

**Illustrative Problem.**—Determine the theoretical and practical lengths of the reinforcing plates in the problem of Art. 12, if the span is 25 ft.

Net section modulus required at center, with one $\frac{3}{8}$-in. flange hole out = 53.6.

Effective section modulus of 12-in., 31.8-lb. I with two $\frac{3}{8}$-in. holes out of tension flange, according to Table 4 = $(0.808)(36) = 29.1$.

Required net section modulus $s_1'$, of two plates to satisfy requirements at the ends of these plates $= 53.6 - 29.1 = 24.5$.

Theoretical length of flange plates required,

$$x = l \sqrt{\frac{s_1'}{S}} = 25 \sqrt{\frac{24.5}{53.6}} = 16.9 \text{ ft.}$$

By making the plates 18 ft. long, or a little more than 6 in. longer at each end, two rows of rivets, 3 in. apart, may be driven outside the theoretical point of cut off and at the same time sufficient end distance allowed for the plate. The relation of the plates to the primary beam may be seen in Fig. 9.
It is possible, therefore, to plot, as has been done in Fig. 8, a curve of required section modulus at each point of the half span and, to superimpose on this, a diagram of provided section modulus. Such diagram may be prepared for any system of loading, although that of Fig. 8 is for uniform loading. The maximum ordinate S represents the required net section modulus of the reinforced beam at the point of maximum moment, and the depths of the various strips required to cover the area beneath the curve of required section modulus represent the net section moduli of the punched beam and the successive flange plates with respect to the total cross-section existing at the point. The minimum length of these strips required to cover completely the area mentioned may be easily scaled from the diagram. The theoretical length, $x$, of any flange plate is twice the scaled distance from the center of the span to the point where the inner (lower) horizontal bounding line for the appropriate strip cuts the curve. To this length a certain addition is made, for reasons set forth below.

If it happens that by reason of staggered rivet spacing in the central portion of the reinforcing plates (or for any other cause), the net section modulus, $s_3'$ of the primary beam is less near the ends of the flange plates (where the rivets will be opposite each other) than the net section modulus, $s_3$, near the center, the upper horizontal bounding lines for the successive strips should be jogged down inside the end of the first (outside) flange plate, as shown in Fig. 8. For approximate work the net section modulus of the beam may be taken as the same throughout—that is, at the least value—and the jogging of the horizontal bounding lines thereby obviated. In general, the strip representing the net section modulus of the pair of outside flange plates will be wider than is actually required to cover the remaining area of the diagram, but this does not affect the graphical determination of length of the outside flange plates.

To determine the theoretical lengths by analytical means, let $x_1$, $x_2$, etc., Fig. 8, be the required half lengths of the flange plates numbered consecutively from the outside. Let the actual effective section modulus of the punched beam at the center be $s_3$ with respect to the maximum section and at the ends of the flange plates be $s_3'$ with respect to the beam section alone. Let $s_2$ be the actual effective section modulus at the center of the span for the two flange plates in immediate contact with the punched beam with respect to the maximum section, and $s_2'$ be the corresponding quantity at the ends of these plates with respect to an assemblage consisting of the beam and these two plates. Let $s_1$ be the required net section modulus of the two outer (first) flange plates with respect to the maximum section, and $s_1'$ be the required net section modulus of these plates at their
ends. Let $S$ be the total required net section modulus at the center of the reinforced beam. The relation of these quantities will be clear from Fig. 8. Then, since the curve $BD$ is a parabola

$$\left(\frac{x_1}{2}\right)^2 - \left(\frac{l}{2}\right)^2 = \frac{s_1'}{S}$$

or

$$x_1 = l \sqrt{\frac{s_1'}{S}}$$

In a similar manner the length of any plate may be derived; thus,

$$x_2 = l \sqrt{\frac{s_1' + s_2'}{S}}$$

$$x_n = l \sqrt{\frac{s_1' + s_2' + \cdots + s_n'}{S}}$$

To ensure that the reinforcing plates are able to take stress at the points where they are first needed, they should be carried past the points of theoretical ending a distance sufficient to accommodate at least two transverse rows of rivets. The addition of 9 in. at each end of the flange plates will make this possible, for the close spacing of rivets usually adopted at the ends of the reinforcing plates. Closer spacing at the ends of the plates than at the center is desirable since the increment of flange stress per lineal inch of girder is greater near the ends than near the center, and since it is well to transfer stress to the flange plates as near their ends as possible in order that they may be fully and uniformly stressed at the points where they are most needed.

![Diagram](image)

**Fig. 9.—Details of flange reinforcement for I-beam.**

**Illustrative Problem.**—Determine the theoretical and practical lengths of the reinforcing plates in the problem of Art. 12, if the span is 25 ft.

Net section modulus required at center, with one \(\frac{7}{8}\)-in. flange hole out = 53.6.

Effective section modulus of 12-in., 31.8-lb. I, with two \(\frac{7}{8}\)-in. holes out of tension flange, according to Table 4 = \((0.808)(36)\) = 29.1.

Required net section modulus $s_1'$ of two plates to satisfy requirements at the ends of these plates = 53.6 - 29.1 = 24.5.

Theoretical length of flange plates required,

$$x = l \sqrt{\frac{s_1'}{S}} = 25 \sqrt{\frac{24.5}{53.6}} = 16.9 \text{ ft.}$$

By making the plates 18 ft. long, or a little more than 6 in. longer at each end, two rows of rivets, 3 in. apart, may be driven outside the theoretical point of cut off and at the same time sufficient end distance allowed for the plate. The relation of the plates to the primary beam may be seen in Fig. 9.
14. Riveting of Reinforcing Plates.—For the adequate attachment of the plates to the primary beam, enough rivets should be employed to develop between the ends and the center of the span, the total stress that exists in the plates at that point. It is sufficiently accurate to assume that the extreme fiber stress obtains over the entire net cross-section of the plate at the span center, although at the fibers nearest the neutral axis, the intensity of stress is somewhat less than at the extreme fibers. The total force to be developed between the end and the center of a tension reinforcing plate will, therefore, be the net area multiplied by the stipulated extreme fiber stress. On the compression side the area of the plates will not be reduced by rivet holes and hence if the plates be the same as on the tension side, the average stress over the plate section will be lower, but the total force to be provided for will be the same. The number of rivets found for the tension side will, therefore, apply also to the compression side. So few are they in relation to the length of the plates generally needed that the actual spacing is usually dictated by practical requirements such, for example, as that rivet spacing in the line of stress must not exceed 16 times the thickness of the outside plate nor 6 in. in any case.

Illustrative Problem.—Determine the correct rivet spacing in the reinforcing plates of the preceding problem, if the safe shearing and bearing stresses on rivets are 10,000 and 20,000 lb. per sq. in. respectively, and the spacing in the line of stress must not exceed 16 times the thickness of the reinforcing plate, nor 6 in.

Net section of one 6 × ¾-in. plate = [6 − 1(0.875)][0.375] = 1.92 sq. in. Hence safe tensile strength = (1.92)(16,000) = 30,700 lb.

Least value of ¾-in. rivet bearing on ¾-in. plate or ¾-in. flange of beam is the single shear value = (0.44)(10,000) = 4,400 lb.

Number of rivets required in one plate from end to center to develop stress at center = 30,700/4,400 = 7. This would give an average spacing of about 1½ ft. even though the rivets are staggered, and hence the rule that the spacing must not exceed 16t = (16)(¾) = 6 in. becomes operative. Beginning at a point 3 in. on each side of the center line, the rivets will be staggered 6 in. apart for 7 ft. 6 in. and then a closer spacing will be used for the remaining distance, as shown in Fig. 9. This closer spacing is desirable since the increment of flange stress per lineal inch is much greater near the ends of the girder than near the center.

15. Proportioning for Flange Buckling.—In proportioning beams for bending moment, it is unsafe to use without reduction, the permissible extreme fiber stress given in specifications if the compression flange is unsupported laterally for a distance exceeding about 10 or 15 times the width of flange. This source of weakness arises from the fact that the compression flange acts in some measure as a column and, like a column, tends to fail by buckling sidewise between points of lateral support. It is not free to do so, however, to the same extent as a column, by reason of the fact that both the web and the tension flange tend, with an effect that depends largely on the depth of the beam and the web thickness, to hold it in a straight line. If the web is very deep and thin, it obviously could afford relatively small support to the compression flange and could transmit but little support from the tension flange.

The compression flange differs from a column also in the manner of loading. A column receives its loading at one point, or a group of points, near the top. If the load be concentric, there is, then, neglecting the buckling stress, a uniformly distributed stress over the cross section from one end to the other. The com-
pression flange of a beam, however, receives its loading at innumerable points from one end to the dangerous section, or if it be a restrained beam, from the point of inflection to the dangerous section.

Early efforts to formulate rules for reducing the permissible flexural stress to compensate for the buckling tendency in compression flanges were based upon unsatisfactory experimental evidence, as was pointed out by R. Fleming in an excellent discussion of the subject in Engineering News, April 6, 1916.

The Pencoed rule, adopted in the Bethlehem, Jones & Laughlin, Phoenix and several other handbooks was based more on judgment than on actual tests. As will be seen from Table 5, no reduction of bending strength is required by this rule where the ratio of unsupported length to width of compression flange does not exceed 20. A reduction of 10 per cent for each additional 10 flange-widths of length is recommended up to 70 flange-widths where the table stops. This rule is the most lenient in common use.

A basis for the Cambria formula, \( p = \frac{-18000}{l^2} \), given in Table 5, was found by considering the top flange a strut of rectangular cross section of width \( b \), sufficiently supported by the web and tension flange to warrant a fiber stress of 18,000 lb. per sq. in. for short lengths. Applying the Rankine formula for fixed ends, \( p = \frac{-18000}{l^2} \), and replacing \( r^2 \) by \( \frac{b^2}{12} \), the Cambria formula was derived. According to this formula, a fiber stress of 16,000 lb. per sq. in. would be attained at a value of \( \frac{l}{b} = 19.37 \) and therefore no reduction is necessary with ratios of less than approximately 20 flange-widths.

The Carnegie formula, recommended since 1913, \( p = 19,000 - 300 \frac{l}{b} \) is an approximate application of the column formula \( p = 19,000 - 100 \frac{l}{r} \) to the compression flange, considering it as a rectangle. Reduction begins at 10 flange-widths and no beam is allowed to have an unsupported length exceeding 40 flange-widths, a restriction of greater severity than had till that time been customarily imposed.

Two characteristic reduction formulas for railway bridge work are; the American Railway Engineering Association formula, \( p = 14,200 - 200 \frac{l}{b} \) and the Canadian Engineering Standards Association formula, \( p = 16,000 - 200 \frac{l}{b} \). No maximum allowable value of \( \frac{l}{b} \) without reduction is specified in either case, but beams of greater length than 20 ft. without lateral support are not allowed. The first is included in Table 5 for purposes of comparison, although it is intended only for plate girder flanges. The second applies to either.

R. Fleming recommends in Engineering News-Record, Feb. 24, 1921, that the permissible flexural stress for beams over 10 in. deep, and for plate girders without cover plates, be \( p = 19,000 - 250 \frac{l}{b} \), while for beams 10 in. deep, and
### Table 5.—Permissible Extreme Fiber Stresses in Steel Beams with Unsupported Compression Flanges

(lb. per sq. in.)

\( l = \) unsupported length in inches, \( b = \) breadth of flange in inches

<table>
<thead>
<tr>
<th>Rule or formula</th>
<th>Maximum allowable ( \frac{l}{b} ) without reduction</th>
<th>Ratio of unsupported length to flange width</th>
<th>Maximum allowable ( \frac{l}{b} ) specified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bethlehem</td>
<td>20.0</td>
<td>15  20  25  30  40  50  60  70  80</td>
<td>Table stops</td>
</tr>
<tr>
<td>Jones &amp; Laughlin</td>
<td>19.37</td>
<td>16,000 16,000 15,200 14,400 12,800 11,200 9,600 8,000</td>
<td>Table carried to 110</td>
</tr>
<tr>
<td>Cambria:</td>
<td>( p = \frac{18,000}{1 + \frac{b}{3,000}} )</td>
<td>16,000 15,880 14,900 13,850 11,740 9,820 8,180 6,840 5,750</td>
<td>None</td>
</tr>
<tr>
<td>Carnegie:</td>
<td>( p = 19,000 - 300 \frac{l}{b} )</td>
<td>10.0 14,500 13,000 11,500 10,000 7,000</td>
<td>40</td>
</tr>
<tr>
<td>A.R.E.A.:</td>
<td>( p = 14,200 - 200 \frac{l}{b} ) for plate girders</td>
<td>11,200 10,200 9,200 8,200</td>
<td>Max. unsupported length = 20 ft.</td>
</tr>
<tr>
<td>C.E.S.A.:</td>
<td>( p = 16,000 - 200 \frac{l}{b} ) for beams or girders.</td>
<td>13,000 12,000 11,000 10,000</td>
<td>Max. unsupported length = 20 ft.</td>
</tr>
<tr>
<td>Fleming:</td>
<td>( p = 19,000 - 250 \frac{l}{b} ) for beams not over 10 in. and plate girders without cover plates.</td>
<td>12.0 15,250 14,000 12,750 11,500 9,000 6,500</td>
<td>50</td>
</tr>
<tr>
<td>Fleming:</td>
<td>( p = 19,000 - 225 \frac{l}{b} ) for beams 10 in. and under and plate girders with cover plates.</td>
<td>13.3 15,630 14,500 13,440 12,250 10,000 7,750</td>
<td>50</td>
</tr>
<tr>
<td>Young:</td>
<td>( p = 16,000 - 120 \frac{l}{b} )</td>
<td>14,200 13,600 13,000 12,400 11,200 10,000,</td>
<td>50</td>
</tr>
</tbody>
</table>
under, and for plate girders with covers, $p = 19,000 - 225 \frac{l}{b}$ is recommended. If the compression flange is rigidly held at the ends, the length to be taken is $\frac{3}{4}$ the span.

A valuable study of flange buckling made by H. F. Moore was reported in 1913 in Bulletin 68 of the University of Illinois Engineering Experiment Station. From theoretical considerations and from a study of all available tests, Mr. Moore concluded that the ultimate fiber stress for steel I-beams not restrained against sidewise buckling of the compression flange, is given by the formula $f_1 = 40,000 - 60 \frac{ml}{r'}$, where $f_1$ = extreme fiber stress in lb. per sq. in. computed by the usual flexure formula, $l$ = span of beam in inches, $r'$ = radius of gyration of the I-section about a gravity axis parallel to the web, and $m$ = a coefficient dependent upon the end conditions and method of loading, $ml$ being an equivalent column length. Values of $m$ vary from 1.0 for a cantilever beam with an end load to 0.25 for a fixed-ended beam with a mid-point load. For a simple beam with uniform load, it is 0.667, while for a simple beam with a single concentrated load at any point of the span, it is 0.50.

From the above formula for ultimate flexural stress, it is possible to derive a working formula with a fairly satisfactory basis in actual experimental results. If $r'$ be considered as equal to 0.20 $b$, which is a very close average value, and the maximum listed value of $m$ be taken, that is 1, then the formula for ultimate buckling strength becomes $f_1 = 40,000 - 300 \frac{l}{b}$. Applying a factor of safety of 2.12—a reasonable one in column design and sufficiently severe in the present case if $m$ be taken at its extreme value of 1—there results the working formula $p = 16,000 - 120 \frac{l}{b}$. The test results indicate that such a formula should be applied for all ratios of $\frac{l}{b}$ up to the arbitrary limit for the ratio set by good practice. It is recommended that the latter do not exceed 50.

In Table 5 the permissible stresses on compression flanges given by the formulas discussed above are listed for values of $\frac{l}{b}$ from 15 to the upper limit allowed. From this it is seen that for short unsupported lengths, disregarding the A.R.E.A. formula which applies to plate girders only, the C.E.S.A. formula is the most severe, while for long lengths the Carnegie formula is the most severe. In the case of the A.R.E.A. formula, considerable weight was given to recent experimental evidence of low strength of short length columns and to the fact that the top flanges of through plate girder spans may be indifferently braced. In view of Moore's investigations, the low values given by the Carnegie formula for high width ratios appear unwarranted. In the case of the recommended formula, the necessity for applying the reduction to beams without lateral support for all values of $\frac{l}{b}$ from zero to 50, might appear to be severe for beams with width ratios deour 10 or 15, for which some formulas permit the specified working stress without reduction. In practice, however, if a beam is supported at intervals of less than 10 or 15 times the flange width, it is likely to be supported continuously,
by flooring or other construction furnishing effective restraint against lateral buckling, and justifying the employment of the full working stress.

To apply the recommended formula when using tables of safe capacity of beams based on an extreme fiber stress of 16,000 lb. per sq. in., the tested loads may be multiplied by the following reduction factors:

<table>
<thead>
<tr>
<th>$\frac{l}{b}$</th>
<th>Reduction factor</th>
<th>$\frac{l}{b}$</th>
<th>Reduction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>30</td>
<td>0.774</td>
</tr>
<tr>
<td>10</td>
<td>0.924</td>
<td>35</td>
<td>0.737</td>
</tr>
<tr>
<td>15</td>
<td>0.887</td>
<td>40</td>
<td>0.700</td>
</tr>
<tr>
<td>20</td>
<td>0.850</td>
<td>45</td>
<td>0.662</td>
</tr>
<tr>
<td>25</td>
<td>0.812</td>
<td>50</td>
<td>0.625</td>
</tr>
</tbody>
</table>

**Illustrative Problem.**—A 20-in., 65.4 lb. girder of 21-ft. span carries a load of 20,000 lb. at each of the third points. If the concentrated loads be considered as including the weight of the girder and there be no lateral restraint between the two points of loading, express an opinion as to the safety of the girder if $p = 16,000 - 120 \frac{l}{b}$.

Moment $= (20,000)(7)(12) = 1,680,000$ in.-lb.
Extreme fiber stress $= M/S = 1,680,000/116.9 = 14,400$ lb. per sq. in.
Permissible stress by formula
\[
p = 16,000 - \frac{(120)(84)}{6.25} = 14,390 \text{ lb. per sq. in.}
\]
As this is almost exactly equal to the existing stress the beam is safe.

**Illustrative Problem.**—If the total capacity of an 18-in., 60-lb. I, 17 ft. in span, is listed in a handbook as 58,700 lb. with a fiber stress of 16,000 lb. per sq. in., find the safe capacity assuming that the beam is without lateral support between its bearings. $p = 16,000 - 120 \frac{l}{b}$.

Width ratio, $\frac{l}{b} = (17)(12)/6.087 = 33.5$

Interpolating in the table of reduction factors, above, 74.8 per cent of the tabular load should be regarded as safe, or $(0.748)(58,700) = 43,900$ lb.

Wherever a beam must be employed with any considerable portion of its length without lateral support, it is advantageous in order to reduce the flange buckling stress to select a section with a relatively wide flange. By so doing a higher permissible flexural stress may be used than for beams with narrower flanges and an important economy effected.

Care must be taken not to assume a beam as supported against lateral buckling unless the lateral restraint is known to be effective. Separators between the webs of I-beams cannot be regarded as fully supporting the compression flange, particularly if they be of the gas-pipe type. Tie rods and sag rods have small value in preventing the compression flange from buckling under overload.

16. **Proportioning for Shear.**—When a rolled beam or channel is proportioned to be sufficiently strong in flexure, it will generally be adequate also for both vertical and horizontal shearing stresses, so that there is usually no necessity of investigating the shearing capacity of the beam. If, however, the span of the beam be short in relation to its depth and it is loaded to capacity in bending, the shearing stresses may be excessive. Such beams should consequently be investigated for shear.
Analysis of the internal stresses in a beam\(^1\) shows that the intensity of either the vertical or the horizontal shearing stress at any point on any cross section may be expressed by the formula

\[
v = \frac{QV}{I_t} \tag{1}\]

where \(Q\) = the statical moment of the area as one side of the point considered, about the neutral axis; \(V\) = total vertical shear; \(I\) = moment of inertia of the section; and \(t\) = thickness of section at the point. It has also been established that for an I-section, by far the greater part of the total vertical shear is resisted by the web and that no material error is committed by considering that the web resists all of it.

**16a. Vertical Shearing Stress.**

While for many purposes it is sufficiently accurate to assume the total vertical shear to be uniformly distributed over the web, considering the web area to be the extreme depth of the beam multiplied by the web thickness, the error involved, which is on the side of weakness, must be offset by the use of a low working stress in shear. In cases of close designing or investigation of seriously overloaded beams, the design for shear should be on the basis of the correct theory.

**Illustrative Problem.**—Find the maximum intensity of the shearing stress on the web of a 12-in., 28.5-lb. Bethlehem I-beam of 12-ft. span carrying a total uniformly distributed load of 32,000 lb., and compare it with the average stress assuming the total shear to be uniformly distributed over the web.

As the maximum stress will be at the neutral axis, \(Q\) is to be taken for half the cross-sectional area.

- \(Q\) for flange rectangle (Fig. 10) = \(6\ 12\times(0.33)\times(5.835) = 11.80\)
- \(Q\) for 2 flange triangles = \((2.935)\times(0.30)\times(5.57) = 4.90\)
- \(Q\) for half web = \((5.67)\times(0.25)\times(2.835) = 4.02\)

\[
\begin{align*}
\text{Total static moment} & = 20.72 \\
\text{Moment of inertia, } I \text{ of beam} & = 216.2 \\
\text{End shear} & = 32,000/2 = 16,000 \text{ lb.}
\end{align*}
\]

Shearing stress, horizontal or vertical, at neutral axis

\[
v = \frac{(20.72)\times(16,000)}{(216.2)\times(0.25)} = 6,130 \text{ lb. per sq. in.}
\]

Assuming the total shear as uniformly distributed over the web

\[
v = \frac{16,000}{(12)\times(0.25)} = 5,333 \text{ lb. per sq. in.}
\]

The actual stress is, therefore, 15.0 per cent in excess of the average.

To simplify the design of beams for shear by the accurate formula, Table 6 has been prepared. It contains for a wide range of rolled I-beams, girder beams and channels the following quantities:

\(^1\) See Sec. 1, Art. 81.
Table 6.—Relation of True Maximum Shearing Stress to Average Shearing Stress for Rolled Beams, Girders and Channels

Maximum stress, $v_m = \frac{Q}{It}$, $V$

Average stress, $v_a = \frac{1}{dt} \cdot V$

<table>
<thead>
<tr>
<th>Section (in.) (lb.)</th>
<th>Statical moment of $\frac{1}{2}$ gross area about neutral axis, $Q$</th>
<th>$\frac{Q}{It}$</th>
<th>$\frac{1}{dt}$</th>
<th>Excess of $v_m$ over $v_a$ (per cent)</th>
<th>Section (in.) (lb.)</th>
<th>Statical moment of $\frac{1}{2}$ gross area about neutral axis, $Q$</th>
<th>$\frac{Q}{It}$</th>
<th>$\frac{1}{dt}$</th>
<th>Excess of $v_m$ over $v_a$ (per cent)</th>
</tr>
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<tbody>
<tr>
<td>Standard and Carnegie I-beams</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>0.96</td>
<td>2.26</td>
<td>1.96</td>
<td>15.3</td>
<td>26 × 90.0</td>
<td>133.4</td>
<td>0.085</td>
<td>0.083</td>
<td>17.4</td>
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<td>0.36</td>
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<td>0.32</td>
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<td>0.46</td>
<td>14.8</td>
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<td>67.5</td>
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<td>0.16</td>
<td>14.8</td>
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<td>0.28</td>
<td>0.24</td>
<td>22.1</td>
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<td>0.10</td>
<td>0.083</td>
<td>18.5</td>
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<td>0.38</td>
<td>14.9</td>
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<td>0.12</td>
<td>14.6</td>
</tr>
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<td>0.15</td>
<td>23.0</td>
<td>20 × 112.0</td>
<td>134.7</td>
<td>0.10</td>
<td>0.091</td>
<td>14.8</td>
</tr>
<tr>
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<td>0.37</td>
<td>0.32</td>
<td>15.0</td>
<td>20 × 140.0</td>
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<td>0.090</td>
<td>0.078</td>
<td>15.3</td>
</tr>
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<td>19.4</td>
<td>0.17</td>
<td>0.14</td>
<td>22.8</td>
<td>24 × 120.0</td>
<td>172.9</td>
<td>0.090</td>
<td>0.079</td>
<td>14.9</td>
</tr>
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<td>12 × 31.8</td>
<td>20.8</td>
<td>0.28</td>
<td>0.24</td>
<td>15.6</td>
<td>24 × 140.0</td>
<td>201.2</td>
<td>0.080</td>
<td>0.070</td>
<td>14.7</td>
</tr>
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<td>12 × 55.0</td>
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<td>0.13</td>
<td>0.10</td>
<td>21.8</td>
<td>26 × 150.0</td>
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<td>0.071</td>
<td>0.061</td>
<td>15.7</td>
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<td>0.19</td>
<td>0.16</td>
<td>16.2</td>
<td>26 × 160.0</td>
<td>248.8</td>
<td>0.070</td>
<td>0.061</td>
<td>14.9</td>
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<tr>
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<td>0.093</td>
<td>0.077</td>
<td>21.4</td>
<td>28 × 165.0</td>
<td>271.9</td>
<td>0.063</td>
<td>0.054</td>
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<td>0.12</td>
<td>16.9</td>
<td>30 × 180.0</td>
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<td>0.056</td>
<td>0.048</td>
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<td>0.083</td>
<td>0.070</td>
<td>18.6</td>
<td>30 × 200.0</td>
<td>353.3</td>
<td>0.052</td>
<td>0.044</td>
<td>16.0</td>
</tr>
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<td>0.10</td>
<td>17.2</td>
<td>Standard</td>
<td></td>
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<tr>
<td>20 × 100.0</td>
<td>99.4</td>
<td>0.069</td>
<td>0.057</td>
<td>20.5</td>
<td>channels</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24 × 79.9</td>
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<td>0.083</td>
<td>16.2</td>
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<td>2.40</td>
<td>1.96</td>
<td>22.1</td>
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<tr>
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<td>0.056</td>
<td>20.4</td>
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<td>0.86</td>
<td>1.15</td>
<td>0.93</td>
<td>24.5</td>
</tr>
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<td>0.082</td>
<td>0.07</td>
<td>15.4</td>
<td>6 × 8.2</td>
<td>2.6</td>
<td>0.99</td>
<td>0.83</td>
<td>18.5</td>
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<td>Bethlehem I-beams</td>
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<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>15.0</td>
<td>7 × 19.75</td>
<td>6.1</td>
<td>0.30</td>
<td>0.23</td>
<td>29.8</td>
</tr>
<tr>
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<td>0.30</td>
<td>0.26</td>
<td>18.2</td>
<td>8 × 11.5</td>
<td>4.8</td>
<td>0.67</td>
<td>0.57</td>
<td>18.4</td>
</tr>
<tr>
<td>12 × 28.5</td>
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<td>0.38</td>
<td>0.33</td>
<td>15.0</td>
<td>8 × 21.25</td>
<td>7.6</td>
<td>0.28</td>
<td>0.22</td>
<td>28.7</td>
</tr>
<tr>
<td>12 × 36.0</td>
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<td>0.31</td>
<td>0.27</td>
<td>15.4</td>
<td>9 × 13.4</td>
<td>6.2</td>
<td>0.57</td>
<td>0.48</td>
<td>18.3</td>
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<tr>
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<td>0.27</td>
<td>0.23</td>
<td>15.3</td>
<td>9 × 25.0</td>
<td>10.1</td>
<td>0.23</td>
<td>0.18</td>
<td>28.4</td>
</tr>
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<td>62.9</td>
<td>0.15</td>
<td>0.13</td>
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<td>0.49</td>
<td>0.42</td>
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<td>49.2</td>
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<td>0.17</td>
<td>10.8</td>
<td>10 × 35.0</td>
<td>15.2</td>
<td>0.16</td>
<td>0.12</td>
<td>31.3</td>
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<td>50.3</td>
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<td>0.11</td>
<td>15.0</td>
<td>12 × 20.7</td>
<td>12.7</td>
<td>0.35</td>
<td>0.29</td>
<td>21.0</td>
</tr>
<tr>
<td>20 × 59.0</td>
<td>68.0</td>
<td>0.15</td>
<td>0.13</td>
<td>15.9</td>
<td>12 × 40.0</td>
<td>21.2</td>
<td>0.14</td>
<td>0.11</td>
<td>29.0</td>
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<td>0.10</td>
<td>0.088</td>
<td>18.1</td>
<td>15 × 33.9</td>
<td>25.2</td>
<td>0.20</td>
<td>0.17</td>
<td>21.0</td>
</tr>
<tr>
<td>24 × 73.0</td>
<td>101.0</td>
<td>0.12</td>
<td>0.11</td>
<td>15.8</td>
<td>15 × 55.0</td>
<td>36.0</td>
<td>0.11</td>
<td>0.082</td>
<td>28.9</td>
</tr>
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<td>0.091</td>
<td>16.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(1) The statical moment, \( Q \), of one-half of the gross area of the section about the neutral axis.

(2) The coefficient \( \frac{Q}{It} \), by which the total vertical shear, \( V \), is to be multiplied to give the maximum shearing stress, \( v_m \), according to the exact theory.

(3) The coefficient \( \frac{1}{dt} \), by which the total vertical shear, \( V \), is to be multiplied to give the approximate, or average, shearing stress, \( v_a \), assuming that the total shear, \( V \), is uniformly distributed over the web area \( dt \).

(4) The percentage by which the true maximum shearing stress exceeds the approximate or average shearing stress.

While in general only two places of decimals have been given for the coefficients \( \frac{Q}{It} \) and \( \frac{1}{dt} \), the percentages in the last column were calculated from coefficients with three places of decimals. The sections for which the quantities have been listed are, in most cases, the minimum and maximum weights rolled for each depth. The values for intermediate sections can be approximated by interpolating in the following manner:

Where sections are of the same section index or number, being produced by the same rolls through slightly varying their distances apart, the figures for sections of the same depth as those listed, but of intermediate weight, may be determined on the basis of relative weights. Thus, the widening of the rolls a distance \( \Delta t \), increases the statical moment by

\[
\Delta Q = \frac{1}{2} \Delta t \left( \frac{d}{2} \right)^2
\]

where \( d \) is the depth of the beam, or decreases it if \( \Delta t \) be regarded as the narrowing of the rolls from the width required for an upper weight section. Having the figures in the table for any beam of the same section number as that being investigated, the percentage change in \( Q \), or the value of \( Q \) for the section in hand, may be found. The value of the coefficient \( \frac{Q}{It} \) is then easily determined by dividing \( Q \) by quantities found in the handbooks. As the coefficient \( \frac{1}{dt} \) is readily found for any beam, the excess of the maximum over the average shearing stress is easily determined.

Table 6 may be used in any one of several ways. The true maximum shearing stress, \( v_m \), may be obtained by multiplying the total shearing force, \( V \), by the appropriate coefficient \( \frac{Q}{It} \), interpolating as explained between the values given for intermediate weights of sections. The average shearing stress, \( v_a \), may be obtained in similar manner by multiplying the shearing force, \( V \), by the coefficient \( \frac{1}{dt} \). From this approximate stress the true maximum stress may be obtained by increasing the former by the appropriate percentage in the last column, or by a percentage obtained by proper interpolation.

An examination of the table shows that the true maximum stresses exceed the average or approximate stresses by from about 10 to 30 per cent, and generally from 15 to 20 per cent. The error involved in using the approximate method of
design is greater for the maximum weights of sections than for the minimum weights. This is because the former section approaches the rectangle more nearly than the latter. Particularly large differences occur for the maximum weights of channels in each depth, reaching in one case over 31 per cent. By reason of the fact that there is less difference in weight between the minimum and maximum Bethlehem sections for a given depth, the percentage excess of true over approximate stresses is more nearly constant than for standard sections.

Illustrative Problem.—The maximum shear on a 9-in., 25-lb. I-beam is 35,000 lb. Compute the true maximum shearing stress on the cross-section.

Since for a 9-in., 25-lb. I the increment to the statical moment 10.8 (Table 6) for a 9-in., 21.8-lb. I is

$$\Delta Q = \frac{1}{2} \Delta \alpha \left(\frac{d}{2}\right)^2 = \left(\frac{1}{2}\right)(0.107)(4.5)^2 = 1.1$$

the statical moment for the heavier beam becomes $10.8 + 1.1 = 11.9$.

The coefficient $Q_{lt}$ is, therefore,

$$\frac{11.0}{(91.4)(0.397)} = 0.328$$

and the maximum shearing stress

$$v_m = (0.328)(V) = (0.328)(35,000) = 11,531 \text{ lb. per sq. in.}$$

16b. Horizontal Shearing Stress.—Since the horizontal and vertical shearing stresses at a point are equal, the maximum horizontal shearing stress occurs, as does the maximum vertical shearing stress, at the neutral axis of the beam. The calculation of its intensity at any point may be made in precisely the same manner as for the vertical shearing stress. Unlike timber, steel is able to take shearing stresses almost equally well in all directions—at least near enough so for purposes of ordinary design. The presence of possible horizontal lines of rivet holes along or near the neutral axis may render the beam weaker in horizontal shear than in vertical shear.

16c. Permissible Shearing Stress.—In proportioning steel beams for shear, it is commonly specified that the shearing stress on the gross section of the web considered as uniformly distributed shall not exceed 10,000 lb. per sq. in., or 62.5 per cent of the customary permissible tensile stress. Since the ultimate shearing strength of structural steel is about 75 per cent of the ultimate tensile strength, the shearing unit stress might appear too low. However, the shearing stress is really not uniformly distributed and since the presence of holes in the web somewhat reduces its strength, the unit is seen to be justifiable. More conservative specifications fix the safe shearing stress at 10,000 lb. per sq. in. on net area. In either case the gross area is taken as $dt$, where $d = \text{extreme depth of beam and } t = \text{thickness of web.}$

The effect of lines of holes in the plane of shear considered is, of course, to weaken the section to a degree proportionate to the number and size of the holes. If, for example, a vertical line of 1-in. holes 4 in. apart center to center lies on the section being investigated, the area has been reduced 25 per cent and the statical moment by an equal amount. The moment of inertia of the web, if deep, is reduced in approximately the same ratio, so that the shearing stresses would be increased approximately in the same ratio as the area is decreased. For a further discussion of this point see Art. 41.
To facilitate the design of beams for shear by the approximate or average method, tables of the safe shearing capacity of rolled beams are inserted in the handbooks. In the tables of safe bending loads the upper limit of loads beyond which excessive shearing stresses (or really web crippling stresses) would be produced are indicated, thus making it easy to avoid sections weak in shear. No provision for loss of section by holes is made except in the lowness of the prescribed working stress.

Illustrative Problem.—If the permissible shearing stress on the webs of beams is 10,000 lb. per sq. in., gross area considered as uniformly distributed, report on the safety in shear of a 15-in., 42.9-lb. I-beam supporting a total uniformly distributed load of 150,000 lb.

Total end shear, \( V = (1/4)(150,000) = 75,000 \text{ lb.} \)

Gross area of web = \((15)(0.410) = 6.15 \text{ sq. in.} \)

Average shearing stress on web = \( 75,000/6.15 = 12,200 \text{ lb. per sq. in.} \) As this is greater than the prescribed shearing stress, the beam is unsafe in shear. A 15-in., 50-lb. 1 with a \( 1/4 \text{-in.} \) web would be adequate.

17. Diagonal Buckling of Web.—Although the web of a beam may be safe so far as either vertical or horizontal shearing stresses are concerned, it may be unsafe for the resistance of the diagonal compression resulting from a combination of shearing and flexural stresses. As has been shown elsewhere in this volume, both the magnitude and the direction of the resultant compressive stress at a point depends on the relative intensities of these two kinds of stress. If \( f_m \) = the maximum compressive or tensile stress at any point on either side of the neutral axis; \( f \) = the flexural stress at the point; and \( v \) = the shearing stress; then, as has been shown,\(^1\)

\[
f_m = \frac{f}{2} \pm \sqrt{\frac{1}{2}f^2 + v^2}
\]

The positive sign is to be used for compressive stresses above the neutral axis and for tensile stresses below it, the negative sign is for tensile stresses above and compressive stresses below the neutral axis.

At the center section of a uniformly loaded beam, the shear is zero and the resultant stress at any point on the section is horizontal. On the other hand, near the support, the flexural stress, \( f \), approaches zero and the resultant diagonal stress makes an angle of nearly 45 deg. with the horizontal throughout the greater part of the depth of the beam. For beams carrying concentrated loads, the shear may be practically constant, and very near its maximum for a large portion, or perhaps all, of its length. In such cases, therefore, the highest diagonal stresses are likely to occur near the section of maximum moment. Such is also true for cantilever, restrained, and continuous beams.

The maximum diagonal compression existing in the web is particularly likely to arise at some point near the junction of the web with the flanges, where a large flexural stress is augmented by the shearing stress. At the ends of beams of ordinary length, the diagonal compressive stress may not be so large as exists at points near the center, but if the beam be short and the shearing stresses heavy, the critical region so far as web crippling is concerned is likely to be near a support. Near this point the diagonal compression throughout the depth of the beam may be regarded as equal in intensity to the vertical or hori-

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\(^1\) See Sec. 1, Art. 53.
zontal shearing stress and as making an angle of 45 deg. with the neutral axis relations that are exactly true at the neutral axis.

Because of its relative thinness, the web of a beam tends to buckle or cripple under the action of the diagonal compressive stresses, but such action cannot proceed in the same manner as if the strip of web under consideration were an isolated bar of metal. At right angles to the strip there are tensile stresses which at the neutral axis have a magnitude equal to the compressive stresses, tending to restrain the strip from buckling, and it is therefore in a more favorable condition than a true column.

A rational method of proportioning a beam so that the compressive stresses in the web will not cause failure through buckling, is that followed in the Cambria handbook. It is assumed that the safe stress on a strip of web making an angle of 45 deg. with the neutral axis is represented fairly by the Rankine formula for fixed-ended columns,

\[ p = \frac{12,000}{1 + \frac{l^2}{36,000r^2}} \]

where \( p \) = safe compressive stress in lb. per sq. in.
\( l \) = length of diagonal strip between fillets.
\( r \) = radius of gyration of the web normal to its plane.

If \( h \) = the clear vertical distance between fillets, and \( t \) = thickness of web,

\[ p = \frac{12,000}{1 + \frac{h^2}{1,500 \bar{t}^2}} \]  \( \text{(1)} \)

If the equality of shearing and diagonal compressive stresses which exists at the neutral axis is assumed to hold throughout the depth of the beam, the average shearing stress in the beam web should also incidentally not exceed \( p \), and so web crippling may be conveniently provided for by ensuring that the average shearing stress comes within the requirement of Formula (1). This stress is to be regarded as uniformly distributed over the area \( dt \), where \( d \) = the depth of the beam.

By applying the Carnegie column formula, \( p = 19,000 - 100 \frac{l}{r} \) to a diagonal strip of web in the same manner as above, the formula for the safe shearing stress based on diagonal buckling becomes

\[ p = 19,000 - 490 \frac{h}{l} \]  \( \text{(2)} \)

The American Railway Engineering Association formula, \( p = 15,000 - 50 \frac{l}{r} \) becomes

\[ p = 15,000 - 245 \frac{h}{l} \]  \( \text{(3)} \)

The A.R.E.A. formula for safe compressive resistance of webs on which the formula for stiffener spacing is based may also be adapted to apply to the webs of beams. This formula, to which reference is made in Art. 52, is

\[ p = 12,000 - 40 \frac{d}{l} \]
where \( d \) = the distance between rivet lines of stiffeners in inches, and \( p \) and \( t \) are as previously defined. For the average case, \( d \) is approximately 1.07 \( h' \), where \( h' \) is the clear distance between stiffeners, and if the measurement be a vertical one, \( d \), the distance between near lines of rivets, is also approximately 1.07\( h \), where \( h \) is the clear distance between flange angles. Consequently, the above formula when expressed in terms of \( \frac{h}{t} \) becomes

\[
p = 12,000 - 43 \frac{h}{t}
\]

(4)

A study of web crippling of I-beams was made by H. F. Moore in Bulletin No. 68 of the University of Illinois Engineering Experiment Station in connection with a general inquiry into the strength of I-beams in flexure. This study was continued by Mr. Moore and W. M. Wilson in Bulletin 86 of the same series. From the records of web failures there presented, the diagonal compressive stresses at mid-web accompanying failure and the ratio of the depth of the web between fillets to its thickness are seen to be as follows:

<table>
<thead>
<tr>
<th>Size of beam</th>
<th>Thickness ratio of web ( \frac{h}{t} )</th>
<th>Computed diagonal compressive stress at mid-web (lb. per sq. in.)</th>
<th>Size of beam</th>
<th>Thickness ratio of web ( \frac{h}{t} )</th>
<th>Computed diagonal compressive stress at mid-web (lb. per sq. in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulletin 68:</td>
<td></td>
<td></td>
<td>Bulletin 86:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12-in., 31.8-lb. I</td>
<td>30.1</td>
<td>25,800</td>
<td>12-in. (web planed thin)</td>
<td>48.0</td>
<td>18,700</td>
</tr>
<tr>
<td>12-in., 31.8-lb. I (web planed thin)</td>
<td>37.4</td>
<td>27,200</td>
<td>12-in. (web planed thin)</td>
<td>55.0</td>
<td>19,500</td>
</tr>
<tr>
<td>12-in., 31.8-lb. I (web planed thin)</td>
<td>55.3</td>
<td>27,400</td>
<td>12-in. special built-up girder</td>
<td>101.0</td>
<td>20,000</td>
</tr>
<tr>
<td>12-in., 31.8-lb. I (web planed thin)</td>
<td>65.0</td>
<td>21,400</td>
<td>12-in. (web planed thin)</td>
<td>56.0</td>
<td>17,300</td>
</tr>
<tr>
<td>12-in., 31.8-lb. I (web planed thin)</td>
<td>65.0</td>
<td>21,400</td>
<td>12-in. (web planed thin)</td>
<td>53.0</td>
<td>17,100</td>
</tr>
<tr>
<td>30-in., 17.5-lb. Bethlehem girder beam</td>
<td>38.6</td>
<td>14,800</td>
<td>12-in. (web planed thin)</td>
<td>52.0</td>
<td>18,300</td>
</tr>
<tr>
<td>20-in. special built-up girder</td>
<td>99.7</td>
<td>26,500</td>
<td>12-in. (web planed thin)</td>
<td>54.0</td>
<td>18,800</td>
</tr>
</tbody>
</table>

In Fig. 11 the diagonal compressive stresses at failure divided by 2\( \frac{3}{4} \)—a reasonable factor of safety for columns—have been plotted with respect to the thickness ratio of the web, \( h/t \). Except for the extraordinary strength of the special built-up girders, the general trend is clearly downward with increasing thickness ratio.

On this diagram several formulas for permissible stresses in web crippling have been plotted. The Euler formula for fixed ends, assuming \( E = 28,000,000 \) lb. per sq. in., has been expressed in terms of the thickness ratio \( h/t \) and divided by 2\( \frac{3}{4} \), to give the plotted working formula

\[
p = \frac{16,700,000}{(\frac{h}{t})^2}
\]

(5)

This formula fits the results for the lower values of \( h/t \) very well, but gives stresses that are too low for the higher values of \( h/t \).
The Cambria formula (1), when plotted in relation to the tests appears to be too severe.

The adapted Carnegie formula (2) when plotted shows relatively very low values and the adapted A.R.E.A. formula (3) values that are nearly as low. The A.R.E.A. stiffener formula (4) gives values that appear too high for I-beams. None of these formulas conform at all closely with all the test results shown.

A straight line formula with two segments might be made to conform very well to the test results for I-beams without departing too radically from accepted practice. Such a formula has also, as compared with those of the Euler or the Rankine type, the merit of greater simplicity. It is recommended that for values of \( h/t \) up to 60, the following working formula be used:

\[
p = 15,000 - 150 \frac{h}{t}
\]  

(6)

For values of \( h/t \) over 60, a reasonable formula would be

\[
p = 10,200 - 70 \frac{h}{t}
\]  

(7)

The relation of the graph of these formulas to the tests and to other formulas is shown in Fig. 11.

If the shearing stress on a given beam is found to be in excess of that permitted by the particular web crippling formula used, the difficulty is best met by selecting a beam with a thicker web. It is not economical to use stiffeners to stay the webs of rolled beams, although it may be for plate girders, and the reinforcement of the web by riveting plates to it is only justifiable where the rolled section used is the heaviest available.
Illustrative Problem.—A 12-in., 31.8-lb. I carries a total uniformly distributed load of 70,000 lb. Investigate its safety against web crippling using the Cambria formula.

Thickness of web = 0.35 in.

Average shearing stress on web = \( \frac{35,000}{(12)(0.35)} = 8,340 \text{ lb. per sq. in.} \)

Clear distance between fillets = 9.75 in.

Safe shearing stress to provide against web crippling is

\[
p = \frac{12,000}{1 + \left( \frac{9.75}{1,500}\frac{0.35}{12} \right)^2} = 7,900 \text{ lb. per sq. in.}
\]

Hence the beam is not safe, according to the specification. A beam with a thicker web should be selected.

Illustrative Problem.—Select a 15-in. I-beam capable of taking safely an end shear of 50,000 lb. without giving rise to dangerous web crippling stresses, according to Formulas (6) and (7).

Assume a 15-in., 37.3-lb. Carnegie I. Area of web = \((15)(0.332) = 4.98 \text{ sq. in.} \)

Average shearing stress = 50,000/4.98 = 10,050 lb. per sq. in.

Permissible shearing stress (based on web crippling) = \( p = 15,000 - (150) \left( \frac{12.25}{0.332} \right) = 9,460 \text{ lb. per sq. in.} \), using Formula (6) which applies, as \( \frac{h}{t} \) is less than 60.

The section is, therefore, not quite adequate.

Assume a 15-in., 41-lb. Bethlehem I.

Average shearing stress = \( 50,000/(15)(0.34) = 9,800 \text{ lb. per sq. in.} \). Permissible shearing stress = \( 15,000 - (150) \left( \frac{12.875}{0.34} \right) = 9,320 \text{ lb. per sq. in.} \). This section is also slightly below the requirement for web crippling.

Try a 15-in., 42.9-lb. Carnegie I. Average shearing stress = \( 50,000/(15)(0.41) = 8,130 \text{ lb. per sq. in.} \). Permissible shearing stress = \( 15,000 - (150) \left( \frac{12.50}{0.41} \right) = 10,430 \text{ lb. per sq. in.} \). This section is adequate.

18. Vertical Buckling of Web.—While the diagonal buckling effect in the web considered above exists and must be provided for even at points remote from the supports or from concentrated loads, a beam to be safe, so far as the web is concerned, must be capable of safely withstand ing concentrated loads or loads distributed over only a short length of the beam. Concentrated loads may be applied to the compression flange, to the web by means of brackets or connection angles, or, as occurs in every beam no matter how loaded, as a vertical (and usually upward) load at the support.

Based upon a series of unpublished tests on beams of various depths and web thicknesses, the safe end reaction \( R \) and the safe interior concentrated load \( W \) are given in the Carnegie Pocket Companion. The formulas, with slight modification, are as follows:

\[
R = pt \left( a + \frac{d}{4} \right) \tag{1}
\]

\[
W = pt \left( a_1 + \frac{d}{2} \right) \tag{2}
\]

In these formulas, \( t \) = web thickness, \( d \) = depth of beam, \( a \) = distance over which reaction is applied, \( a_1 \) = distance over which concentrated load is applied, \( p = \text{safe compressive resistance of web} = 19,000 - 173 \frac{d}{t} \). This permissible stress is not limited to 13,000 lb. per sq. in. in the tables of allowable buckling resistance given in Carnegie.
An examination of these formulas indicates that they are based on the conception of the vertical loads \( R \) or \( W \) being resisted through column action by a section of the web of height \( d \) and width parallel to the beam equal to the distance \( a \) or \( a_1 \) over which the load is applied plus one-quarter of the depth of the beam in the case of the reaction, or one-half the depth of the beam in the case of an interior load. Formula (2) is supposed to apply strictly only to the case of a single load concentrated at the center of the span. Some designers prefer the more conservative rule of considering the effective strip of the web as equal to not over the length of bearing of the load. The permissible compressive stress in the above formulas is based on the Carnegie column formula

\[
p = 19,000 - 100 \frac{l}{r},
\]

\( l \) being taken as \( d/2 \), since the compression in the strip of web is not constant throughout its depth, but varies from a maximum at one end to zero at the other. It is, however, more convenient to base the formula on \( d/l \) than on \( d/r \).

In a similar manner other web buckling formulas might be evolved using any accepted column formula as the basis—for example, the formula of the American Railway Engineering Association

\[
p = 15,000 - 50 \frac{l}{r}
\]

Letting \( l = d/2 \) and \( r = \sqrt{\frac{d}{l}} \), the safe buckling resistance of the web would be

\[
p = 15,000 - 87 \frac{d}{l}
\]

(3)

The width of the column might be assumed as the length of bearing of the load plus any approved fraction of the depth of the beam.

By replacing \( d \) in the Carnegie formula, \( p = 19,000 - 173 \frac{d}{l} \) and in the adapted A.R.E.A. formula \( p = 15,000 - 87 \frac{d}{l} \) by its approximate average value in terms of \( h \)—that is 1.25 \( h \)—these formulas in terms of \( \frac{h}{l} \) become respectively

\[
p = 19,000 - 216 \frac{h}{l}
\]

(4) and

\[
p = 15,000 - 109 \frac{h}{l}
\]

(5)

Although Formulas (6) and (7) of Art. 17 are intended to give only the safe diagonal buckling stress, they are sufficiently severe to be applied to vertical buckling. Up to a value of \( \frac{h}{l} = 60 \), Formula (6) of Art. 17, that is

\[
p = 15,000 - 150 \frac{h}{l}
\]

gives smaller values of the permissible compressive stress than do Formulas (4) and (5) of this article.

Due to exceptionally heavy concentrated loads applied to the compression flange, or perhaps even to the web itself, the thickness of web, although possibly adequate for ordinary diagonal compression arising from the combination of
shearing and flexural stresses, may need to be increased. An alternative procedure is to use stiffeners or to reinforce the web immediately under the load. For rolled beams it is generally desirable to use a beam with a thicker web, where such does not involve a very great increase in weight, rather than to use stiffeners. Reinforcement of the web is adopted only where certain restrictions respecting depth or availability of sections make the employment of a given section necessary.

Illustrative Problem.—A rolled beam resting on two columns 20 ft. apart supports two symmetrically placed loads of 70,000 lb., each 1.5 ft. from the supports, as shown in Fig. 12. Find the required size of the beam, if the permissible stresses are as follows: Flexure, 16,000 lb. per sq. in.; maximum shearing stress at neutral axis, 12,000 lb. per sq. in.; average shearing stress based on diagonal web buckling, \( p = \frac{12,000}{\frac{1}{2} h^2} \); maximum vertical compression at support to be according to Carnegie formula (1) of this article. Assume a 12-in. support.

Assume an 18-in., 48.2-lb. I, having web thickness of 0.38 in. and section modulus of 81.9.

Maximum moment due to concentrated loading = (70,000)(1.5) = 105,000 ft.-lb.

Maximum moment due to weight of beam = \( (\frac{W}{12})(48.2)(20)^2 \) = 2,410 ft.-lb.

Total maximum moment = (107,410)-

(12) = 1,289,000 in.-lb.

Section modulus required 1,289,000 -

16,000 = 80.5. The section selected is, therefore, adequate for moment.

Maximum end shear = 70,000 + (10)-

(48.2) = 70,480 lb.

Average shearing stress

\[ \tau_a = \frac{70,480}{(18 \times 0.38)} = 10,310 \text{ lb. per sq. in.} \]

Increasing this by correction of 17 per cent (Table 6, Art. 16), the maximum shearing stress

\[ \tau_m = (10,310)(1.17) = 12,079 \text{ lb. per sq. in.} \]

The section is, therefore, sufficiently strong for shear also.

Safe shearing stress based on diagonal buckling of web

\[ p = \frac{12,000}{1 + \frac{(15.25)^2}{(1.500)(0.547)^2}} = 6,000 \text{ lb. per sq. in.} \]

This is very much below the existing average shearing stress of 10,310 lb. per sq. in. and the section must be increased.

Try an 18-in., 60-lb. I which has a 0.547-in. web.

Average shearing stress, taking the revised end shear as 70,700 lb., is

\[ \tau_a = \frac{70,700}{(18)(0.547)} = 7,180 \text{ lb. per sq. in.} \]

Safe shearing stress based on buckling diagonally

\[ p = \frac{12,000}{1 + \frac{(15.25)^2}{(1.500)(0.547)^2}} = 7,900 \text{ lb. per sq. in.} \]

This section is, therefore, adequate for diagonal buckling.
Vertical compressive stress at support, from Formula (1)

\[ f = \frac{R}{(a + \frac{d}{t})t} = \frac{70,700}{(12 + \frac{18}{4})(0.547)} = 7,820 \text{ lb. per sq. in.} \]

Permissible vertical compression in web

\[ p = 19,000 - 173 \frac{d}{t} = 19,000 - (173) \left( \frac{18}{0.547} \right) = 13,310 \text{ lb. per sq. in.} \]

which is much greater than the existing stress.

The 18-in., 60-lb. I is, therefore, adequate in all respects.

It is assumed that the web is reinforced, if necessary, to take the concentrated superimposed loads.

**Illustrative Problem.**—A 6-in. H-column with a load of 85,000 lb. is supported on the top flange of a 20-in., 59-lb. Bethlehem I as shown in Fig. 13. Determine whether the web will carry the concentrated load without stiffening or reinforcement if the concentration be considered to be distributed over a length equal to the width of the column shaft plus one-half the depth of the beam. \[ p = 19,000 - 173 \frac{d}{t} \] (Carnegie formula).

Length of web over which concentration of 85,000 lb. is concentrated \[ = 6 + \frac{20}{2} = 16 \text{ in.} \]

Compressive stress on web

\[ f = \frac{85,000}{(16 \times 0.50)} = 10,630 \text{ lb. per sq. in.} \]

Permissible compressive stress

\[ p = 19,000 - (173) \left( \frac{20}{0.375} \right) = 9,780 \text{ lb. per sq. in.} \]

Stiffeners are therefore required under the load. It is recommended that two \( 3 \times 2\frac{1}{2} \times \frac{3}{4} \)-in. angles be placed vertically on each side of the web so that the outstanding 3-in. legs are directly under the flanges of the column. Four \( \frac{3}{4} \)-in. rivets in each angle are sufficient, as with the stiffeners ground to fit the beam flanges, most of the load in them is transferred by end bearing.

**Illustrative Problem.**—A 15-in., 33.9-lb. channel carries at one point the ends of two 12-in. I-beams in the manner shown in Fig. 14. The reaction of each beam is 13,000 lb. Investigate the compressive stresses in the web under the combined load, assuming that the flange. Permissible compressive stress \[ p = 15,000 - 109 \frac{h}{t} \] the adapted A.R.E.A. formula (5).

Total concentrated load on 5 in. of web \[ = 26,000 \text{ lb.} \]

Vertical compressive stress in web due to concentration, and neglecting compression due to combination of shearing and flexural stresses

\[ = \frac{26,000}{(5)(0.40)} = 13,000 \text{ lb. per sq. in.} \]
Permissible compressive stress on unstiffened web = \( p = 15,000 - (109) \left( \frac{12.25}{0.40} \right) \) = 11,660 lb. per sq. in. It will, therefore, be necessary to stay the web by two stiffener angles under the shelf angle, as shown in Fig. 14. These would not be necessary for stiffening the shelf angle under the existing load, with an angle \( \frac{1}{4} \) in. thick or over, but they are required to prevent the web from buckling. Two \( \frac{3}{4} \)-in. rivets in each angle under the shelf angle will be sufficient, under any ordinary specification. The stiffeners may be two \( 3 \times 2\frac{1}{2} \times 5\frac{1}{2} \) in. angles.

**Illustrative Problem.**—A double-tier steel grillage carrying a total load (including the weight of the 20- \( \times \) 20-in. column base) of 600,000 lb. has to be made up of 12-in., 31.8-lb. I-beams, reinforced if necessary. Three beams, 5 ft. long, constitute the upper tier, and 7 lines of 8-ft. beams of the same size the lower tier, as shown in Fig. 15. Investigate the web crippling and vertical compressive stresses in the beams of the two tiers, assuming that the beams without reinforcement are sufficient for bending moment. Permissible diagonal web crippling and web vertical compressive stress = \( p = 15,000 - 150 \frac{h}{l} \). Assume the direct compression to be distributed over a length of web equal to the length of bearing.

Total shear across beams of upper tier at edge of column base = \( \left( \frac{1}{3} \right) \times (600,000) = 200,000 \) lb.

Average shearing stress on unreinforced beam webs

\[
\frac{200,000}{(3)(12)(0.35)} = 15,900 \text{ lb. per sq. in.}
\]

Total area of webs of beams of upper tier resisting 600,000-lb. vertical compression, \( a_{1} \) being 20-in. = \( (3)(20)(0.35) = 21 \text{ sq. in.} \)

Compressive stress on beam webs \( f = 600,000/21 = 28,600 \text{ lb. per sq. in.} \)

Permissible shearing stress in order that web crippling stresses may not be excessive, and also permissible vertical compressive stress under concentrated loading = \( p = 15,000 - (150) \left( \frac{0.75}{0.35} \right) = 10,820 \text{ lb. per sq. in.} \) It is thus evident that the beams of the upper tier, without reinforcement, would be overstressed by ordinary web crippling due to the combination of shearing and flexural stresses, and very seriously overstressed by the direct compression of the column load.
Add one 1/4-in. plate—the thinnest practicable plate—to each side of the web, of sufficient width, when ground at the edges, to fit tightly against both upper and lower flanges. Thickness of reinforced web = 0.35 + (2)(0.3125) = 0.97 in.

Average shearing stress on reinforced web now becomes
\[ \tau_a = \frac{200,000}{(3)(12)(0.97)} = 5,720 \text{ lb. per sq. in.} \]
or much below that allowed.

Compressive stress on reinforced webs under column base
\[ f = \frac{600,000}{(3)(20)(0.97)} = 10,320 \text{ lb. per sq. in.} \]

Permissible compressive stress for reinforced web, \(p = 15,000 - (150)(\frac{9.75}{0.97})\) = 13,490 lb. per sq. in. The reinforcement provided is adequate.

The reinforced plates may be dispensed with, in theory, at a distance out from the edge of the column base where the total shear is such as to give a shearing stress on the reinforced web within the permissible stress given by the formula specified.

Shearing capacity of three webs = (3)(12)(0.35)(10,820) = 136,400 lb. This shear would exist at a distance from the edge of the column base
\[ x = 20 - \left( \frac{136,400}{200,000} \right)(20) = 6.4 \text{ in.} \]

The plates should, however, be carried far enough beyond this point of theoretical ending to accommodate at least one vertical row of rivets, so that the projection would probably be at least 11 in. In such cases the plates are often carried to the ends of the beams.

As the vertical compressive stress at the junction of the web and flange of the unrebuckled beam is 28,000 lb. per sq. in. and in excess of the usual permissible stress in bearing — 20,000 to 24,000 lb. per sq. in.—the horizontal edges of the reinforcing plates should be ground to fit both the upper and lower flanges of the beams.

Since the 5-in. flanges of the 3 lines of beams of the upper tier cross all 7 lines of the lower tier, the area of webs of the beams of the lower tier resisting the 600,000-lb. load = (21)(5)(0.35) = 36.8 sq. in.

Compressive stress, \(f = \frac{600,000}{36.8} = 16,300 \text{ lb. per sq. in.} \)

To provide for this vertical compressive stress due to the load from the upper tier, the beams of the lower tier would need to be reinforced similarly to those of the upper tier for a central length of about 27 in. If beams with heavier webs were available, it would be much more economical to employ them.

The diagonal web crippling stress in this case of the beams of the lower tier is well within the permissible limit.

The reinforcing plates on the webs of the beams should be attached by rivets spaced vertically not over 5 in. apart and preferably less.


19a. Beams with Constant Section.—Although a beam may be strong enough to ensure that under the greatest loads ever likely to be applied, it will not fail, the proportions may be such as to bring about objectionable and even alarming deflection. No harm may come to the beam itself because of the excessive deformation, but any very apparent sag, especially if it visibly increases during the imposition of a load, is likely to convey the impression of weakness to the observer. It is possible, too, that through large deflections, plastered ceilings may be cracked, tile, stone, or concrete floors may open out at the beam supports in a direction transverse to the beams, supported walls may crack, glass in nearby windows may be broken, doors may jam, or shafting or attached equipment or machinery may be thrown seriously out of line or level. If the deflection due to live load is large and frequently occurring, it promotes excessive vibration which may rock the structure, loosen rivets and necessitate constant and troublesome repairs.
Observation and experience show that the maximum deflection that it is safe to allow in a beam supporting plastered ceilings, after the ceilings have been plastered, is $\frac{1}{200}$ in. per ft. or $\frac{1}{2000}$ of the span. Stone or tile floors are likely to crack when the deflection of the supporting beams is less than this. Where such floors are carried, it is desirable to limit the deflection to about $\frac{1}{200}$ of the span. Obviously, only the deflection produced by loads applied to the beam after the ceiling is plastered, or the floor laid, need be considered, so far as the possibility of cracking is concerned. Wherever the situation will permit, it is desirable to keep down the deflection by using the deepest practicable beams. If no dependent construction will be affected by deflection—as, for example, the interior panels of a building with timber flooring not supporting mechanical equipment—the amount of deflection permissible is entirely a matter of appearance. It might for example, be $\frac{1}{200}$ or $\frac{1}{2000}$ of the span.

The maximum deflection of a beam is

$$
\Delta = K \cdot \frac{Wl^3}{EI}
$$

where $K$ is a constant depending upon the nature of the supports at the ends and the system of loading. Knowing $K$ and the other quantities involved, it is easy to compute the deflection. Wherever a steel handbook is available, however, it is much more convenient to calculate the deflection by means of coefficients than to use the deflection formula for the type of beam and loading under consideration. For example, if in the case of the simply supported uniformly loaded beam, for which the maximum deflection

$$
\Delta = \frac{5}{384} \cdot \frac{Wl^3}{EI}
$$

the load $W$ be replaced by its value $\frac{8fl}{lc}$, and the span be expressed in feet, that is $l = 12L$, the deflection may be expressed as

$$
\Delta = \frac{30fl^2}{E} \cdot \frac{1}{2c}
$$

Values of the coefficient $\frac{30fl^2}{E}$ for various spans are tabulated for the simply supported, uniformly loaded beam in the handbooks for $E = 29,000,000$ and for various common values of $f$, so that to determine the maximum deflection of a uniformly loaded beam of either symmetrical or unsymmetrical section it is only necessary to divide the appropriate coefficient by twice the distance from the neutral axis to the extreme fiber. For beams of symmetrical section, the divisor is obviously the depth of the beam.

Illustrative Problem.—A lintel consisting of two $6 \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles, placed with the 6-in. legs vertical and back to back, has a span of 10 ft., and carries a uniformly distributed load which produces an extreme fiber stress of 16,000 lb. per sq. in. What is the center deflection?

From the table in either Carnegie, Cambria or Bethlehem, the coefficient of deflection for a uniformly loaded beam simply supported at the ends and stressed in flexure to 16,000 lb. per sq. in. = 1.655.

Distance from neutral axis to extreme fiber = 3.96 in.

Hence center deflection

$$
\Delta = \frac{1.655}{(2)(3.96)} = 0.209 \text{ in.}
$$

1 See Sec. 1, Art. 66
For conditions other than those assumed in the tables, either the coefficients, or the resulting deflections, may be readily adjusted. From the nature of the coefficient it is seen that the deflection varies directly as the fiber stress, directly as the square of the span and inversely as \( E \). For other systems of loading than the uniform load and for other end conditions, the deflection found in the tables may be multiplied by the factors given in Table 7 to give the deflection under the same fiber stress for the same span.

**Table 7.—Relation of Maximum Deflections of Typical Beams Under Same Fiber Stress and for Same Span**

<table>
<thead>
<tr>
<th>System of loading</th>
<th>Factor by which deflection for simply supported uniformly loaded beam is to be multiplied</th>
<th>System of loading</th>
<th>Factor by which deflection for simply supported uniformly loaded beam is to be multiplied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply Supported Beam</td>
<td></td>
<td>Cantilever Beam</td>
<td></td>
</tr>
<tr>
<td>Uniform load</td>
<td>1.00</td>
<td>Uniform load</td>
<td>2.40</td>
</tr>
<tr>
<td>Single central load</td>
<td>0.80</td>
<td>Single end load</td>
<td>3.20</td>
</tr>
<tr>
<td>Two loads at ( \frac{1}{2} ) points</td>
<td>1.20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Useful tables are given in the Cambria handbook for simplifying the calculation of the deflection of a beam of any section having been given the load and the span. The coefficients \( N \) and \( N' \) there tabulated are the deflection for a simply supported beam 1 ft. long, loaded respectively by a uniformly distributed load of 1,000 lb. and a concentrated central load of the same amount. The deflection for a beam supporting any load of either of these types, and of any span, is found by multiplying the appropriate coefficient by the number of 1,000-lb. units in the load and by the cube of the span in feet.

In fixing the sizes of beams for a given situation, it is most desirable to select a depth that under the adopted working stress will be sure to give a deflection within the prescribed limit. This result will be attained if the depth of the beam is made not less than a certain fraction of the span, depending on the nature of the material, the end conditions, and the system of loading. If the limiting deflection be \( \frac{1}{360} \) of the span, and this be equated to the deflection expressed in terms of the fiber stress, the limiting depth ratio may be readily obtained. Thus, for uniform loading

\[
\Delta = \frac{5}{48} \cdot \frac{fl^2}{Ec} = \frac{l}{360}
\]

from which the maximum permissible span for the stated deflection \( l_{360} \) is found to be

\[
l = \frac{Ec}{37.5f}
\]

If \( E = 29,000,000, f = 16,000 \) lb. per sq. in., and the beam has a symmetrical cross-section, so that \( c = \frac{d}{2} \),

\[
l = 24.1d
\]

If the section be unsymmetrical,

\[
l = 48.2c
\]

\( c \) being the distance from the neutral axis to the extreme fiber.
For other types of loading, and for the values of $E$ and $f$ given above, the maximum span for which the deflection will be $\frac{l}{360}$ is given in Table 8 for beams of symmetrical cross-section.

<table>
<thead>
<tr>
<th>System of loading</th>
<th>Ratio of $\frac{l}{d}$ for $\Delta = \frac{l}{360}$</th>
<th>System of loading</th>
<th>Ratio of $\frac{l}{d}$ for $\Delta = \frac{l}{360}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simply Supported Beam</td>
<td></td>
<td>Cantilever Beam</td>
<td></td>
</tr>
<tr>
<td>Uniform load</td>
<td>24.1</td>
<td>Uniform load</td>
<td>10.1</td>
</tr>
<tr>
<td>Single central load</td>
<td>30.2</td>
<td>Single end load</td>
<td>7.05</td>
</tr>
<tr>
<td>Two loads at $\frac{l}{2}$ points</td>
<td>23.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In order that the maximum permissible flexural stress and the maximum permissible deflection of $\frac{l}{360}$ may be attained at the same time, the span must, in general, not exceed

$$l = \frac{Ec}{Kf}$$

the constant $K$ depending on the type of beam and loading. For a simply supported load carrying a uniform load $K$ has been shown to be $= 37.5$, while for the same type of beam carrying a single central load, $K = 30$, and for a beam loaded at the third points it is $38.3$.

Limitation of the depth ratio is frequently prescribed in structural specifications. For example, in Schneider’s “General Specifications for Structural Work of Buildings,” the following clauses occur:

The depth of rolled beams in floors shall be not less than one-twentieth of the span, and, if used as roof purlins, not less than one-thirtieth of the span.

In case of floors subject to shocks and vibrations, the depth of beams and girders shall be limited to one-fifteenth of the span. If shallower beams are used, the sectional area shall be increased until the maximum deflection is not greater than that of a beam having a depth of one-fifteenth of the span, but the depth of such spans and girders shall in no case be less than one-twentieth of the span.

The building code of Philadelphia contains the following restrictions:

The allowable deflection for beams or girders shall not exceed one-thirtieth of an inch per foot of span where the ceiling is to be plastered, or one-twenty-fifth of an inch per foot of span, where the ceiling is not to be plastered.

In what has been said above concerning the calculation of maximum deflection, the effect of the shear in producing deflection has been neglected. This is justifiable for all except precise calculations and for short beams and girders carrying heavy loads. As may be shown, the calculation of shearing deflection for rectangular, or nearly rectangular, sections must take into account the fact that the shearing stress is not uniformly distributed. For I-sections—the most commonly employed ones for flexural members—it may be assumed as pointed
out in Art. 16 that the shearing stress is uniformly distributed over the web only, and the shearing deflection computed accordingly.

A useful comparison of the deflections resulting from flexure and shear, made by R. Fleming in Engineering News-Record, May 27, 1920, is reproduced in Table 9 with some modifications and additions. The deflection due to shear was computed for uniformly loaded beams by the formula

$$\Delta_s = \frac{6WL}{40E \cdot dt}$$

and for the centrally loaded beams by the formula

$$\Delta_s = \frac{6PL}{20E \cdot dt}$$

In these expressions

- $W =$ total uniformly distributed load.
- $P =$ concentrated load at center of span.
- $l =$ span in inches.
- $E_s =$ shearing modulus of elasticity ($= 12,000,000$ lb. per sq. in.).
- $d =$ depth of beam.
- $t =$ thickness of web.

An examination of the last column of Table 9 shows that for very short spans —five or six times the depth of the beam—loaded to capacity in bending, the deflection due to shear may be between 30 and 50 per cent of that due to flexure. It is relatively more important for beams carrying concentrated loads than for those carrying uniformly distributed loads. For beams with a span of from 20 to 24 times the depth (a ratio that is likely to be closely approached in most designs), the shearing deflection is in the neighborhood of 2 or 3 per cent of the deflection due to flexure. It is therefore evident that only for short spans loaded to capacity in bending is there necessity of taking the shearing deflection into account. Should it be desired to include it, for spans of ordinary proportion a close approximation to the total deflection may be made by increasing the deflection due to flexure by a percentage taken from Table 9, interpolating if necessary. Another method of taking account of the shearing deflection is to compute the deflection due to flexure by using a value of the modulus of elasticity somewhat lower than the usual value assumed, say from 10 to 25 per cent.

Since for beams of the usual depth ratios, the shearing deflection is relatively small as compared with that due to flexure, the shearing deflection may with sufficient accuracy be calculated on the assumption that the shearing stress is uniformly distributed over the web and is entirely borne by the web.

19b. Beams with Variable Section.—In computing the deflection of reinforced steel beams, account must be taken of the fact that the moment of inertia is not constant throughout the length of the beam. For such cases the total deflection may be computed by summing a number of partial deflections. The beam is first divided up into a number of short segments, so chosen that any abrupt changes in sectional area or moment will take place at the dividing lines between segments. If $E$ be constant, the deflection due to flexure only is found by applying to the whole reinforced beam the summation

$$\Delta = \frac{1}{E} \sum \frac{M}{I}(m)(dx)$$
### Table 9.—Relation of Shearing and Flexural Deflections for Simply Supported Rolled I-Sections. $f = 16,000$ lb. per sq. in.

<table>
<thead>
<tr>
<th>Section (in.)</th>
<th>Span (feet)</th>
<th>Ratio of depth to span</th>
<th>Total load (pounds)</th>
<th>Distribution of load</th>
<th>Deflection due to flexure $\Delta_f$ (inches)</th>
<th>Deflection due to shear $\Delta_s$ (inches)</th>
<th>$100\Delta_s/\Delta_f$ (per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard I-beams</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$8 \times 18.4$</td>
<td>5</td>
<td>$\frac{1}{7.5}$</td>
<td>30,300</td>
<td>Uniform</td>
<td>0.050</td>
<td>0.011</td>
<td>22.0</td>
</tr>
<tr>
<td>$8 \times 18.4$</td>
<td>10</td>
<td>$\frac{1}{15}$</td>
<td>15,150</td>
<td>Uniform</td>
<td>0.200</td>
<td>0.011</td>
<td>5.5</td>
</tr>
<tr>
<td>$8 \times 18.4$</td>
<td>15</td>
<td>$\frac{1}{22.5}$</td>
<td>10,100</td>
<td>Uniform</td>
<td>0.449</td>
<td>0.011</td>
<td>2.5</td>
</tr>
<tr>
<td>$8 \times 18.4$</td>
<td>5</td>
<td>$\frac{1}{7.5}$</td>
<td>15,150</td>
<td>Middle</td>
<td>0.040</td>
<td>0.011</td>
<td>27.5</td>
</tr>
<tr>
<td>$8 \times 18.4$</td>
<td>10</td>
<td>$\frac{1}{15}$</td>
<td>7,600</td>
<td>Middle</td>
<td>0.160</td>
<td>0.011</td>
<td>6.9</td>
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<tr>
<td>$8 \times 18.4$</td>
<td>15</td>
<td>$\frac{1}{22.5}$</td>
<td>5,050</td>
<td>Middle</td>
<td>0.359</td>
<td>0.011</td>
<td>3.1</td>
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<tr>
<td>$12 \times 31.8$</td>
<td>5</td>
<td>$\frac{1}{5}$</td>
<td>76,700</td>
<td>Uniform</td>
<td>0.063</td>
<td>0.014</td>
<td>42.5</td>
</tr>
<tr>
<td>$12 \times 31.8$</td>
<td>10</td>
<td>$\frac{1}{10}$</td>
<td>38,350</td>
<td>Uniform</td>
<td>0.133</td>
<td>0.014</td>
<td>10.5</td>
</tr>
<tr>
<td>$12 \times 31.8$</td>
<td>20</td>
<td>$\frac{1}{20}$</td>
<td>19,175</td>
<td>Uniform</td>
<td>0.534</td>
<td>0.014</td>
<td>2.6</td>
</tr>
<tr>
<td>$12 \times 31.8$</td>
<td>5</td>
<td>$\frac{1}{5}$</td>
<td>38,350</td>
<td>Middle</td>
<td>0.027</td>
<td>0.014</td>
<td>51.9</td>
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<tr>
<td>$12 \times 31.8$</td>
<td>10</td>
<td>$\frac{1}{10}$</td>
<td>19,175</td>
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<td>0.107</td>
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<td>$12 \times 31.8$</td>
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<td>9,580</td>
<td>Middle</td>
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<td>0.014</td>
<td>3.3</td>
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<tr>
<td>$20 \times 65.4$</td>
<td>10</td>
<td>$\frac{1}{6}$</td>
<td>124,700</td>
<td>Uniform</td>
<td>0.080</td>
<td>0.019</td>
<td>23.8</td>
</tr>
<tr>
<td>$20 \times 65.4$</td>
<td>20</td>
<td>$\frac{1}{12}$</td>
<td>62,350</td>
<td>Uniform</td>
<td>0.321</td>
<td>0.019</td>
<td>5.9</td>
</tr>
<tr>
<td>$20 \times 65.4$</td>
<td>40</td>
<td>$\frac{1}{24}$</td>
<td>31,175</td>
<td>Uniform</td>
<td>1.281</td>
<td>0.019</td>
<td>1.5</td>
</tr>
<tr>
<td>$20 \times 65.4$</td>
<td>10</td>
<td>$\frac{1}{6}$</td>
<td>62,350</td>
<td>Middle</td>
<td>0.064</td>
<td>0.019</td>
<td>29.7</td>
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<tr>
<td>$20 \times 65.4$</td>
<td>20</td>
<td>$\frac{1}{12}$</td>
<td>31,175</td>
<td>Middle</td>
<td>0.256</td>
<td>0.19</td>
<td>7.4</td>
</tr>
<tr>
<td>$20 \times 65.4$</td>
<td>40</td>
<td>$\frac{1}{24}$</td>
<td>15,580</td>
<td>Middle</td>
<td>1.024</td>
<td>0.19</td>
<td>1.9</td>
</tr>
<tr>
<td><strong>Bethlehem I-beams</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$30 \times 120.0$</td>
<td>15</td>
<td>$\frac{1}{6}$</td>
<td>248,400</td>
<td>Uniform</td>
<td>0.124</td>
<td>0.035</td>
<td>28.2</td>
</tr>
<tr>
<td>$30 \times 120.0$</td>
<td>30</td>
<td>$\frac{1}{12}$</td>
<td>124,200</td>
<td>Uniform</td>
<td>0.496</td>
<td>0.035</td>
<td>7.1</td>
</tr>
<tr>
<td>$30 \times 120.0$</td>
<td>50</td>
<td>$\frac{1}{20}$</td>
<td>74,000</td>
<td>Uniform</td>
<td>1.379</td>
<td>0.035</td>
<td>2.5</td>
</tr>
<tr>
<td>$30 \times 120.0$</td>
<td>15</td>
<td>$\frac{1}{6}$</td>
<td>124,200</td>
<td>Middle</td>
<td>0.099</td>
<td>0.035</td>
<td>35.4</td>
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<tr>
<td>$30 \times 120.0$</td>
<td>30</td>
<td>$\frac{1}{12}$</td>
<td>62,100</td>
<td>Middle</td>
<td>0.390</td>
<td>0.035</td>
<td>8.8</td>
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<tr>
<td>$30 \times 120.0$</td>
<td>50</td>
<td>$\frac{1}{20}$</td>
<td>37,300</td>
<td>Middle</td>
<td>1.102</td>
<td>0.035</td>
<td>3.2</td>
</tr>
<tr>
<td><strong>Bethlehem girder beams</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$30 \times 190.0$</td>
<td>15</td>
<td>$\frac{1}{6}$</td>
<td>388,480</td>
<td>Uniform</td>
<td>0.124</td>
<td>0.042</td>
<td>33.9</td>
</tr>
<tr>
<td>$30 \times 180.0$</td>
<td>30</td>
<td>$\frac{1}{12}$</td>
<td>194,240</td>
<td>Uniform</td>
<td>0.496</td>
<td>0.042</td>
<td>8.5</td>
</tr>
<tr>
<td>$30 \times 180.0$</td>
<td>50</td>
<td>$\frac{1}{20}$</td>
<td>116,600</td>
<td>Uniform</td>
<td>1.379</td>
<td>0.042</td>
<td>3.1</td>
</tr>
<tr>
<td>$30 \times 180.0$</td>
<td>15</td>
<td>$\frac{1}{6}$</td>
<td>194,240</td>
<td>Middle</td>
<td>0.099</td>
<td>0.042</td>
<td>42.4</td>
</tr>
<tr>
<td>$30 \times 180.0$</td>
<td>30</td>
<td>$\frac{1}{12}$</td>
<td>97,120</td>
<td>Middle</td>
<td>0.396</td>
<td>0.042</td>
<td>10.6</td>
</tr>
<tr>
<td>$30 \times 180.0$</td>
<td>50</td>
<td>$\frac{1}{20}$</td>
<td>58,300</td>
<td>Middle</td>
<td>1.102</td>
<td>0.042</td>
<td>3.8</td>
</tr>
</tbody>
</table>
In this expression,

\[ \Delta = \text{required maximum deflection.} \]
\[ E = \text{modulus of elasticity.} \]
\[ M = \text{bending moment at the center of any segment distant } x \text{ from a support for simply supported beams and from the free end in the case of cantilevers.} \]
\[ I = \text{moment of inertia of the beam at the center of any segment.} \]
\[ m = \text{bending moment at center of any segment due to load of 1 lb., acting at the point where the deflection is required.} \]
\[ dx = \text{length of any short segment.} \]

![Fig. 16.—Calculation of deflection of reinforced or built-up beam.](image)

Applying this to a uniformly loaded beam with several reinforcing plates on each flange, as shown in Fig. 16, the summation by means of the Calculus for the entire span gives a deflection of

\[
\Delta = \frac{wL^3}{6EI_1} \left[ l_1^3 + l_2^3 - l_1^3 + l_3^3 - l_2^3 \right] - \frac{wL}{8EI_1} \left[ l_1^4 - l_2^4 - l_1^4 + l_3^4 - l_2^4 \right]
\]

where

\[ w = \text{uniform load per unit of length.} \]
\[ E = \text{modulus of elasticity of material.} \]
\[ L = \text{span length.} \]
\[ l_1, l_2, l_3 = \text{distances of successive points of change of moment of inertia from support.} \]
\[ I_1, I_2, I_3 = \text{moment of inertia of successive sections from support.} \]

This formula may be used to cover any number of abrupt changes by the inclusion of more terms.

For purposes of computing deflections, the moment of inertia of the gross section, which is the predominant section, should be used.

Computations made for beams of constant depth and section so varied as to give constant strength show deflections from 20 to 100 per cent greater than for beams with constant moment of inertia.

**Illustrative Problem.**—Compute the maximum center deflection of a 25-ft. 12-in., 31.8-lb. I with one 6 × 3½-in. plate, 18 ft. long, riveted to each flange. (See Fig. 9 and the problem under Art. 12.) The beam carries a total uniformly distributed load of 900 lb. per lin. ft. \( E = 29,000,000 \) lb. per sq. in.

The half beam will be divided into two segments, the first of which comprises the 3.5-ft. unreinforced portion of the end, and the second the remaining 9-ft. portion of the half span. The values of \( l_1 \) and \( l_2 \) are, therefore, 3.5 and 12.5 ft. respectively.

Moment of inertia of gross unreinforced section, \( I_1 = 215.8. \)

Moment of inertia of gross reinforced section, \( I_1 = 388.2. \)

For the case in hand, Formula (1) becomes

\[
\Delta = \frac{wL}{6EI_1} \left[ l_1^3 + \left( \frac{l_1}{2} \right)^3 - l_1^3 \right] - \frac{wL}{8EI_1} \left[ l_1^4 - \left( \frac{l_1}{2} \right)^4 - l_1^4 \right]
\]
Using the inch as unit of length, and inserting the appropriate numerical quantities,
\[
\Delta = \frac{75}{215.8 + \frac{42}{388.2}} \left[ \frac{(150)^4 - (42)^4}{(6)(29,000,000)} \left( \frac{300}{215.8 + \frac{42}{388.2}} \right) \right] \left( \frac{(29,000,000)(388.2)}{(5)(29,000,000)} \right)
\]
\[
= 0.721 \text{ in.}
\]

If it be assumed that the gross moment of inertia of the reinforced section applies for the whole span, the deflection would be
\[
\Delta = \frac{5}{384} \cdot \frac{Wl^3}{EI}
\]
or, for the beam under consideration,
\[
\Delta = \frac{5}{384} \cdot \frac{(900)(25)(300)^3}{(29,000,000)(388.2)} = 0.702 \text{ in.}
\]
or but slightly less than the deflection found by the correct method. The close correspondence of these results is due to the fact that the flange plates run nearly the full length of the beam and the stresses in the central reinforced section influence the deflection much more than those in the unreinforced section near the ends.

20. Combined Stresses.—Cases frequently arise in practice of members subjected to flexure and at the same time to an axial tensile or compressive force. These are in most cases, however, primarily tension or compression members and are discussed as such in this volume. One characteristic case of a flexural member being subjected to axial loading is the trussed beam. This type of member is discussed in Art. 21.

21. Trussing of Beams.—If it happens that the heaviest rolled section available is not sufficiently strong to carry the stipulated load, and there is no restriction with respect to headroom, a rolled section may be trussed so as to enable it to carry a load. Two common methods of trussing are used, the king-post, Fig. 17(a) and the queen-post, Fig. 17(b) and (c). With the first, a single strut is connected to the primary beam at the center and a rod is carried from the bottom of it to each end. With the second, two struts are used, dividing the beam into three segments not necessarily equal. The struts may be of angles or castings and the ties may be single or multiple rods. Where cast struts are used, they may be at right angles to the top chord, as in Fig. 17(b), but if angles are used they should be battered so as to bisect the angle between the horizontal and sloping sections of the tie rod to give axial stress only in the struts.

A common use of trussed steel beams is in roof construction, as rafters or purlins. If they are used as purlins, the bending will not be in a principal plane of the trussed section and hence in designing the principles pertaining to unsymmetrical bending (Art. 22) must be observed.

While the accurate design of a trussed beam should be carried out in accordance with the method of least work, a sufficiently accurate procedure for most purposes is to regard the structure as a beam continuous over the struts. This involves the erroneous assumption that the beam does not settle at the struts with respect to the end supports—an assumption that is, however, justified for approximate design.

1 See chapter on "Bending and Direct Stress" in Sec. 1.
In accordance with this assumption, the primary beam is not only subjected to the moments and shears existing in a continuous beam of the same number of spans as there are panels, but must also resist the axial thrust due to the pull of the tie rod. The end connections of this tie should be such that the thrust is applied centrically, thus avoiding secondary stresses.

The approximate method of design outlined above, may best be studied by means of an example.

**Illustrative Problem.**—An opening of 15 ft. center to center of bearings is to be spanned by a beam carrying a total uniformly distributed load of 600 lb. per lin. ft. For this situation there are available only minimum weight channels of depths up to 9 in., angles, and soft steel rods. There is no restriction as to headroom. Lateral support to the beam is afforded at the center and at points 3 ft. from each end. Design a trussed beam to carry the load if the permissible stresses are as follows:

Bending, 16,000 lb. per sq. in.

Compression on struts, \( p = 19,000 = 100 \frac{l}{r} \) where \( l = \) unsupported length and \( r = \) least radius of gyration.

Combined compression and bending, \( p = 19,000 - 300 \frac{l}{b} \) where \( l = \) unsupported length of flange and \( b = \) breadth of flange.

Shear, 10,000 lb. per sq. in., gross area of web.

Tension on soft steel rods, 15,000 lb. per sq. in.

Bearing, on soft steel, 15,000 lb. per sq. in.

As only very light channel sections are available, an arrangement will be adopted favorable to the primary beam, or what is really the top chord of the resulting truss.

![Diagram](image-url)

**Fig. 18.**—Details of trussed channel.

Two struts will therefore be used, symmetrically placed and 6 ft. apart at their intersection with the center line of the channels, as shown in Fig. 18, and the depth from the center of the top chord to the center of the tie rod will be 2 ft., giving a slope of the end sections of tie rod of 1 vertical to 2.83 horizontal, with the struts bisecting the angle between the horizontal and inclined portions of the tie rod, which is desirable in order to give only axial stress in the struts.

**Shear.**—From the theory of continuous beams, the maximum shear in the top chord occurs at the two struts on the sides nearest the end supports and is

\[
V = \frac{6}{10} wp
\]

where \( w = \) total uniform load per unit of length; and \( p = \) panel length.

For this case

\[
V = (\frac{6}{10})(600)(6) = 2,160 \text{ lb.}
\]

Assuming one 8-in., 11.5-lb. channel as the top chord, the average shearing stress on the web is

\[
\tau_w = \frac{2,160}{(8)(0.220)} = 1,230 \text{ lb. per sq. in.}
\]

which is very much below the allowed limit.
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Combined Bending and Compression.—For a continuous beam of 3 equal spans, assuming no restraint at the end supports, the maximum moment occurs at an intermediate support and is

\[ M = \frac{1}{10}w p^2 \]

which for this problem becomes

\[ M = \left(\frac{1}{10}\right)(600)(8)^2(12) = 25,920 \text{ in.-lb.} \]

Extreme fiber stress assuming the top chord to be one 8-in., 11.5-lb. channel with a section modulus of 8.1 is

\[ f_1 = \frac{M}{S} = \frac{25,920}{8.1} = 3,200 \text{ lb. per sq. in.} \]

Horizontal or axial compression in top chord, neglecting the horizontal component of the stress in the strut,

\[ H = \frac{11}{10}w p \cot a, \]

where \( a \) = angle of slope with the horizontal of the end sections of the tie rod. Numerically,

\[ H = \left(\frac{11}{10}\right)(600)(6)(2.83) = 11,200 \text{ lb.} \]

Maximum axial compressive stress,

\[ f_z = \frac{H}{A} = \frac{11,200}{3.36} = 3,340 \text{ lb. per sq. in.} \]

Total maximum compressive stress,

\[ f_1 + f_z = 3,200 + 3,340 = 6,540 \text{ lb. per sq. in.} \]

Permissible compressive stress on chord

\[ p = 19,000 - (300)\left(\frac{72}{2.26}\right) = 9,450 \text{ lb. per sq. in.} \]

Since the effect of the necessarily eccentric application of the axial thrust has been neglected, the margin of safety is not too great.

Tie Rod.—Tension in tie rod

\[ T = \frac{11}{10}w p \csc a = \left(\frac{11}{10}\right)(600)(6)(2.99) = 11,850 \text{ lb.} \]

Required area = 11,850/15,000 = 0.79 sq. in.

Use one 1-in. rod upset, having an area of 0.79 sq. in. in the body and of 1.054 sq. in. at the root of the thread of the 1\(\frac{1}{4}\)-in. upset ends. A turnbuckle will be needed at the center of the span for adjustment.

Struts.—Compression in struts,

\[ P = \left(\frac{11}{10}\right)w p = \left(\frac{11}{10}\right)(600)(6) = 3,960 \text{ lb.} \]

Assume one 3 \(\times\) 3 \(\times\) 5\(\frac{1}{6}\)-in. angle, for which

\[ A = 1.78 \text{ sq. in. and least } r = 0.59 \text{ in.} \]

Compressive stress = 3,960/1.78 = 2,220 lb. per sq. in.

Permissible stress,

\[ p = 19,000 - \frac{100}{r} = 19,000 - (100)\left(\frac{24}{0.59}\right) = 14,930 \text{ lb. per sq. in.} \]

The outstanding leg of the lower end of the struts will be notched to semi-circular form so as to receive the rod.

Bearing area required for rod,

\[ A = \frac{P}{p} = \frac{3,960}{14,930} = 0.26 \text{ sq. in.} \]

Area provided = (1.00)(0.3125) = 0.31 sq. in., which is adequate.

Details.—Details may be arranged as shown in Fig. 18. The connection of the tie rod and the struts to the top chord must be sufficient in strength to transmit the stresses in them to the channel.
22. Proportioning for Unsymmetrical Bending.—Beams subjected to bending not operating in the plane of one of the principal axes cannot properly be designed by the simple flexure formula

\[ f = \frac{M}{S} \]

in which \( S \) is the ordinary section modulus about the principal axis most nearly at right angles to the plane of loading. If, however, the true section modulus applicable under the circumstances be employed, either the maximum stress at the extreme fiber or the safe capacity may be computed accurately by the common flexure formula. This quantity known as the flexural modulus is, as has been shown elsewhere in this volume,

\[ S' = I_{y/y} \frac{I_z I_y}{\sin \theta + I_x \cos \theta} \]

where
- \( I_z \) = moment of inertia of section about the \( x \)-axis.
- \( I_y \) = moment of inertia of section about the \( y \)-axis.
- \( x, y \) = coordinates of the most highly stressed fiber.
- \( \theta \) = angle between the plane of the moment and the \( x \)-axis.

For the purposes of practical design it is more convenient, however, to resolve the moment into two components, parallel respectively to the two principal axes of the section, and then add together the stresses produced by them at the critical fiber. The correctness of this method of procedure has been established elsewhere in this volume.

A frequent case of unsymmetrical bending is that of a beam subjected to both vertical and transverse moment, as a floor beam supporting a vertical load and at the same time resisting the thrust of an arch. Here a resultant oblique moment really exists in the form of two principal components. Investigation of such a beam may, therefore, be carried out as explained above.

Illustrative Problem.—A 6-in., 8.2-in. channel purlin of 15-ft. span with web inclined 30° to the vertical, as shown in Fig. 19, carries a vertical roof load of 100 lb. per lin. ft. Express an opinion as to its safety if the permissible stress in bending is 16,000 lb. per sq. in.

Vertical moment on purlin

\[ M = \frac{Wl}{8} = \frac{(160)(15)(180)}{8} = 54,000 \text{ in.-lb.} \]

Component of moment in plane of purlin web, or about the axis of \( x \)

\[ M_x = (54,000)(\sin 60^\circ) = 46,800 \text{ in.-lb.} \]

Component of moment at right angles to plane of web, or about the axis of \( y \)

\[ M_y = (54,000)(\cos 60^\circ) = 27,000 \text{ in.-lb.} \]
The fiber at point $A$ is evidently the most highly stressed one. Its coordinates are: $x = 1.4$ and $y = 3.0$. The moment of inertia about the $x$-axis is $I_x = 13.0$ and about the $y$-axis it is $I_y = 0.70$. Resultant fiber stress at point $A$ is therefore

$$f_x + f_y = \frac{M_{xy}}{I_x} + \frac{M_{yx}}{I_y}$$

$$= \frac{(40,800)(3.0)}{13.0} + \frac{(27,000)(1.4)}{0.70}$$

$$= 10,800 + 54,000$$

$$= 64,800 \text{ lb. per sq. in.}$$

The purlin is therefore stressed to its ultimate strength. By supporting it laterally at short intervals the stress $f_y$ could be greatly reduced and the resultant stress $f_x + f_y$ brought within the safe limit.

Had the loading been assumed as acting in the plane of the web, as is sometimes erroneously done, the fiber stress obtained would be 12,470 lb. per sq. in. The stress calculated in this manner may, therefore, give no real indication as to the actual existing stress.

**Illustrative Problem.**—A floor beam of 18-ft. span, consisting of one 12-in., 31.8-lb. I, carries a total uniformly distributed vertical load of 900 lb. per lin. ft. and a resultant horizontal arch thrust of 500 lb. per lin. ft. If the beam is divided into three 6-ft. segments by tie rods, as shown in Fig. 20, find the maximum fiber stress, assuming perfect lateral restraint at the points of attachment of the tie rods.

Section modulus $S_z$ of 12-in., 31.8-lb. I about $x$-axis (normal to web) $\approx 36.0$, and section modulus about axis lying in center of web, $S_y = 3.8$. Flange width = 5 in.

Vertical moment at center of span,

$$M_x = \frac{wt^2}{8} = \frac{(900)(18)^2(12)}{8} = 437,400 \text{ in.-lb.}$$

Horizontal moment at center of span

$$M_y = \frac{ts^2}{24} = \frac{(500)(6)^2(12)}{24} = 9,000 \text{ in.-lb.}$$

where $t =$ lateral thrust per lin. ft., and $s =$ spacing of the rods.

Maximum fiber stress on fiber at $A$ or $B$, at center of span,

$$f_x = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{437,400}{36.0} + \frac{9,000}{3.8} = 11,540 \text{ lb. per sq. in.}$$

Vertical moment at a tie-rod connection,

$$M_y' = \frac{ts^2}{12} = \frac{(500)(6)^2(12)}{12} = 388,800 \text{ in.-lb.}$$

Horizontal moment at the rod connection, assuming perfect restraint,

$$M_y' = \frac{ts^2}{12} = \frac{(500)(6)^2(12)}{12} = 18,000 \text{ in.-lb.}$$

Maximum fiber stress on fiber $A'$ or $B'$, at tie rod connection,

$$f_{x'} = \frac{M_x'}{S_x} + \frac{M_y'}{S_y} = \frac{388,800}{36.0} + \frac{18,000}{3.8} = 15,530 \text{ lb. per sq. in.}$$

The beam is, therefore, more seriously stressed at the tie rod connections than at the center.
23. Proportioning for Torsion.—Wherever beams are curved horizontally or are of such shape in plan that the applied loads do not lie on a straight line joining the two supports, a torsional moment is set up.

A typical case of this kind is the circular girder supporting an elevated tank. The arc of the girder between two adjacent posts must withstand a torsional moment of the magnitude that may be computed by the methods explained in discussions of elevated tanks.

If long, flexible beams connect to one side only of a girder, the girder is thereby subjected to torsional stresses which in severe cases should be investigated. A girder with a narrow flange, such as a single channel, is likely to be highly stressed in torsion. The torsional moment produced in a girder by a beam attached to it by a web connection may be considered as equal to the moment of restraint of the beam at the end. While such moment of restraint is disregarded in fixing the section of steel beams, the practice is common to assume that there is a moment of restraint that offsets the apparent moment of eccentricity in the end connection and renders it necessary to proportion the rivets through the beam web for direct shear only. Based on the character of the end connections of the beams framing into the girder subjected to torsion, an estimate may be made of the probable torsional moment applied at each loading point. Such torsional moments may be regarded as divided between the two segments of the beam on the two sides of the loading point in the inverse ratio of their length. With two symmetrically-applied torsional moments, there will be equal torsions in the two end segments and zero torsion in the center segment.

In determining the maximum existing torsional shearing stress on the cross section of an I-beam or channel section, it is incorrect to assume that the common torsion formula for circular shafts applies.

This formula is
\[ \frac{q}{c} = \frac{T}{J} \]  

where \( q \) = torsional shearing stress at the extreme fiber.
\( c \) = radial distance from center of gravity of section to extreme fiber.
\( T \) = torsional moment.
\( J \) = polar moment of inertia (see treatise on mechanics).

Experimental determination of the torsional elastic limit of I-beams made by the author indicate that this is reached at a torque less than 20 per cent as great as Formula (1) would indicate. The relatively thin metal of the web has little torsional resistance itself and does not effectively prevent the flanges from twisting around under a combination of shear and bending. Beams designed for torsion should only be proportioned by Formula (1), provided the allowable stress selected is not over 20 per cent the usual permissible stress in shear.

MULTIPLE BEAM GIRDERS

BY C. R. YOUNG

24. Types and Uses.—Where a single rolled beam or girder with adequate bending capacity for the situation in hand is not available, it is frequently advantageous to use two or more rolled sections placed side by side a short distance.
apart and suitably connected together. Such construction is particularly useful for the support of walls, on account of the broad bearing offered for the load. The number of sections varies from two or three, used for the support of walls, to as many as 10 or 12 in the case of a tier in a grillage foundation.

While the component sections are frequently of the same type, depth and weight, it is by no means necessary that they should be so. If three sections are used it may be advantageous to make the outer two somewhat lighter than the inner one; or if the latter section be an I-beam, to make the outer two channels of the same depth. Characteristic sections for multiple beam girders are shown in Fig. 21. Those shown in (a), (b) (c) and (d) are frequently employed for the support of walls, beams, and columns, while the use of a large number of sections as in (e), is confined to grillage tiers. Rolled beams in groups of from two to four are frequently employed as girders supporting timber decks in railway bridges. One group is placed under each rail. Modification of some of the types shown by the addition of shelf angles at the bottom is frequently made in order to adapt multiple beam girders to use as lintels or spandrel girders. Examples of these are shown in Fig. 24.

In making up the section of a multiple beam girder, regard must be had to the character of the determining stress. If the bending moment is relatively more important than the shear, I-beams should be employed, rather than channels, since the flexural efficiency of the I-beam is greater than that of the channel, as has been pointed out in the discussion of beams, Art. 6. On the other hand, if the shear is large enough to influence the design, channels are preferable for economic reasons as the amount of shearing area per square inch of total section is greater for channels than for I-beams. This latter condition also gives channels an advantage in resisting local transverse compression or web crippling.

In selecting the sections to be utilized in a multiple beam girder, the bearing area that must be provided for the applied load should be considered. For well-bonded brick walls, there is no reason why the brick work in a wall of any thickness likely to be carried on multiple beam girders should not arch laterally over the clear space between the flanges of two supporting beams or channels.

25. Advantages and Disadvantages.—The use of multiple beam girders is only advantageous where large flexural strength with small depth is required. The broad bearing afforded by such a girder for the support of walls and for the transmission of loads to the end supports is also an advantage, as is the consider-
able lateral stiffness of the combined beam. It is, too, very convenient to be able to utilize a series of available light beams for building up a girder for the support of heavy loads.

On the other hand, the use of shallow beams or girders is highly uneconomical so far as flexure is concerned, as has been pointed out in the discussion of beams in Art. 5. Where the shear is relatively more important than the moment, as in short, heavily-loaded tiers of grillage beams, sections of small depth may be found more desirable because of their greater aggregate web area. They are deficient in vertical stiffness, however, and unless care is taken to limit the ratio of span to depth the deflection may be so large as to be objectionable in appearance or, in the case of foundation girders, to lessen the bearing at the outer ends of the beams. In no case should girders of the multiple beam type be used in damp situations without being properly protected from corrosion on the interior surfaces. Such protection is naturally afforded by the encasing concrete in grillages, but girders above ground are frequently left without such protection, and from the nature of the construction cannot be inspected or painted after erection.

26. Separators.—In order that the assemblage of sections may act as a unit in the support of loads and may possess adequate lateral rigidity, separators of various types are employed. These maintain the spacing of the component beams, and when loads are not applied equally to all of them should be able to distribute it equally amongst the various elements composing the girder, unless they are designed for unequal loads.

Three types, indicated in Fig. 21, are in common use—gas pipe, cast iron, and built-up or riveted separators.

The gas pipe type, Fig. 21 (a) and (c), consists of a series of short lengths of gas pipe fitting closely between the webs, with bolts passing through them from one side of the girder to the other. The number of tiers of bolts and pipe sections used, varies with the depth of the girder, but in average practice conforms approximately to the following table:

<table>
<thead>
<tr>
<th>Depth of Girder (inches)</th>
<th>Number of Tiers of Bolts</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-10</td>
<td>1</td>
</tr>
<tr>
<td>12-18</td>
<td>2</td>
</tr>
<tr>
<td>20-30</td>
<td>3</td>
</tr>
</tbody>
</table>

Separators of the gas-pipe type are cheaper than any others, but are incapable of transferring any load from one beam of the compound girder to another. Wherever the applied load is known to be equally applied to the component sections or where the individual sections are designed for definite parts of the load, as in some lintels or spandrel girders, gas pipe separators may be used to advantage. They are particularly desirable in tiers of grillage beams since they do not interfere with the placing of the encasing concrete or break it up, as would cast iron or built-up separators. The American Bridge Company's standards call for gas pipe separators for all girders composed of beams under 6 in. in depth. The size of gas pipe and bolts should conform to the size of the prevailing rivets in the work. Generally, \( \frac{3}{4} \) in. bolts and 1-in. gas pipe are used.

Cast iron separators, Fig. 21 (b) and (d), consist of cast plates, usually from \( \frac{3}{8} \) to \( \frac{5}{8} \) in. thick, with one or two lugs cast on the face of the plates to receive
the bolts which secure the separator in a transverse position between the connected beams. Two types of separator are commonly used, (1) a rectangular plate of width such as to maintain the component beams at the desired distance apart, and of height sufficient to clear the fillets (Fig. 21), and (2) a plate shaped to fit tightly against the webs and flanges of the beams (Fig. 22). If the second type be made to bear properly against the flanges, it is superior to the first, for the transfer of load from one beam to the other would then not depend solely upon the shearing and bending value of long flexible bolts, as with gas pipe separators. If a load be applied to one beam and not to the other, as in Fig. 22, the deflection of the loaded beam causes the top flange to transfer part of the load in bearing through the separator to the bottom flange of the other beam. With this type of separator, properly fitted, the tendency is for applied loads to be equally distributed amongst the component sections if they be of equal stiffness. If the sections are of unequal stiffness, the stiffest would receive the greatest loads. In situations where distribution of the load by separators is counted upon, therefore, cast iron separators of the second type may be advantageously employed. They should not be used in grillages for the reasons already given.

According to the standards given in the handbooks of the steel companies, one bolt only may be used in each separator if the beams be not over 10 or 12 in. deep. Two bolts are used for beams from 12 to 24 in. deep, and 3 bolts for beams over 24 in. The prescribed spacing of the bolts and the dimensions and weights of separators and bolts is given in the standards mentioned. The width of the separators is so fixed that when they are used with the maximum weight of beams for the depth to which they conform, the flanges will clear.

Built-up separators or diaphragms are employed in situations where very rigid bracing is required between the component sections of a girder or where provision must be made for distributing unequally applied loads. They may be made up of a plate and two or four angles to form a built-up channel or I-section with flanges riveted to the webs of the beams, or if the desired spacing of the component sections will permit, of a piece of channel or I-beam placed with its flanges vertical and in contact with the webs of the connected sections. This riveted construction ensures the action of the assembled sections as one unified girder.

While separators serve to stay the top flanges of the component sections of the girder to some extent, their effectiveness in this regard, except in the case of the shaped cast iron separator, is considerably reduced through the attachment being to the web rather than to the flanges. The spacing of separators is therefore generally less than would be obtained by applying such rules as that the compression flange of beams must be stayed at intervals of 10 or 20 flange-widths if the customary flexural working stress is to be employed. It is good practice to place separators at the ends of the girder and at, or near, all points of concentrated loading. In addition they are placed at intermediate points, distances
apart varying with the depth of the beams. The spacing adopted where the position of points of concentrated loading does not determine it, is usually about as follows:

<table>
<thead>
<tr>
<th>Depth of Girders (inches)</th>
<th>Spacing of Separators (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3–10</td>
<td>3</td>
</tr>
<tr>
<td>12–15</td>
<td>4</td>
</tr>
<tr>
<td>18–30</td>
<td>5</td>
</tr>
</tbody>
</table>

27. Proportioning of Multiple Beam Girders.—The design of multiple beam girders differs in no way from that of single beams. Having found the maximum bending moment and maximum vertical shear, such component sections must be selected as will give the desired width for the effective support of the applied load and will supply the total section modulus and shearing area required. Deductions for any flange holes that may be near the critical section for moment should be made as described for beams in Arts. 8 to 11 inclusive, but web holes may be neglected so far as moment is concerned. If a built-up separator chances to

be at or slightly inside the plane of maximum shear, account may need to be taken of the lessened shearing resistance of the web produced by the vertical lines of holes.

In calculating the load for which multiple beam girders must be designed, regard must be had to the arching effect of any brick, tile or masonry walls that may be supported. Observations of the cracking of such walls above a supporting girder or lintel that has partially failed, or sagged excessively, show that under certain favorable conditions only a relatively small triangular portion of masonry is really carried by the girder. The height of this triangle is variously assumed as from $\frac{1}{4}$ to $\frac{3}{4}$ of the span. While the cracks mentioned trend upward and inward from the junction of the top of the girder with the faces of the supports, as shown in Fig. 23, it is more convenient and just as accurate, to assume the height of the supported triangle as based on the center to center span—the span on which the calculation of moments and shears must be based.

It is only safe to assume the arching effect as relieving the girder of all wall load, except the weight of the triangular portion mentioned, (1) when the supports
are capable of taking thrust in a direction parallel to the girder, or the supported wall continues past the supports for some distance, (2) when there is a height of brickwork not weakened by openings, for a distance above the girder about equal to the span length, (3) when the depth ratio of the girder is large enough to prevent excessive and disrupting deflection, and (4) when the masonry is well seasoned. Under such conditions the area of wall supported may safely be taken as contained within a triangle having base equal to the center to center span and height above the top of girder equal to \(\frac{1}{2}\) of the span. If the existing conditions depart measurably from those outlined, the full height of wall to the next support above should be taken. Piers or concentrated loads carried into the wall above the opening must be especially provided for. If the loads be relatively large, it is not safe to depend much on arch action in the masonry.

Care must be taken to add to the weight of the wall any floor loads that may be carried into it or into the girder direct.

If the wall above the opening spanned is cut up by windows or other openings, the weight of the existing sections of wall must be computed and the point of application of such weights carefully determined.

**Illustrative Problem.**—A solid, well-seasoned 13-in. brick wall weighing 120 lb. per cu. ft. is to be carried over a clear opening of 17 ft. The wall continues on for some distance past the supports on either side. Design a suitable multiple beam girder to carry the wall. Permissible stresses in bending and shear = 10,000 and 10,000 lb. per sq. in., respectively, the shearing stress to be the average on gross section of the web. Permissible
stress to safeguard against web crippling = \(\sigma = 15,000 - 150h\).

To ensure that the deflection will not be great enough to destroy the arching effect which the stated conditions would permit, the depth of beam, assuming the center to center span to be 18 ft. should not be less than about \(\frac{1}{2}\) of \(\frac{13}{12}\) of 120 = 130 lb. per sq. ft., is

\[
W_1 = (18)(\frac{13}{12})(120) = 10,530 \text{ lb.}
\]

Moment due to brickwork, allowing for triangular loading, is

\[
M_1 = \frac{W_1d}{6} = \frac{(10,530)(18)}{6} = 31,500 \text{ ft.-lb.}
\]

Moment due to weight of girder, assuming it to be made up of two 9-in., 21.8-lb. I-beams, the whole including gas-pipe separators, weighing 44 lb. per lineal ft., is

\[
M_2 = (\frac{3}{4})(44)(18)^2 = 1,780 \text{ ft.-lb.}
\]

Total moment, \(M_t = 31,500 + 1,780 = 33,370 \text{ ft.-lb.} = 400,400 \text{ in.-lb.}

From tables of bending capacity, it is seen that a girder of two 9-in., 21.8-lb. I-beams would have a moment of resistance of \(2 \times 25,160 = 50,320 \text{ ft.-lb.} \) at a fiber stress of 16,000 lb. per sq. in. A girder built up of two such beams would therefore be much stronger in bending than is necessary. Two 10-in., 15.3-lb. channels with a combined bending capacity of 35,680 ft.-lb. will be sufficiently strong and weigh less than the I-beams. Two such channels spaced 5 in. back to back, as shown in Fig. 21a will be adopted, subject to their being adequate in shear.

Total end shear = \(\frac{1}{2}(W_1 + W_2) = \frac{1}{2}(10,530) + (18)(44) = 5,660 \text{ lb.}
\]

Average shearing stress on webs = \(v_a = \frac{5,660}{(2)(9)(0.24)} = 1,310 \text{ lb. per sq. in.}
\)

The girder is therefore evidently ample against both shear and diagonal buckling of the web. A single tier of 1-in. gas pipe separator spaced 3 ft. apart, will be used.

A type of multiple beam girder requiring great care in design is a tier of beams in a grillage. In this case shearing and web crippling stresses are likely to be
very important, if not the determining factor in the design of the tier. The compressive stresses in the webs due to the application of heavy concentrated loads to the flanges must also be investigated. For a problem of this kind see Art. 18.

METALLIC LINTELS

BY C. R. YOUNG

28. Types and Uses.—Beams which carry walls over openings and deliver their loads to masonry walls or piers rather than to columns are called lintels. While structurally simple, their design is rendered somewhat uncertain by differences of opinion as to how much of the weight of the wall supported is really borne by the lintel, and how much that does go to the lintel is borne by the component sections thereof. The matter of loading from masonry walls has been discussed in detail in Art. 27. The clear spans may vary from the width of an ordinary window or door to more than 20 ft. Metallic lintels may be of structural steel or of cast iron.

29. Steel Lintels.—Some types of structural steel lintels commonly employed are shown in Fig. 24. An essential feature of these members is that they must

![Typical steel lintels](image)

Fig. 24.—Typical steel lintels.

be so constructed as to give proper support to every part of a supported body of masonry, the bottom of which may be irregular in outline and at different levels, as shown in Fig. 24 (c). The support offered may be in part through hook bolts or anchors attached to convenient flanges of angles specially riveted to the primary elements of the lintel. Several shapes, specially arranged for each particular case, are often required for the support of walls with stone or terra cotta facing which must be tied in to the mass.

In the design of steel lintels the same principles are observed as in the design of multiple beam girders. The angles riveted to the sides of primary shapes, as in Fig. 24 (b), (c), and (d), are attached by sufficient rivets to support the
column of masonry bearing on them for only a few feet above, or up to such height as the projecting masonry may be considered as thoroughly bonded into the principal mass and deriving its support therefrom. These angles are not regarded as contributing to the flexural strength of the lintel as a whole. Provision must be made for any floor loads applied to the lintel or to the wall carried by it.

In fixing the composition and lateral dimensions of the lintel, regard must be had to the necessity for supporting the mass of masonry above so that no cracking will occur. A continuous surface for the bearing of the supported wall is not required, as the masonry will arch laterally over a space of several inches between the component sections. The shapes employed must not be so shallow as to deflect to such an extent as to prove unsightly or cause cracking in the masonry. For a discussion of deflection see Art. 19.

30. Cast Iron Lintels.—Occasionally use is made of cast iron for lintels, although much less frequently than a few years ago, due to the greater reliability of structural steel. Common forms of such lintels are shown in Fig. 25. They consist essentially of a flat plate, or soffit, surmounted by a vertical rib or ribs. The number of ribs required will depend on the span and loading. These ribs are encased in the masonry or form exposed surfaces which may be ornamented (Fig. 25d and e). While theoretically they should be so proportioned that the factor of safety against compression on the upper fibers of the ribs would be the same as the factor of safety against tension on the lower fibers of the soffit, the liability to shrinkage cracks at the junction of highly unequal masses of cast iron, prompts designers to use the same thickness of rib as of soffit. The dimensioned requirements and limiting deflections also tend to modify any proportions that might be fixed by the stresses and the properties of the material. The New York and Boston building codes both specify that cast iron lintels shall not be less than 3/4 in. thick and shall not be used for spans exceeding 6 ft.

In calculating the capacity of cast iron lintels, account should be taken of the fact that the flexural capacity may be fixed by the tensile or the compressive stresses on the corresponding extreme fibers. However, with ordinary proportions the capacity is much more likely to be limited by tensile than by compressive flexural stresses.

Typical permissible working stresses for gray cast iron are those prescribed in the New York building code, namely: bending on extreme compressive and tensile fibers = 16,000 and 3,000 lb. per sq. in., respectively; shear = 3,000 lb. per sq. in.

Due to the lack of symmetry of the section in a vertical direction, it is necessary to find the center of gravity of the section preliminary to finding the moment of inertia. The procedure to be followed in fixing a typical lintel section is one of trial and error—that is, a section is assumed and its capacity found. If it is
inadequate, the section is increased until it is sufficient to carry the specified load. The method is best elucidated by an example.

Illustrative Problem.—Find the total safe uniform load for a cast-iron lintel of 6-ft. span, center to center, having the section shown in Fig. 26, if the permissible stresses in flexure on the extreme compressive and tensile fibers are 16,000 and 3,000 lb. per sq. in. respectively, and in shear 3,000 lb. per sq. in.

![Fig. 26.—Design of a cast-iron lintel.](image)

To find the center of gravity of the cross section—that is, the position of the neutral axis—it is convenient to take moments about the center line of the ribs. The calculations are then as follows:

<table>
<thead>
<tr>
<th>Part</th>
<th>Area</th>
<th>ARM</th>
<th>Statical Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 webs</td>
<td>15.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Flange</td>
<td>24.0</td>
<td>-3.0</td>
<td>-72.0</td>
</tr>
<tr>
<td>39.0</td>
<td></td>
<td></td>
<td>-72.0</td>
</tr>
</tbody>
</table>

Position of center of gravity below center line of webs is $\frac{7}{2} \times 9 = 1.85$ in.

Moment of inertia of 3 webs about the neutral axis, or the gravity axis of whole section = $3(I_0 + A_ya^2) = 3[(\frac{1}{4})24(1)(5)^2 + (5)(1.85)^2] = 82.5$.

Moment of inertia of flange about neutral axis = $[(\frac{1}{4})(24)(1)^2 + (24)(1.15)^2] = 33.8$.

Total moment of inertia = 116.3.

Section modulus, $S_y$, with respect to extreme compressive fiber = $116.3/4.35 = 26.7$.

Section modulus, $S_h$, with respect to extreme tensile fiber = $116.3/1.65 = 70.4$.

Safe resisting moment based upon permissible extreme fiber stresses in compression and tension is $(26.7)(16,000) = 427,000$ in.-lb. and $(70.4)(3,000) = 211,000$ in.-lb. respectively.

The safe capacity is, therefore, dependent on the tensile stress in flexure and the total safe uniform load is

$$ W = \frac{8M}{l} = \frac{(8)(211,000)}{(6)(12)} = 23,400 \text{ lb.} $$

To facilitate the selection of cast iron lintel sections, tables may be prepared giving the properties of all sections likely to be employed.

**BOX GIRDERs**

**By C. R. Young**

31. Types and Uses.—In situations where a broad, comparatively shallow beam of great strength is required, such as for the support of walls or columns, and a multiple beam girder of sufficient capacity cannot be devised, resort is had to the box girder. This may consist of two or more rolled beams or channels arranged as in a multiple beam girder, with cover plates on their flanges, or it may be composed of an assemblage of built-up channels or beams with cover plates, as shown in Fig. 27.

The form of section adopted depends on the character and magnitude of the load carried. For moderate wall loads and for spans of such length that the
design depends on bending moment rather than on shear, the sections shown in Fig. 27 (a) and (b) utilizing channels and I-beams are satisfactory. By the use of the deeper and heavier I-beams, reinforced by one or more cover plates, a large moment of resistance may be developed with comparative cheapness. As has been pointed out in Arts. 6 and 24, the use of I-beams is preferable where the predominating stress is flexural, but where shear and web crippling are determining factors, channels may be employed to advantage. In the latter case, the girder sections shown in Fig. 27 (a) and (c) are suggested.

If the situation calls for a special depth that cannot be made up by the use of reinforced rolled sections, or the required resistance cannot be readily developed in that manner, built-up channels or I-beams, as shown in Fig. 27 (e) and (f), are employed with the necessary number of cover plates. By this method it is possible to place the material where it will be most effectively used for moment and shear. Only one angle can be employed to connect the cover plates to the outer webs of the girder shown in Fig. 27 (f), since, unless the girder is large enough to allow a man to crawl through it, the riveting to the inside angles could not be done. The flanges are riveted to the center web before the center webs are assembled in place.

It is with the object of resisting very heavy shears, rather than moments, that the sections with three webs, Fig. 27 (c), (d), and (f), are employed. Within certain limits, an increased bending moment might be met by increasing the number or thickness of the flange plates, but, for increased shear, additional webs or thicker ones must be used. If the load applied to the top flange, or cover plates, be uniformly distributed laterally, it is reasonable to assume that, for a section such as (d), the three I-beams would bear the load equally, if they are of equal strength and stiffness. If the outer ones are lighter, or if the section be as in (c), with channels on the outside, the lesser stiffness of the outer component parts would, under the same load as could be borne by the center section, bring about a greater yielding in them and a transfer of a larger proportion of the load to the center section. Consequently, it is common practice to make the center web of girders of the type of (f) twice the thickness of each of the outer webs. Two angles at both the upper and the lower edges of this web are needed to receive and transfer the flange stress that is passed on to them by the web. Under the above assumption, it is reasonable to assume that one-half the cover plate area is tributary to the center web and one-quarter to each of the outer webs. Since the center web is also twice as thick as the side webs, the values of the flange rivets through it will be twice as great as for those through the outer webs. It is thus possible by this arrangement to keep the rivet spacing equal in the inner and outer flange angles.
It is desirable to keep the composition of the flanges practically alike so that the neutral axis may not be in any case far from the center of the webs. By so doing the rivet spacing may in general be made the same in the two flanges.

32. Advantages and Disadvantages.—The advantages and disadvantages of box girders are the same as those pertaining to multiple beam girders. They give high strength with shallow depth; they afford broad bearing for applied loads and at the supports; and they are stiff laterally. On the other hand, they are uneconomical of material; their pound cost is greater than for single or multiple rolled sections because of the extra work of fabrication involved; they lack vertical stiffness because of their small depth; and they are subject to corrosion on the interior faces in damp situations.

Box girders are superior to multiple beam girders because of the better tie between the compression flanges and the better bearing for the applied load. The rigid connection of parts reduces the tendency to flange buckling and increases the factor of safety in compression.

33. Proportioning for Moment.—Whether a box girder consists of rolled beams or channels with flange plates riveted thereto, Fig. 27, (a), (b), (c) and (d), or of an assemblage of plates and angles, Fig. 27 (e) and (f), it should, because of the rigid attachment of the parts to each other be regarded as essentially a built-up beam. Its moment of resistance, or capacity to resist bending moment should therefore, be computed from the common flexure formula, \( f/c = M/I \), or \( M = fI/c \). This necessitates the computation of the moment of inertia, \( I \), of the section, unless this fortunately chances to be listed in available tables, as in those given in Cambria Steel. Because of the innumerable combinations of shapes and plates worked into box girder sections, the properties of the particular section most suitable for the work in hand are frequently not listed and so must be specially determined.

It is of great advantage to use an approximate method of design at times, particularly in making rough estimates or in making the first trials for an exact design. For such purpose the approximate method of designing plate girders (Art. 44) may be used. If the box girder be much over 3 ft. deep, such a method may be sufficiently accurate for final design.

In proportioning box girders, the permissible working stress is commonly assumed at either 15,000 or 16,000 lb. per sq. in. The tables of capacity of box girders given in Cambria Steel are based on the former stress, but the rivet holes are assumed as only \( \frac{3}{16} \) in. larger than the diameter of the rivet. The capacity tables for riveted beam girders given in the Carnegie Pocket Companion are based on a working stress of 16,000 lb. per sq. in., but the section modulus used is that for the gross section.

In making exact designs by what is called the "moment of inertia" method, no reduction in section modulus need be made for any type of beam on account of holes in the compression part of the section. The neutral axis of a box girder, therefore, lies somewhat above the center of gravity of the gross section, if the flanges have the same gross area, but for the reasons set forth in Art. 7, the shift cannot be so great as consideration only of the net area through the weakest section would indicate. In view of the uncertainty as to the exact position of the neutral axis, and in view of the simplification of work introduced by computing the net section modulus with respect to the neutral axis of the gross section, this
method will be adopted, the results being corrected as recommended in Art. 10 to compensate for the error in the assumption of the position of the neutral axis. For further substantiation of the percentage corrections specified in the latter article, see the second problem under Art. 37.

For girders with heavy flanges so supported laterally that no allowance need be made for flange buckling, it is desirable to make the net area of the tension flange as nearly as possible equal to the gross area of the compression flange, which means that the gross area of the tension flange must be greater than the gross area of the compression flange. By so doing, the neutral axis is kept practically at the center line of the webs, thus improving the flexural efficiency of the girder and making it possible to keep the rivet spacing in the tension and compression flanges equal (except for local transverse loading). An approximate compensation for the loss of section due to rivet holes on the tension side may be made by adding to the cover plates on the tension side sufficient area to offset the rivet holes in the flange material. The high working stress of the material added to the plates will roughly offset the neglect of any web holes.

In the computation of net section modulus of box girders built up with rolled sections as their primary component parts, such as shown in Fig. 27 (a), (b), (c) and (d), the work may be carried out by making use of Table 3. To the net section modulus in the compound section of the beams or channels employed, based on the stationary axis theory, may be added the net section modulus of the added plates, deducting holes for the plates on the tension side only. The sum may then be reduced by the appropriate percentage to compensate for the erroneous assumption of fixed neutral axis.

**Illustrative Problem.**—Calculate the net section modulus of the compensated, 3-web, built-up box girder section shown in Fig. 28, assuming that rivet holes are to be deducted on the tension side only and that the neutral axis is at the center line of the webs. Rivets \( \frac{3}{8} \) in. and rivet holes 1 in. diameter.

In finding the net moment of inertia of the compensated section, it is convenient to tabulate the quantities as done in Table 10, the gross areas being taken first and the moment of inertia of the holes being listed below. The moment of inertia of the holes about their own gravity axes is neglected, since it is relatively too small to affect the result appreciably. Areas of metal are marked “plus” and areas of holes “minus.” Distances above and below the assumed neutral axis, which is at the center line of the webs, are “plus” and “minus” respectively. The summations at the foot of the second, fourth, fifth and sixth columns are algebraic. The summation of the fourth and fifth columns when added should equal the summation of the sixth.

Net section modulus of girder = net moment of inertia divided by the distance from the assumed neutral axis to the extreme fiber, which in this case (because of the thicker flange plates on the tension flange) is to the extreme tensile fiber.

Hence

\[
S = \frac{I}{c} = \frac{27,028.5}{15.75} = 1,717
\]
### Table 10.—Net Section Modulus of Compensated Built-Up Box Girder

<table>
<thead>
<tr>
<th>Part</th>
<th>Area of part (sq. in.)</th>
<th>Distance (y) from assumed neutral axis to gravity axis of part (in.)</th>
<th>Moment of inertia (I) of part about its gravity axis (in.⁴)</th>
<th>Area (in.*)</th>
<th>I = Iₙ + Auy² (in.*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 webs</td>
<td>+ 42.00</td>
<td>0.0</td>
<td>+2,744.0</td>
<td>0.0</td>
<td>+ 2,744.0</td>
</tr>
<tr>
<td>4 top angles</td>
<td>+ 28.44</td>
<td>+12.52</td>
<td>+ 96.8</td>
<td>+ 4,460.0</td>
<td>+ 4,556.8</td>
</tr>
<tr>
<td>4 bottom angles</td>
<td>+ 28.44</td>
<td>-12.52</td>
<td>+ 96.8</td>
<td>+ 4,460.0</td>
<td>+ 4,556.8</td>
</tr>
<tr>
<td>3 top covers</td>
<td>+ 36.75</td>
<td>+14.91</td>
<td>+ 5.3</td>
<td>+ 8,175.0</td>
<td>+ 8,180.3</td>
</tr>
<tr>
<td>3 bottom covers</td>
<td>+ 42.00</td>
<td>-15.00</td>
<td>+ 7.9</td>
<td>+ 9,450.0</td>
<td>+ 9,457.0</td>
</tr>
<tr>
<td>3 Holes A</td>
<td>- 1.50</td>
<td></td>
<td></td>
<td></td>
<td>- 1.50</td>
</tr>
<tr>
<td>3 Holes B</td>
<td>- 1.50</td>
<td>- 3.25</td>
<td></td>
<td></td>
<td>- 159</td>
</tr>
<tr>
<td>3 Holes C</td>
<td>- 1.50</td>
<td>- 6.50</td>
<td></td>
<td></td>
<td>- 63.4</td>
</tr>
<tr>
<td>3 Holes D</td>
<td>- 4.00</td>
<td>-11.75</td>
<td></td>
<td></td>
<td>- 552.0</td>
</tr>
<tr>
<td>3 Holes E</td>
<td>- 8.50</td>
<td>-14.69</td>
<td></td>
<td></td>
<td>- 1,835.0</td>
</tr>
<tr>
<td>+100.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+24,077.7</td>
</tr>
</tbody>
</table>

As compensation has been made by the extra thickness of the flange plates on the tension flange for the loss of section occasioned by rivet holes on the tension side of the neutral axis, no correction need be made because of the assumption that the neutral axis is at the center line of the webs.

**Illustrative Problem.**—Find the total safe uniformly distributed load that may be carried by a box girder of 21-ft. span of the form shown in Fig. 29, consisting of one 15-in., 65-lb. I, two 15-in., 33.9-lb. channels, and two 16 X 3/8-in. flange plates on each flange. If 3/4-in. rivets are used, and two lines are employed in each flange of the I-beam in addition to one line in each flange of the channels, compute the effective section modulus of the combined section, assuming the neutral axis as at the center of gravity of the gross section, and correcting the result as explained in Art. 10. f = 16,000 lb. per sq. in.


Gross moment of inertia of two pairs of 16 X 3/8-in. flange plates

\[
= 2 \left[ \frac{1}{12} (10)(0.75)^4 + (12.0)(7.875)^2 \right] = 1,490
\]

Total gross moment of inertia = 1,257 + 1,490 = 2,747.

I of 2 holes diam. holes through tension flange of beam and flange plates, if grip of beam is 3/6 in. = (2)(1.63)(0.88)(7.44)^2 = 158.

I of two holes through channel flanges and plates = (2)(1.38)(0.88)(7.56)^2 = 138.

Total I of 4 holes = 296.

Net I of entire section = 2,747 - 296 = 2,451.


This corrected by the coefficient 0.95, as recommended in Art. 10 is (0.95)(297) = 282.

Hence total safe uniformly distributed load

\[
W = \frac{8\sqrt{f}}{I} = \frac{(8)(282)(16,000)}{(21)(12)} = 143,000 \text{ lb.}
\]

34. Length of Flange Plates.—As in the case of rolled beams reinforced for bending, discussed in Art. 12, it is possible, if there is more than one plate on each
flange, to vary the section of a box girder by terminating some of the flange plates where they are no longer needed. The theoretical length of any flange plate, of a uniformly loaded girder is, as was established in Art. 13,

$$x_n = l \sqrt{\frac{s_1 + s_2 + \ldots + s_n}{S}}$$

(1)

where \(x_n\) theoretical length of the \(n\)th flange plate from the outside.

\(l = \) span of girder.

\(s_1, s_2, \ldots, s_n = \) section moduli contributed to the total required section modulus of the girder by successive pairs of cover plates from the outside beginning with the second plate.

\(s_n = \) section modulus required to be contributed by outside plate.

\(S = \) total required net section modulus of girder.

![Graphical determination of length of flange plates for box girder.](image)

In a box girder one cover plate on each flange must run full length to form the necessary tie between the main component parts of the girder, and often in the case of the top flange, to receive and distribute the applied load.

While it is commonly specified that the thinnest of the flange plates shall be put on the outside, there appear to be better reasons for placing the thinnest on the inside, as pointed out in Art. 46.

The practical rule respecting the addition of 9 or 12 in. to the theoretical length of the cover plates at each end stated for reinforced beams in Art. 13 applies also to the box girder.

If the loading on a box girder be not uniform, it is necessary to make use of a graphical method for the determination of the length of cover plates. The bending moment is computed at critical or determining points and from it the required section modulus is derived. A diagram is then prepared for the half span if the loading be symmetrical, and for the full span if it be unsymmetrical, showing the requirement for section modulus at all points of the span. On this diagram is
laid off vertically from the base the section modulus provided (1) by the primary component parts of the box girder, (2) by each of the successive plates from the inside plates outward. The point where the upper horizontal boundary line of the rectangle laid off to represent a component part of the girder cuts the curve will locate the point of theoretical cut-off for the next (outside) part represented.

Illustrative Problem.—Assuming that the loading to which the 21-ft. girder of Fig. 29 is subjected, is not uniform, but is as shown in Fig. 30, and that the span is 21 ft., find the theoretical and practical lengths of the flange plates.

The moments and required section moduli at the points of concentrated loading and certain intermediate points are as shown in the accompanying table.

<table>
<thead>
<tr>
<th>Distance of point from left support (ft.)</th>
<th>Uniform load moment (ft.-lb.)</th>
<th>Moment from concentrated (ft.-lb.)</th>
<th>Combined moment (ft.-lb.)</th>
<th>Required section modulus (in.²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.15</td>
<td>4,460</td>
<td>158,500</td>
<td>162,960</td>
<td>122</td>
</tr>
<tr>
<td>4.3</td>
<td>7,895</td>
<td>317,000</td>
<td>324,895</td>
<td>244</td>
</tr>
<tr>
<td>10.5</td>
<td>12,100</td>
<td>342,000</td>
<td>354,100</td>
<td>266</td>
</tr>
<tr>
<td>18.3</td>
<td>5,400</td>
<td>369,500</td>
<td>374,900</td>
<td>282</td>
</tr>
<tr>
<td>19.65</td>
<td>2,900</td>
<td>185,100</td>
<td>188,000</td>
<td>141</td>
</tr>
</tbody>
</table>

Plotting the required section moduli vertically on a diagram for the full span, Fig. 30, and laying off also vertically the section moduli provided by the primary beam and channels, and the successive cover plates, the theoretical required lengths of the plates may be readily scaled off.

In the previous problem on the girder of Fig. 29, the effective section modulus at the maximum section was found to be 282.

The effective section modulus of the beam and channels plus one cover plate on each flange needs to be found.

Gross I of beam and channels = 1,257.
Gross I of two plates (one on each flange) is approximately \( (2)(9)(7.1875)^2 = 620 \).
Total gross I = 1,877.
I of two holes through beam flange and plate and two through the channel flanges and plate
\[
\]
Net I of section with two flange plates only = 1,877 - 220 = 1,657.
Corrected net section modulus = \( (0.95)(1,657)/7.875 = 199 \).

Laying off this distance vertically on the required section modulus diagram of Fig. 30 and drawing a horizontal line across the diagram, the points of theoretical cut-off are found where this line cuts the curve. Since the stress which these outer plates must carry must be transferred to them in distances of 4.3 and 2.7 ft. at the left and right hand ends respectively, the plates will need to be carried full length to accommodate the necessary rivets.

35. Stiffeners.—In order that concentrated loads may be transferred to the webs of a box girder without exceeding the permissible buckling stress in the webs, it may be necessary to use stiffeners, as in the case of beams, Art. 18. If it is not practicable to make the thickness of the webs great enough to obviate danger from crippling due to ordinary diagonal compression, stiffeners spaced at suitable intervals throughout the length of the girder will need to be used. For the principles governing their proportioning and spacing, see Art. 52.

36. Diaphragms.—To ensure that the principal component elements of a box girder act together as a unit and that any excess of loading received by one ele-
ment is distributed to the others, diaphragms should be inserted at certain points. It is not prudent to count on the stiffness of the cover plates in a vertical direction to transfer load from one vertical rib to another. Diaphragms are essentially the same as the built-up separators, described in Art. 26, being attached to the webs of the box girder preferably by rivets, or if such cannot be driven, then by bolts if possible. If web stiffeners are used for the girder, the attachment of the diaphragm plates may be to them. If stiffeners are not used, the attachment may be by vertical connection angles.

The difficulty of fastening diaphragms to the webs arises from the fact that unless the girder be very large, rivets cannot be driven through the outside webs of a three-web girder or through either web of a two-web girder after the flange plates are riveted on. For a two-web girder the diaphragms may in theory at least, be riveted to the webs before the flange plates are riveted on, though there are shop difficulties entailed by this procedure. For a three-web girder, the cover plates must be connected to the inner web before the outer webs are assembled in place, thus making it impossible to rivet the diaphragms to the outer webs. One way of overcoming the difficulty is to rivet the diaphragms to the inner web, as shown in Fig. 31, and then provide on the inside of each of the outer webs (at top and bottom where diaphragms occur), short bracket angles "A" between which the diaphragm would fit. Before these bracket angles are riveted in place, the diaphragms should be fitted in between them to ensure close bearing when the whole girder is riveted up. By this device, the excessive deflection of one web with respect to the others could be obviated—that is, the load might be distributed transversely.

Where the spacing of the girder webs permit, single pieces of channel run vertically, as shown in Fig. 21c, may be used. This is usually only practicable for girders composed of rolled sections as the primary elements, and is not desirable for the heavier girder because of the lack of stiffening of the horizontal edges of the diaphragm. For such girders, diaphragms with both horizontal and vertical edge angles are desirable. The thickness of the diaphragm plate is commonly fixed by experience, though an indication of a suitable thickness may be gained by considering a load equal to one-quarter of the total load carried by the girder, divided by the number of diaphragms connecting to one outer web as applied at the upper outside corner of each diaphragm. The web should then be proportioned by a web crippling formula such as

\[ p = 15,000 - 150 \frac{h}{t} \]

or any other of those mentioned in Art. 17. The connection and stiffening angles should be \( \frac{5}{6} \) in. for the lighter girders and \( \frac{3}{8} \) in. for the heavier ones.

Diaphragms should be placed at all points of concentrated loading to ensure the proper lateral distribution of the load. They should be placed also at the
ends to give lateral support where the web buckling tendency is pronounced, and at such other points as they might appear desirable in view of probable inequalities of the loading.

37. Flange Riveting.—A simple but indirect method of determining the spacing of rivets in the flanges of a box girder is that followed in the problem on the reinforced beam, Art. 14. This consists of finding the difference in total stress in the added flange material at two sections, and placing between the sections sufficient rivets to develop the difference in stress. The two sections may be conveniently taken at the end of the attached flange element and at the point of maximum stress therein. For most box girders, such as those with only one or two plates of moderate thickness, where the theoretical spacing is much greater than would be permissible by the practical restrictions relating to the maximum rivet spacing, this method is sufficiently accurate. In computing the total stress in the plate, the stress per sq. in. may, without material error, be taken as the maximum permissible fiber stress in bending.

For box girders with relatively heavy flanges, in which the adopted rivet spacing will depend on stress conditions rather than on practical rules for maximum spacing, it is desirable to employ a more exact method than that described in the previous paragraph. Although in the case of deep box girders, the flange rivet spacing may be determined by the approximate methods usually adopted for plate girders (discussed in Art. 51), the generally applicable method is that based on the true horizontal shear between faces of connected parts.

It has been established in Sec. 1, Art. 51b, that the intensity of horizontal shearing stress at any point in a beam or girder is given by the formula

\[ v = \frac{QV}{It} \]

where \( v \) = intensity of horizontal (or vertical) shearing stress in lb. per sq. in.

\( Q \) = statical moment of area on either side of the point considered, taken about the neutral axis.

\( V \) = total vertical shearing force at the section considered.

\( I \) = moment of inertia of the area of the entire section about the neutral axis.

\( t \) = thickness of the section at the point considered.

For a lin. in. of girder the horizontal shearing area = \( (1)(t) = t \), and hence the total horizontal shear per lin. in. is

\[ H = vt = \frac{QV}{I} \]

While the applicability of this formula to joints that lie in a horizontal plane is clear, it may not be so evident that it applies to joints in a vertical plane. For example, let it be required to determine the total horizontal shear per lin. in. between the web plates of the girder shown in Fig. 32 and the flange angles riveted thereto. The total horizontal shear between that portion of the section which lies above the horizontal plane \( BB \), and the portion lying below it, is obviously

\[ H_b = \frac{Q_bV}{I} \]
Part of $H_B$ is borne by the portion of the two web plates above the level $BB$ and part by the flange angles and cover plate, the division being on the basis of relative statical moments. Consequently if in $Q_B$ only the statical moment of the angles and cover plate is included, the result, $H_B$, will be the amount of total horizontal shear between the flange angles and the web plates.

Having found the horizontal shear per lin. in., $H_1$ between an element of the flange which is riveted to the remainder of the girder, the rivet pitch in the flange element, provided it does not bear any local transverse load, may be expressed by the formula

$$p = \frac{r}{H_1}$$

where $r =$ safe resistance of one rivet in the situation under consideration.

**Illustrative Problem.**—If, at a certain section, the box girder shown in Fig. 32 is subjected to a total vertical shear of 348,000 lb., find the required pitch of the rivets in the tension flange. Rivets, $\frac{7}{8}$ in. diameter. Safe shearling and bearing stresses = 12,000 and 21,000 lb. per sq. in., respectively.

Total horizontal shear per lin. in. on planes of contact between web plates and angles of tension flange riveted thereto is

$$H_1 = \frac{QV}{l}$$

The statical moment of the net area of the tension flange about the assumed neutral axis (the center line of the webs) may be readily determined from the figures given for this girder in Table 10, p. 236, respecting the moment of inertia of this section. Arranged in tabular form, the computation is as below. For this purpose the areas of holes are considered negative, but all distances, though measured downward from the neutral axis, are taken as positive. From the summation of the last column, the statical moment is seen to be 831.8.

<table>
<thead>
<tr>
<th>Part</th>
<th>Area of part (sq. in.)</th>
<th>Distance ($y_0$) from assumed neutral axis to gravity axis of part (in.)</th>
<th>Statical moment of part ($Ay_0$) (in.$^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 bottom angles</td>
<td>+28.44</td>
<td>+12.52</td>
<td>+356.0</td>
</tr>
<tr>
<td>3 bottom covers</td>
<td>+42.00</td>
<td>+15.00</td>
<td>+630.0</td>
</tr>
<tr>
<td>Part 3 holes ($D$)</td>
<td>−2.50</td>
<td>+11.75</td>
<td>−29.4</td>
</tr>
<tr>
<td>4 holes ($E$)</td>
<td>−8.50</td>
<td>+14.69</td>
<td>−124.8</td>
</tr>
<tr>
<td></td>
<td>+50.44</td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

Net moment of inertia of section, from Table 10 = 27,028.5.

Horizontal shear, per lin. in. of girder, transferred to flanges by web

$$H_1 = \frac{(831.8)(348,000)}{27,028.5} = 10,700 \text{ lb.}$$
Since the outer webs are each one-half the thickness of the center web, each will be assumed as taking one-quarter of the total horizontal sheaf per lin. in.—that is,

$$H_1 = 0.25(10,700) = 2,675 \text{ lb.}$$

The minimum value of one \(\frac{7}{8}\)-in. rivet connecting the flange angles to the center web is its single sheafing value \(t = 0.60(12,000) = 7,200 \text{ lb.}

Hence theoretical pitch should be

$$p = \frac{t}{H_1} = \frac{7,200}{2,675} = 2.69 \text{ in.}$$

To give a well distributed connection between the web and the flange angles it is best to use two gage lines in the vertical legs of the angles, so that the pitch of 2.69, or say \(\frac{7}{8}\) in., would be a staggered pitch.

**Illustrative Problem.**—Determine the necessary section for a symmetrically made up double-beam box girder, of the form shown in Fig. 33 (illustrating a girder designed in accordance with assumption \(D\)), to the data given below under the following alternative assumptions respecting allowance made for rivet holes and the position of the neutral axis:

(A) No deduction for rivet holes; neutral axis at center of gravity of gross section.

(B) Rivet holes in both flanges deducted neutral axis at center of gravity of net section.

(C) Rivet holes in tension flange only deducted; neutral axis at center of gravity of net area.

(D) Rivet holes in tension flange only deducted; neutral axis at center of gravity of gross section.

**Data.**—Span, 22 ft. clear, or 23 ft. 4 in. center to center of bearings.

Section must be not over 23 in. deep nor over 14 in. wide.

Load to be 10,270 lb. per lin. ft., uniformly distributed, consisting of 10,000-lb superimposed load and 270 lb. due to weight of girder.

Permissible stresses to be as follows: Bending on extreme fiber = 10,000 lb. per sq. in., shearing on beam webs = 10,000 lb. per sq. in. on gross area.

**Vertical buckling stress on beam webs**

$$f = \frac{R}{\left(a + \frac{d}{4}\right)}$$

must not exceed \(\mu = 10,000 - \frac{173}{l}\), where

\(R = \) end reaction of one beam in lb.
\(a = \) length of bearing in in.
\(d = \) depth of beam.
\(t = \) thickness of beam web.

**Rivets** = \(\frac{7}{8}\) in. dia.; holes for stress calculations = \(\frac{7}{8}\) in. diam.

**Design A**

**Shear.**—End reaction, or end shear,

$$V_1 = \left(\frac{7}{8}\right)(23.33)(10,270) = 119,800 \text{ lb.}$$

Required section for shear = 119,800/10,000 = 11.98 sq. in.

As the limitation for depth would permit 20-in. beams, it would be in the interests of flexural economy to use beams of this depth, although shallower ones with thicker webs would be desirable if the shear happened to determine the design. Assume two 20-in., 65.4-lb. Y's. Shear area provided = \(2dt = (2)(20)(0.5) = 20 \text{ sq. in.}$$
These are, therefore, sufficient.

Web Buckling.—Existing vertical compressive stress on the web of one beam at the support, assuming a 14-in. bearing,

\[ f = \frac{(14)(119,800)}{(14 + \frac{20}{4})(0.5)} = 6,300 \text{ lb. per sq. in.} \]

Permissible buckling stress on web,

\[ p = 19,000 - 173 \left( \frac{20}{0.5} \right) = 12,100 \text{ lb. per sq. in.} \]

Hence, assumed section is safe against web buckling.

Bending.—Maximum bending moment,

\[ M = \frac{Wl}{8} = \frac{(257)(233)(280)}{8} = 8,387,000 \text{ in.-lb.} \]

Required maximum section modulus,

\[ S = \frac{M}{f} = \frac{8,387,000}{16,000} = 52 \]

Assume as section the following:

Two 20-in., 65.4-lb. I's
Four 14 × \( \frac{5}{16} \)-in. plates,

arranged as shown in Fig. 33 (which in detail applies only to case D), the outer plate on each flange to be cut off at the point where it is no longer necessary.

Gross moment of inertia of two 20-in., 65.4-lb. I's about neutral axis of girder = \((2)(1169.5) = 2,339.\)

Gross moment of inertia of two pairs of 14 × \( \frac{5}{16} \)-in. plates,

\[ I_p = 2(I_0 + A_{p0}) = 2[(\frac{5}{16})(\frac{5}{2})(1.125) + (14)(1.125)(10.5635)] = 2(2 + 1.755) = 3,514 \]

Total gross moment of inertia = 5,853.

Gross section modulus provided = 5,853/11.125 = 526. The section assumed is therefore adequate for bending.

Design B

Shear and Web Buckling.—The stresses and necessary sections are the same as for Design A.

Bending.—To compensate for the loss of area due to rivet holes in both flanges, assume the following section:

Two 20-in., 65.4-lb. I's
Two 14 × \( \frac{5}{6} \)-in. plates (inside)
Two 14 × \( \frac{5}{4} \)-in. plates (outside).

Gross moment of inertia of two I's = 2,339 in.\(^4\)

Gross moment of inertia of two pairs of plates, each pair comprising one 14 × \( \frac{5}{6} \)-in. plate and one 14 × \( \frac{5}{4} \)-in. plate, the thinner plate being on the inside for the reason given in Art. 46,

\[ I_p = 2[(\frac{5}{16})(14)(1.375) + (14)(1.375)(10.0889)] = 4,404 \text{ in.}^4 \]

Total gross moment of inertia = 2,339 + 4,404 = 6,743 in.\(^4\)

Moment of inertia of four \( \frac{5}{6} \)-in. holes through 0.78 + 1.38 = 2.16 in. of metal, neglecting the moment of inertia of the holes about their own gravity axis, \( I_h = (4)(0.875)(2.16)(10.30)^2 = 803 \text{ in.}^4 \)

Net moment of inertia = 6,743 − 803 = 5,940 in.\(^4\)

Net section modulus = 5,940/11.375 = 522.

This is sufficiently near the requirement, 524.

Design C

Shear and Web Buckling.—Same as for Design A.

Bending.—In this case, only the holes on the tension side are to be deducted and the neutral axis is assumed to take up a position at the center of gravity of the resulting net area.
The eccentricity, \( e \), or distance of the center of gravity of the net area from the center of gravity of the gross area may be readily found by the formula,

\[
e = \frac{Q_h}{A_n}
\]

where \( Q_h \) = statical moment of the holes deducted, taken about center of gravity of the gross section;

\( A_n \) = net area of whole section.

For this case assume that the section adopted and the gross area is as follows:

Two 20-in., 65.4-lb. I's = 38.16 sq. in. gross

Four 14 \( \times \) 5/8-in. plates = 35.00 sq. in. gross

Total gross area = 73.16 sq. in.

The statical moment of two holes about the neutral axis of the gross section

\[
Q_h = (2)(0.875)(2.03)(10.24) = 36.4 \text{ in.}^4
\]

Net area of whole section = 73.16 - (2)(0.875)(2.03) = 69.60 sq. in.

Hence, eccentricity

\[
e = 36.4 \quad 69.6 = 0.52 \text{ in.}
\]

Moment of inertia, \( I_n \), of net section about the neutral axis established above may be found from

\[
I_n = I_\varphi + Ax^2 - ay_o^2
\]

where

\( I_\varphi \) = moment of inertia of gross section about neutral axis of gross section.

\( A \) = gross area of section.

\( a \) = area of holes deducted.

\( y_o \) = distance of center of gravity of holes deducted from neutral axis of net section.

\( I_\varphi \) for the present case = 2.339 + \( (\frac{1}{12})(14)(1.25)^3 + (14)(1.25)(10.625)^3 \) = 6,297 in.\(^4\)

\[
A = 73.16 \text{ sq. in.}
\]

\[
e = 0.52 \text{ in.}
\]

\[
a = (2)(0.875 \times 2.03) = 3.55 \text{ sq. in.}
\]

\[
y_o = 10.24 + 0.52 = 10.76 \text{ in.}
\]

Hence,

\[
I_n = 6,297 + (73.16)(0.52)^2 - (3.55)(10.76)^2 = 5,906 \text{ in.}^4
\]

Net section modulus = 5,906/11.77 = 502 in.\(^3\)

As this is somewhat below the requirement, 524, the section will need to be increased. Assume that the outer plates are each increased \( \frac{1}{4} \) in. in thickness. The increased moment of inertia for the addition to the compression flange is approximately

\[
I = (\frac{1}{16})(10.73 + 0.03)^2 = 101.1 \text{ in.}^4
\]

The increase in moment of inertia for the added thickness to the tension flange is

\[
\Delta I' = 14 - (2)(0.875) \frac{(11.77 + 0.03)^3}{16} = 106.4
\]

Total increase in moment of inertia = 207.5.

Total net moment of inertia of increased section = 5,906 + 207.5 = 6,113.5. Hence, net section modulus of increased section = 6,113.5/11.83 = 518, which is the nearest approach that can be made to the requirement, 524. The outer cover plates will therefore each be 14 \( \times \) 5/8 in.

**Design D**

*Shear and Web Buckling.—Same as for Design A.*

*Bending.—Assume as section,*

Two 20-in., 65.4-lb. I's

Four 14 \( \times \) 5/8-in. plates

From Design C, the gross moment of inertia of this section about the gravity axis of the gross section = 6,297 in.\(^4\)

Moment of inertia of two holes about the neutral axis of the gross section (assumed in this case as the neutral axis of the girder as built) is

\[
I_h = (2)(0.875)(2.03)(10.24)^2 = 373 \text{ in.}^4
\]
Net moment of inertia of girder = 6,297 - 373 = 5,924 in.\(^4\)
Net section modulus = 5,924/11.25 = 526. This is adequate.

Comparison of Designs

In order to facilitate the comparison of the results reached by designing according to the four alternative assumptions respecting the effect of rivet holes, the following table has been prepared:

<table>
<thead>
<tr>
<th>Design</th>
<th>Holes deducted</th>
<th>Assumed position of neutral axis</th>
<th>Net I (in.(^4))</th>
<th>Net S (in.(^4))</th>
<th>Max. gross area, (A) (in.(^2))</th>
<th>(S) (A)</th>
<th>Relative efficiencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>None</td>
<td>Center of gravity of gross section</td>
<td>5,853</td>
<td>526</td>
<td>69.66</td>
<td>7.55</td>
<td>1.00</td>
</tr>
<tr>
<td>B</td>
<td>Two from each flange</td>
<td>Center of gravity of gross section</td>
<td>5,940</td>
<td>522</td>
<td>76.66</td>
<td>6.81</td>
<td>0.90</td>
</tr>
<tr>
<td>C</td>
<td>Two from tension flange</td>
<td>Center of gravity of net area section</td>
<td>6,114</td>
<td>518</td>
<td>74.91</td>
<td>6.92</td>
<td>0.92</td>
</tr>
<tr>
<td>D</td>
<td>Two from tension flange</td>
<td>Center of gravity of gross area section</td>
<td>5,924</td>
<td>526</td>
<td>73.16</td>
<td>7.18</td>
<td>0.95</td>
</tr>
</tbody>
</table>

In the next to the last column of the table is given the amount of section modulus developed for each square inch of gross area in accordance with the four basic assumptions of design. In the last column the relative efficiencies are given—that is, the relative amount of section modulus developed per sq. in. of gross area. From this column it is seen that there is a loss of about 10 per cent where all holes are deducted, but only from 5 to 8 per cent, depending on the assumption respecting the position of the neutral axis, when only the holes in the tension flange are deducted.

The figures given in the table show the possibility of saving time in the design of box girders by making the calculation of section modulus by one of the simpler assumptions, such as A or D, and then correcting the results in accordance with the actual assumption made respecting deductions and position of the neutral axis. For example, if the section modulus for a given section were found according to assumption A, reducing it by 8 per cent would give the section modulus for the same gross section according to assumption C. If the section modulus were found by assumption D, reducing it by 4 per cent would give the section modulus according to assumption C. It is best to base the calculation on assumption D, as the amount of correction is less than required if it is based on method A.

**Illustrative Problem.**—Find the theoretical and practical lengths of the cover plates for the box girder designed in the last problem, according to assumption B.

Although the outer cover plates are required for only a fraction of the girder length, the inner cover plate on the compression flange must be curved full length to stay the flanges of the beams against buckling and to provide a satisfactory bearing for the applied load. While in theory the inner cover plate on the bottom flange may be cut off short of the end, it is customary to carry it full length also. This practice has the incidental advantage of keeping the neutral axis near the center of the beam webs.

From Formula (1) of Art. 34, the length of the outer cover is given by the formula

\[ x_1 = l \sqrt{\frac{b' I'}{S}} \]

where
- \(x\) = the theoretical length of the cover plate.
- \(l\) = length of span.
- \(b'\) = section modulus required to be contributed to the girder by the two outer covers.
- \(S\) = total required net section modulus.
Net section modulus of beams with two inner \(14\times\frac{3}{4}\)-in. cover plates only is found by the methods already elucidated to be 328.

Difference between total required section modulus \(S\) and the section modulus provided by two beams and the two inside flange plates is

\[s_1' = 524 - 328 = 196\]

Hence, theoretical length of outer flange plates is

\[x_1 = 23.33\sqrt{\frac{196}{524}} = 14.3\text{ ft.}\]

To this length about 2 ft. would be added for the reasons given in Art. 18, so that the plates would be about 16 ft. long.

**Illustrative Problem.**—Determine the rivet spacing in the cover plates of the girder of the last problem given the following data: Rivets, \(\frac{3}{4}\) in. diam.; permissible shearing and bearing stresses on shop rivets = 10,000 and 20,000 lb. per sq. in. respectively.

The number of rivets required in each flange from the center of the bearing to the center line of the girder is the number necessary to transmit the total stress in the two plates from the beams into those plates.

Since the fiber stress increases uniformly from the neutral axis out to the extreme fiber and the thickness of plates on each flange is \(\frac{5}{8} + \frac{3}{4} = 1.38\) in., the average fiber stress in the flange plates will be

\[f_a = (16,000) \left(\frac{10.69}{11.38}\right) = 15,000\text{ lb. per sq. in.}\]

Total stress in two plates at center of girder equals net area of plates multiplied by average stress per sq. in.—that is,

\[P = [(14) - (2)(0.875)](1.38)(15,000) = 254,000\text{ lb.}\]

Least safe resistance of one rivet is single shearing value,

\[r = (0.442)(10,000) = 4,420\text{ lb.}\]

Number of rivets required from center of bearing to center line of girder

\[N = \frac{254,000}{4,420} = 57\]

or, say, 29 in each gage line.

Considering the outer one of the two cover plates alone—that is, the \(\frac{3}{4}\)-in. plate—the average working stress in it is

\[f_{aw} = (16,000) \left(\frac{11.00}{11.38}\right) = 15,450\text{ lb. per sq. in.}\]

Total stress borne by one \(14\times\frac{3}{4}\)-in. plate with two \(\frac{5}{8}\)-in. holes out is

\[P' = [(14) - (2)(0.875)](0.75)(15,450) = 142,000\text{ lb.}\]

Number of rivets required through outer cover plate from its end to center line of girder

\[N = \frac{142,000}{4,420} = 32\]

or 16 in each gage line.

The actual spacing should be arranged so as not to exceed 6 in. in either line and so that for a distance of about 2 ft. at each end of each plate the spacing is less than this, say 3 or 4 in.

**PLATE GIRDERS**

**By C. R. Young**

**38. General Characteristics.**—Whenever the situation calls for a beam or girder of greater flexural capacity than that of any single rolled beam or beam girder available, and the height conditions permit a girder of economic depth for bending, a single built-up beam, or plate girder, can be used to advantage. Although the pound price of such a girder is greater than for a multiple beam girder, or for a box girder utilizing rolled beams or channels for its primary elements, the saving in metal due to the use of a more favorable depth generally makes the plate girder cheaper for heavy loads.
Essentially, a plate girder is an I-beam built up of plates and angles, as shown in Fig. 34. Unlike the I-beam, which is of uniform section throughout its length, the plate girder may be varied in section should such prove desirable. It is, for example, easy to reduce the flange section at points where the smallness of the bending moment warrants it, and for very large girders to use a thinner web in the region of light shear than in those of heavy shear.

Like the I-beam, the plate girder developed naturally from the I-shaped cast-iron beams and girders that preceded it. An indication of the possibilities of long-span built-up girders was given in the successful completion of the great Britannia and Conway tubular bridges in Wales, the former containing two spans of 460 ft. each, built in 1850. These tubular spans were, in reality, nothing but very large box girders carrying the traffic through, rather than on top of them. The longest span ever built in single web plate girder construction was a through span of 170 ft. in the clear, built in 1864 over the Pilalee River on the Eastern Bengal Railway. The two main girders were 13½ ft. deep and 22 ft. apart. The longest simple plate girder spans in America occur in the double-track bridge of the Lehigh Valley Railroad over the Susquehanna River at Towanda, Pa., which contains 13 spans of 129½ ft., and one of 120 ft.

Although plate girder spans of 130 ft. or over may thus be successfully built, it is usually more economical to employ a truss span for lengths over about 120 ft. The pound price and the maintenance cost of plate girder bridges is relatively low, but beyond the 120-ft. limit the saving of material in truss spans is likely to offset the advantages of plate girder construction.


39a. Web.—The primary element in the make-up of a plate girder is the web. This may be in one piece if the girder is not over about 30 ft. in span, or it may be in several pieces for longer girders. For very long and very deep girders, web splices, such as shown in Fig. 34, may be as close together as 10 ft. due to the difficulties in getting web plates of sufficient width for the depth of the girder.

39b. Flanges.—Flange angles are riveted to the upper and lower edges of the web so as to add to the flexural capacity of the web, and to these angles flange plates are riveted in turn. The most common type of flange is the T-flange, consisting of two angles arranged as a T, with or without attached plates, as in Fig. 35 (a) and (b). The bottom flange of the girder of Fig. 34 is of this type. Angles alone are used for comparatively light girders.
If more than about 50 per cent of the flange material occurs in the form of cover plates, it is customary, if a T-flange is being used, to connect flange plates directly to the web by placing them between the flange angles and the web and letting them extend past the inner edges of the flange angles. A flange of this sort is shown in Fig. 35 (c).

If it is desirable to maintain the top surface of the top flange as a plane—as, for example, in deck plate girder spans for railway work—a four-angle flange, arranged as shown in Fig. 35 (d) may be used to advantage. Although flange, or cover plates, are added to a T-flange on the backs of the outstanding legs, they are added to a four-angle flange in two vertical planes on the outer faces of the vertical legs. In either case, variation of the flange section is easily possible by cutting off the flange plates whereover desired. If it be desired to build up a particularly heavy flange, four angles may be used and flange plates may be added both in horizontal and vertical planes, as shown in Fig. 35 (f). Some of the latter may be placed between the angles and the web, and thus be classed as directly-connected material. Where a considerable lateral moment is exerted on the flange—as for a crane runway girder—or for any reason especial lateral rigidity is required in the flange, a channel with flanges turned down, Fig. 35 (g), is advantageously employed.

If the total length of the girder is over about two car lengths, or about 60 ft., it is usually necessary to count on splices in the flange, as flange angles in single pieces of greater length than mentioned are not likely to be available in the average shop. These splices consist commonly of a short piece of angle of appropriate section riveted to the spliced flange angle, as described in Sec. 3, Art. 17.

39c. Stiffeners.—If the unsupported depth of the web exceeds the limit mentioned in Art. 42, stiffeners are riveted vertically to the web at intervals equal roughly to the depth of the web. These consist usually of a pair of angles, one on each side of the web with one line of rivets serving the two. Where the top flange is of the four-angle type, as in Fig. 35 (d), (e), and (f) pairs of short angles must be used between the upper and lower angles of the top flange to give support to the outstanding legs of the upper angles and to help transfer the concentrated applied loads to the web. These short angles should be ground to fit at both top and bottom, but the main angles, except in the case of the end ones, need to be fitted tightly only at the top. At the ends of the girder two pairs of stiffener angles are employed to prevent the web from buckling and to serve in a measure as a column for the transfer of the end reaction to the support. Fillers are always inserted under these stiffener angles between the flange angles on each side of the web to keep the stiffener angles straight. Fillers may be used under intermediate stiffeners, but it is generally more economical, particularly for the deeper girders, to crimp the ends of the stiffener angles to fit over the flange angles.
39d. Bearings.—At each end of the girder is a sole or shoe plate resting on a bed plate or bed casting. If the span is over about 80 ft. a bolster, such as described in Arts. 56 and 59, is generally used between the shoe and the bed plates to overcome the tendency of the deflection to produce intensified pressures on the supports near the inner edge. Rollers may be used at the sliding end of the span, under the conditions described in Arts. 56 and 60.

40. Stress Conditions to be Met.—Like an ordinary beam, a plate girder must be secure against failure by flexure, flange buckling, shear or web crippling, and at the same time must not deflect to such an extent as to cause damage or unsightliness to any construction or interfere with the easy operation of moving structures. As a plate girder may be built to conform closely to all the stress conditions, unlike a rolled beam which must be of some standard cross-section, greater economies of material are possible with the use of plate girders than by the utilization of rolled beams.

41. Proportioning for Shear.—Although with a rolled I-beam, investigating for shearing stresses is necessary only for short, heavily-loaded spans, with a plate girder the design concerns the web quite as much as the flanges. In the latter case, the relation of web to flange area is subject to almost indefinite variation, while for beams there are a few fixed standard proportions, one of which must be adopted.

The general formula giving the intensity of shearing stress at any point in a beam,

\[ v = \frac{QV}{It} \]

discussed in Art. 16, in connection with beams, applies to plate girders, as does the common approximate assumption that the shearing stress is borne entirely by the web and may be considered as uniformly distributed over the web. It is, of course, true, as has been shown in the discussion of Art. 16, and in Table 6 giving the relation of maximum to average shearing stress in beams, that the above assumptions are in error. For plate girders the error is about the same as for beams—that is, the maximum shearing stress at the neutral axis is ordinarily from 10 or 20 per cent greater than that given by the method of average stress. As, in the case of beams, compensation for the error involved is made by selecting a conservative working stress, one which at the same time makes provision for the ordinary loss of section due to vertical lines of rivet holes.

Although the effect on the shearing strength of a web plate produced by a vertical line of holes filled with rivets is not definitely known, some idea of the extreme limit of this effect may be gathered from considering the effect of a vertical line of open holes. Assume two plates each of thickness \( t \), the first (a) being 32\( t \) deep and the second (b) 64\( t \) deep, as shown in Fig. 36. Let there be holes of diameter 2\( t \) in each, spaced 8\( t \) apart, center to center. For convenience, let half
of one hole in each case be deducted at each of the extreme edges. In each case the loss of area and the loss of statical moment of half the area about the neutral axis is 25 per cent. For case (a) the loss of moment of inertia is 28.2 per cent, and for case (b) it is 25.9 per cent. This shows that the reduction in moment of inertia of a plate due to punching holes at uniform spacing is very nearly proportional to the reduction of area, being almost exactly so for the deeper plates. Since the true shearing stress

\[ \tau = \frac{QV}{It} \]

and both the statical moment, \( Q \), and the moment of inertia, \( I \), may be taken as proportional to the net area of the plate, the shearing stress will be increased in the same ratio as the area is reduced.

If the actual maximum shearing stress is therefore in excess of the average shearing stress on the gross area of the web by, say, 15 per cent on account of the error involved in assuming uniform distribution, and this in turn must be increased by as much as 25 per cent by reason of holes, the actual stress would be 44 per cent in excess of the apparent stress. If a girder web is designed, therefore, for an average stress on gross area of 10,000 lb. per sq. in., and allowance for the effect of lines of fairly closely-spaced holes must be made fully, the actual maximum stress would be over 14,000 lb. per sq. in. With a factor of safety of 4, the safe shearing stress in structural steel is about 12,000 lb. per sq. in., so that the 10,000 lb. on gross area would then be excessive. However, it is probably too severe to assume a line of vertical holes tightly filled with rivets as no better than a line of open holes for the resistance of shear. It is probably not necessary, therefore, to require webs to be designed for an average shearing stress of 10,000 lb. per sq. in. on net area, although there are cases when 10,000 lb. per sq. in. on gross area gives excessive results.

42. Proportioning for Web Buckling.—As has been pointed out in the discussion of web stresses in beams, Arts. 16, 17, and 18, the adequacy of a web is more frequently determined by its capacity to resist crippling or buckling than by its shearing value. The maximum crippling stress at a point arises from a combination of the flexural and the shearing stress. The location of the point where the diagonal compression reaches its absolute maximum is of importance, as well as the actual value of this maximum.

In order to show the nature and magnitude of the diagonal compressive stresses in typical plate girders, the stresses at various sections of two characteristic girders, Fig. 37, have been calculated in the two problems which follow. The intensity of the maximum stresses and their direction are computed by the appropriate formulas of Sec. 1, Art. 53.

\[ f_m = \frac{1}{2} f \pm \sqrt{\frac{1}{4} f^2 + \tau^2} \]

\[ \tan 2\theta = -\frac{2\tau}{f} \]

The flexural stress at any point on a cross-section has been computed by applying to the section, the ordinary flexure formula

\[ f = \frac{My}{I} \]
where

\[ M = \text{moment in inch-pounds} \]

\[ y = \text{distance of fiber from neutral axis of girder} \]

\[ I = \text{gross moment of inertia of section}. \]

The shearing stress was calculated by the exact formula of Art. 16.

\[ v = \frac{QV}{It} \]

**Fig. 37.**—Diagonal compressive stresses at various sections on typical plate girders

**Illustrative Problem.**—A 30-ft. plate girder consisting of one \(36 \times \frac{5}{16}\)-in. web plate, four \(6 \times 4 \times \frac{3}{8}\)-in. flange angles, and two \(13 \times \frac{3}{8}\)-in. cover plates, 20 ft. long, as shown in Fig. 37a, carries a total uniformly distributed load of 5,000 lb. per lin. ft. Find the intensity and direction of the diagonal compressive stress at the points indicated in Fig. 37a at cross sections 1.17, 5, 10 and 15 ft. from one support, neglecting the local effect of the application of the superimposed load.

The bending moments and shearing forces at the section considered are listed in Table 11.

Gross moment of inertia of girder without cover plates = moment of inertia of web + moment of inertia of four flange angles or

\[ I_1 = \left(\frac{1}{12}\right)(0.3125)(36)^3 + \frac{4(4.9) + (3.61)(17.31)^2}{2} = 5,565 \text{ in.}^4 \]

Gross moment of inertia of girder with two cover plates,

\[ I_2 = (5,565) + (2)(4.875)(18.438)^2 = 8,880 \text{ in.}^4 \]

Applying the flexure and shearing stress formulas to each of the four cross-sections, the bending and shearing stresses at the seven stipulated points are calculated. Combining them, the intensities and direction of the maximum resultant stresses at the selected points are then found. In Fig. 37a the intensity of the resultant compressive stress at the seven points on each of the four cross sections is plotted horizontally to the right of the section, and curves drawn through the extremities of the lines so plotted. By scaling off the horizontal distance from the section plane to the appropriate curve at the level of the point
considered, the intensity of the maximum diagonal compression at the point may be found. The direction of the maximum compressive stress is indicated for the chosen points on each cross-section by arrows to the left of the section.

Figures for the actual intensity of the diagonal compressive stress are given in Table 11 in the case of each of the cross-sections for the extreme compressive fiber and for the fiber of the web immediately below the inner edge of the flange angles.

**Table 11.—Comparison of Maximum Compressive Stresses at Selected Points on Various Cross-Sections of Typical Girders**

<table>
<thead>
<tr>
<th>Section no.</th>
<th>Distance from support (ft.)</th>
<th>Moment M (ft.-lb.)</th>
<th>Shear V (lb.)</th>
<th>Moment of inertia I (in.⁴)</th>
<th>Point on cross-section</th>
<th>Flexural stress f (lb. per sq. in.)</th>
<th>Shearing stress v (lb. per sq. in.)</th>
<th>Maximum compressive stress (lb. per sq. in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.167</td>
<td>84,000</td>
<td>69,200</td>
<td>5,563</td>
<td>Extreme compressive fiber</td>
<td>3,320</td>
<td>3,320</td>
<td>3,320</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
<td>312,500</td>
<td>50,000</td>
<td>5,565</td>
<td>Extreme fiber</td>
<td>12,320</td>
<td>5,720</td>
<td>7,150</td>
</tr>
<tr>
<td>3</td>
<td>10.0</td>
<td>500,000</td>
<td>25,000</td>
<td>8,880</td>
<td>Extreme fiber</td>
<td>12,580</td>
<td>4,140</td>
<td>11,150</td>
</tr>
<tr>
<td>4</td>
<td>15.0</td>
<td>562,500</td>
<td></td>
<td>8,880</td>
<td>Extreme fiber</td>
<td>14,150</td>
<td>2,100</td>
<td>10,060</td>
</tr>
</tbody>
</table>

**Girder with two symmetrical concentrated loads, Fig. 37 (b)**

<table>
<thead>
<tr>
<th>Section no.</th>
<th>Distance from support (ft.)</th>
<th>Moment M (ft.-lb.)</th>
<th>Shear V (lb.)</th>
<th>Moment of inertia I (in.⁴)</th>
<th>Point on cross-section</th>
<th>Flexural stress f (lb. per sq. in.)</th>
<th>Shearing stress v (lb. per sq. in.)</th>
<th>Maximum compressive stress (lb. per sq. in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.167</td>
<td>131,000</td>
<td>112,500</td>
<td>5,565</td>
<td>Extreme fiber</td>
<td>4,300</td>
<td>4,300</td>
<td>4,300</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
<td>562,500</td>
<td>112,500</td>
<td>8,880</td>
<td>Extreme fiber</td>
<td>14,150</td>
<td>9,300</td>
<td>11,130</td>
</tr>
<tr>
<td>3</td>
<td>10.0</td>
<td>562,500</td>
<td></td>
<td>8,880</td>
<td>Extreme fiber</td>
<td>14,150</td>
<td>9,460</td>
<td>16,310</td>
</tr>
<tr>
<td>4</td>
<td>15.0</td>
<td>562,500</td>
<td></td>
<td>8,880</td>
<td>Extreme fiber</td>
<td>14,150</td>
<td>14,150</td>
<td>14,150</td>
</tr>
</tbody>
</table>

**Illustrative Problem.**—A 30-ft. girder made up as for the last problem, but with cover plates 27 ft. 8 in. long, carries a 112,500-lb. load at each of two points 5 ft. from the supports. The dead weight of the girder is assumed to be included in these loads. Find the intensity and direction of the maximum compressive stresses at the same sections and points as those selected for the last problem.

The moments and shears for the sections considered are listed in Table 11. On the assumption that the weight of the girder is included in the two concentrated loads, the moment increases uniformly from zero at the supports to a maximum at the points of loading and in loads. The shear being assumed as zero for this section, the maximum stresses are all purely flexural. Diagrams similar to those plotted in Fig. 37 (a) are shown in Fig. 37 (b). The intensities of maximum compressive stress for the two points of particular interest on each cross-section are listed in Table 11. For simplicity, the maximum shear and maximum moment are both assumed to occur at the center of the concentrated load.

**42a. Variation in Web Compression.**—Study of the results obtained in the last two illustrative problems shows:

1. That the diagonal compressive stress tends to become measurably constant throughout the clear depth of the web as the shear becomes relatively large and the moment relatively small.
2. That the direction of the maximum diagonal stresses tends to become approximately at 45 deg. with the neutral axis throughout the clear depth of the web, wherever the shear is relatively large and the moment relatively small.

3. That near the ends of a girder and at points where the shear and moment are both large, the diagonal compressive stress in the web immediately inside the inner edge of the flange angles may be considerably in excess of the flexural stress at the extreme fiber. Such occurs at sections 1 and 2 for both girders considered.

42b. Thickness Ratio of Web.—Although some provision against failure of a girder web by buckling is involved in proportioning for shear according to the usual range of permissible shearing stresses, further limitations must often be made.

To keep the slenderness ratio of a vertical strip of web plate within reasonable bounds, it is sometimes specified that the thickness of the web shall not be less than \( \frac{3}{4} \frac{b_0}{h} \) of the unsupported depth between flange angles or side plates. This, for example, is the requirement in Schneider’s “General Specifications for the Structural Work of Buildings.” The rule adopted in the 1920 specification of the American Railway Engineering Association is that the thickness of the web plate shall not be less than \( \frac{1}{2} \sqrt{b_0} \), where \( b \) is the unsupported distance between flange angles. Other specifications commonly fix the minimum thickness of webs as \( \frac{3}{8} \) in. for girders of railway bridges and \( \frac{5}{16} \) in. for highway bridge and building girders. Occasionally a \( \frac{3}{4} \)-in. web is used for building work, but the percentage loss through corrosion is so large and the possible damage in fabrication, transportation and erection so great that such thin webs are not to be recommended. In most cases a uniform thickness is used throughout the length of the girder, although for very heavy girders an increase of thickness in regions of large shear and diagonal compression is adopted.

A further restriction of the clear length of web plate that may buckle without hindrance is imposed by the requirement of intermediate stiffeners under certain conditions. If the thickness of web plate is less than \( \frac{1}{4} b_0 \) or \( \frac{1}{60} \) the unsupported distance between flanges, stiffeners are commonly specified. These, if spaced sufficiently close together, as explained in Art. 52, break up, or limit the length of diagonal belts of web, along which compressive or buckling stresses may reach a high intensity in regions of large shear, or large shear and large moment combined.

42c. Limiting Buckling Stresses. Although the limiting thickness ratio of a web may be observed and stiffeners used, dangerous buckling stresses may nevertheless arise in the webs at a section of heavy shear, unless the stiffeners are closely spaced. To provide for these, the shearing stress at the section should be computed and compared with the safe shearing stress based on web buckling.

**Illustrative Problem.**—Express an opinion concerning the safety against crippling of the web of the girder of the last problem under an end shear of 112,500 lb. Safe web crippling stress by adapted A.R.E.A. stiffener formula (Art. 17).

\[
p = 12,000 - \frac{b}{42},
\]

Average existing shearing stress

\[
\tau_s = \frac{112,500}{(36)(0.3125)} = 10,040 \text{ lb. per sq. in.}
\]
This being considerably less than the existing average shearing stress on the cross-section, the web would need to be thickened or stiffeners used, spaced a distance apart in the clear about one-half the clear depth of the web, so that the longest unsupported strip of web making an angle of 45 deg. with the neutral axis would be one-half that assumed in the above formula for working stress.

42d. Proportioning for Diagonal Tension.—As has been pointed out in Art. 17, there exist at points below the neutral axis of a simply supported beam, diagonal tensile stresses of the same magnitude as those existing at corresponding points above the neutral axis, acting in a direction at right angles to the maximum compressive stress existing at the point on the tension side being considered. If the upper half of each diagram of Fig. 37 were turned down about the neutral axis as a hinge it would correctly represent the intensity of the maximum tensile stresses below the neutral axis.

In spite of the existence of these high diagonal tensile stresses, however, it is usually unnecessary to investigate them, since if the web can safely resist the shearing stresses in it, it can safely resist any tensile stresses arising from the combination of flexure and shear. This is because the safe strength of steel is a third more in tension than in columnar compression. The only exception to this condition is for the web fibers immediately inside the inner edge of the flange angles in girders with very thin webs. There, as has been already pointed out in this article, the maximum diagonal stresses may exceed the maximum flexural stress at the extreme fiber. In such cases, the web may need to be reinforced along this line of weakness by longitudinal plates placed under the flange angles and extending inside them, or by using angles with wider vertical legs.

Indirectly, some advantage accrues to the compression half of the beam through large diagonal tensile stresses, whether brought about by the combination of ordinary flexural and shear stresses, or by the application of a concentrated load to the tension side. The greater the diagonal tension in the web, the better is the web restrained against buckling. Inadequate webs are thus often kept from failure in buckling by the existence of excessive tensile stresses.

The effect of concentrated loads on the web immediately below or above them is to increase the maximum diagonal compressive stress already existing. If the load is applied above the neutral axis, the diagonal compressive stress of the web below is increased; if it is applied below the neutral axis, the diagonal tensile stress is increased. On the assumption that the shearing stress is uniformly distributed over the web, it may be shown that the intensity of the vertical stress on horizontal sections arising from the concentrated load decreases uniformly with the vertical distance of the horizontal section from the point of loading. If this vertical stress at any selected point be \( q \), the maximum diagonal stress, \( f_m \), at this point may be shown to be

\[
f_m = \frac{1}{2} (f + q) + \sqrt{\frac{1}{2} (f - q)^2 + v^2}
\]

where \( f \) and \( v \) are the flexural and shearing stresses at the point.\(^1\) The angle \( \theta \) which this total maximum stress makes with the vertical is such that

\[
\tan 2\theta = \frac{2v}{f - q}
\]

\(^1\) See Sec. 1, Art. 58.
Adoption of a given web thickness for a girder is influenced by other factors than the capacity to resist shear and crippling without exceeding certain prescribed stresses. If in order to obtain the required area, a thin web be made very deep, the extra width may entail a higher pound cost for the web material. In addition, there is considerable risk of damage to thin webs in fabrication, transportation, and erection and the percentage loss through corrosion is higher than for thicker webs. Wherever an effort is made to utilize a very thin web, it may necessitate stiffeners so closely spaced as to increase the cost of the whole girder materially.

43. Moment of Resistance, Exact Method.—By reason of the rigid attachment of the flanges to the web, a plate girder is essentially a built-up beam and its moment of resistance, or capacity to resist bending moment, should properly be computed from the common flexure formula, \( f = M/S \), or \( M = Sf \). To apply this formula accurately, it is necessary to compute the moment of inertia of the actual section, if such is not available in tables. However, because of the innumerable combinations of plates and shapes worked into plate girder sections, the properties of the particular section in hand are often not listed, and hence the value of \( I \) must be specially determined for the case under consideration.

In allowing for rivet holes on the tension side of the girder, the same differences of practice exist as have been mentioned in connection with the design of beams containing rivet holes. Some designers proportion by the moment of inertia of the gross section, as is recommended by the American Bridge Co. in specifications for steel structures. Some proportion for the gross section on the compression side and net section on the tension side and others for the net section on both sides. Some assume the neutral axis to be at the center of gravity of the gross section and others assume it as at the center of gravity of the net section.

As has been pointed out in the discussion of the moment of resistance of the net section of beams, it is reasonable to assume the gross section as operative on the compression side but only the net section on the tension side. While the neutral axis is shifted upward a small distance by the holes on the tension side, it is more convenient to assume it as at the center of gravity of the gross section and then apply a correction to the net moment of inertia or section modulus to compensate for the erroneous assumption respecting the position of the neutral axis. Since the position of the neutral axis must be somewhere between the gravity axes of the gross section and that of the net section, this correction cannot be large.

Computation of the moment of resistance of a plate girder section is simple, once the correct section modulus has been found. The determination of this quantity is tedious if the net section is considered, since the holes in the web as well as those in the flange must be considered. The necessary operations can best be explained by an example.

Illustrative Problem.—A plate girder section consists of a 36 × ½-in. web plate, four 6 × 6 × ⅜-in. angles spaced 36¼ in. back to back, and four 14 × ⅞-in. cover plates, as shown in Fig. 38 (b). Holes for ⅜-in. rivets (counted as 1 in. diameter) occur in the flanges and web, as shown, those on the compression side not being counted. Compute the section modulus: (1) Of the gross section; (2) of the section with the holes on the tension side only deducted, assuming that the neutral axis is at the center of gravity of the net section; (3) of the net section as above, assuming the neutral axis as at the center of gravity of the gross section.
1. Section Modulus of the Gross Section.—This is found by dividing the moment of inertia, \( I \), of the gross section by the distance, \( e \), from the center of gravity of the gross section to the extreme fiber of the outer cover plate. The method has been already illustrated in the problems on box girders.

Gross \( I \) of section:

\[
\begin{align*}
\text{Web, } (2\frac{1}{2})(0.375)(36)^3 &= 1,458 \\
4 \text{ angles, } 4(I_0 + A_y y_0^2) &= 4[(15.4) + (4.36)(16.61)^2] = 4,882 \\
2 \text{ prs. plates, } I_0 \text{ being negligible} &= (2)(14)(0.75)(18.625)^2 = 7,290 \\
\text{Total } I &= 13,630
\end{align*}
\]

Section modulus of gross area,
\[
S = \frac{13,630}{19.0} = 717.5
\]

![Fig. 38.—Comparison of the section moduli of typical plate girder sections.](image)

2. Section Modulus of Net Section, Neutral Axis at its Gravity Axis.—The distance, \( e \), of the gravity axis of the unsymmetrical net section above the center of gravity of the gross section may be found by the formula

\[
e = \frac{Q_h}{A_n}
\]

where

- \( Q_h \) = statical moment of holes about neutral axis of gross section.
- \( A_n \) = net area of girder section.

* From Fig. 38 (b), \( Q_h \) is seen to be

\[
0.375(0 + 3.5 + 7 + 10.5) + (1.125)(15.75) + (2)(1.125)(18.438) = 67.1
\]

Net area of section = gross area − area of holes on or below gravity axis of gross section = 51.94 − 4.88 = 47.06 sq. in. Hence

\[
e = \frac{67.1}{47.06} = 1.425 \text{ in.}
\]

The moment of inertia of the unsymmetrical net section about its gravity axis may be found readily from the formula

\[
I_n = I + Ae^2 - I'_n
\]

where

- \( I \) = moment of inertia of gross section about its own gravity axis.
- \( A \) = area of gross section.
- \( e \) = distance of gravity axis of net section above gravity axis of gross section.
- \( I'_n \) = moment of inertia of holes about gravity axis of net section.

The latter will, following Fig. 38b, be

\[
0.375(1.425^2 + 4.925^2 + 8.425^2 + 11.925^2) + (1.125)(17.175)^2 + (2)(1.125)(19.863)^2 = 1,313.
\]

Hence

\[
I_n = (13,630) + (51.94)(1.425)^2 - 1,313 = 12,423
\]
Section modulus of net section,
\[ S_n = \frac{12.423}{20,425} = 0.608 \quad S_n/S = 0.848 \]

3. Section Modulus of Net Section, Neutral Axis at Gravity Axis of Gross Section.—Net moment of inertia of unsymmetrical net section,
\[ I_n' = I - I_h \]
where \( I_n \) = moment of inertia of holes about gravity, axis of gross section.

The latter is
\[ 0.375(0^2 + 3.5^2 + 7^2 + 10.5^2) + 1.125(15.75)^2 + (2)(1.125)(18.438)^2 = 1.108 \]
Hence
\[ I_n' = 13,630 - 1,108 = 12,522, \]
and
\[ S_n' = 12,522/19.0 = 659. \quad S_n'/S = 0.92 \]

Comparing \( S_n \) and \( S_n' \) it is seen that \( S_n/S_n' = 0.923 \). That is, the section modulus obtained by assuming the neutral axis to be at the center of gravity of the net section is 92.3 per cent of that obtained assuming the neutral axis as fixed at the neutral axis of the gross section. However, as pointed out in the discussion of beams, Art. 7, the neutral axis probably does not shift to the extreme position of the center of gravity of the net section and consequently if the convenient assumption of fixed neutral axis be made the result in the present instance need be reduced by not over 5 per cent.

Computations similar to those in the last problem have been made for the three other girders shown in Figs. 38a, c, and d and the results obtained are set forth in Table 12. An examination of this table shows that, for the four typical girders analyzed, the loss of section modulus under the most serious assumption—namely, that the neutral axis is at the center of gravity of the unsymmetrical net section—ranges from 12.7 to 17.4 per cent, being greater for sections with heavy flanges than for those with light ones. The loss under the simpler assumption of fixed axis ranges from 6.5 to 8.3 per cent, being, as before, greater for sections with heavy flanges than for sections with light ones. The ratio of section modulus determined under the first assumption to that determined under the second one varies from 0.90 to 0.934, being lowest for girders with heavy flanges. Since the true position of the neutral axis is somewhere between the two positions assumed, a reduction factor of \( K = 0.95 \) applied to \( S_n' \) will give sufficiently close results for even girders with heavy flanges.

### Table 12 — Moments of Inertia and Section Moduli of Typical Plate Girders

<table>
<thead>
<tr>
<th>Girder</th>
<th>Gross section</th>
<th>Unsymmetrical net section</th>
<th>Neutral axis at gravity axis of net area</th>
<th>Neutral axis at gravity axis of gross area</th>
<th>Average reduction factor &quot;K&quot; recommended to be applied to ( S_n' ) to give ( S_n'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Fig. No.</td>
<td>( I )</td>
<td>( S )</td>
<td>( I_n )</td>
<td>( S_n )</td>
</tr>
<tr>
<td>1</td>
<td>38a</td>
<td>5,810</td>
<td>318</td>
<td>5,395</td>
<td>278</td>
</tr>
<tr>
<td>2</td>
<td>38b</td>
<td>13,630</td>
<td>718</td>
<td>12,423</td>
<td>608</td>
</tr>
<tr>
<td>3</td>
<td>38c</td>
<td>19,170</td>
<td>634</td>
<td>17,155</td>
<td>538</td>
</tr>
<tr>
<td>4</td>
<td>38d</td>
<td>40,910</td>
<td>1,319</td>
<td>36,347</td>
<td>1,086</td>
</tr>
</tbody>
</table>

44. Moment of Resistance, Approximate Method.—To obviate the somewhat laborious computation of moment of inertia involved in employing the accurate
method for determining moment of resistance, it is customary to employ an approximate method giving sufficiently precise results for all but the shallower girders. This method is based on the concept of the girder as a truss. Virtual chord areas are determined from the area of the girder section and the disposition of the material therein. Knowing these chord areas and the distances between their centers of gravity, it is easy to compute the moment of resistance, which is merely the product of the total permissible stress in one chord and the distance between chord centers.

Consider the flexure formula

\[ M = fI/c, \]

as applied to any girder section, such as that shown in Fig. 39. The moment of inertia of the whole section is made up of the moment of inertia of the flanges plus the moment of inertia of the web—that is, \( I = I_f + I_w \). Let \( A_f' \) be the net area of one flange proper—that is, the angles and flange plates riveted to the web at one edge as flange material; \( d \) the distance between centers of gravity of flanges, or the effective depth; \( t \) the thickness of the web plate; and \( h \) the depth of this plate. Then, approximately,

\[ M = 2A_f' \left( \frac{d}{2} \right)^2 \frac{1}{12} th. \]

In this expression the moment of inertia of the two flange areas \( A_f' \) about their own gravity axes parallel to the neutral axis of the whole girder have been neglected, since it is relatively unimportant when compared with the term \( 2A_f' \left( \frac{d}{2} \right)^2 \).

Where cover plates are employed, as in Fig. 39, the center of gravity of the flange is not far from the edge of the web plate—that is, \( h = d \), approximately. If at the same time the distance \( c \), be replaced by \( \frac{d}{2} \), or in effect the prescribed stress \( f \) be assumed to act at the center of gravity of the flange, the expression for moment of resistance of the entire section becomes,

\[ M = f \left\{ 2A_f' \left( \frac{d}{2} \right)^2 + \frac{1}{12}ld^3 \right\} \]
or simplifying,

\[ M = fd(A' + \frac{1}{6}A_w) \]  

where \( A_w' = td \) is approximate area of the web.

Formula (1) is an expression for the moment of resistance of a virtual truss, of which the area of one chord or flange is \( A' + \frac{1}{6}A_w \). It is evident, therefore, that the web contributes to the moment of resistance an amount equal to that which would be produced by concentrating approximately one-sixth of its area at the center of gravity of each of the flanges. This amount is commonly known as the web equivalent.

In applying Formula (1), care must be taken to give proper recognition to the presence of rivet holes. \( A_w \), the area of one flange, must be the net, not the gross area, for while it is customary to make no deduction for holes in the compression flange if properly filled with well-driven rivets, the full deduction for a tension member must be made for the tension flange. The actual number of rivets to be deducted in a given case will depend on the number of rows of rivets in the vertical and horizontal legs of the flange angles and the pitch of the rivets. The method of computing the deduction will be in accordance with the provisions of the specification. It may be that outlined in Art. 65.

If there be vertical lines of holes in the web, as at a stiffener or a web splice, the area of web \( A_w' \), should be taken as the net area through the rivet holes. For such girders, it is convenient to make an average approximation of the relation of net to gross web area, and base the formula on gross web area. If it be assumed that \( \frac{1}{6} \)-in. rivets or 1-in. holes are spaced on an average 4 in. vertically apart in the stiffeners, the net area of the web is \( \frac{3}{4} \) the gross area, and hence \( \frac{1}{6} \) of the net area equals \( \frac{1}{8} \) of the gross area. Formula (1) applied to girders with vertical lines of holes in the web then becomes

\[ M = fd \left( A' + \frac{1}{6}A_w \right) \]  

where \( A_w = \) gross area of the web. Some designers permit only \( \frac{1}{4} \) or \( \frac{1}{2} \) of the gross area of the web to be counted as web equivalent, but as no deduction really needs to be made for the compression half, \( \frac{1}{6} \) is not excessive.

Formerly, it was common practice to disregard altogether the value of the web in contributing to the moment of resistance and require all of the flange area to be area in excess of the web. Under such a specification, Formula (2) would become

\[ M = fdA' \]  

The obvious severity of this requirement has led to its abandonment in nearly all specifications.

In computing the moment of resistance of a girder, it is common practice to assume the effective depth of plate girders with two T-flanges carrying cover plates as the depth of the web. No appreciable error is involved in such an assumption as will be shown in examples, but the rule does not apply with sufficient accuracy to girders with four-angle flanges or with T-flanges without cover plates. In these cases the center of gravity of the flange may be several inches inside the edge of the web plate, and its position must therefore be specially calculated.

Since neither the areas of the two flanges nor the permissible stresses in them are necessarily equal, the moment of resistance with respect to the two flanges
will, in general, be unequal. The strength of the girder will, of course, be governed by the lesser of the two.

**Illustrative Problem.**—Compute by the approximate method the moment of resistance of the plate girder section shown in Fig. 39, and made up of one web plate $60 \times \frac{3}{8}$ in., four $6 \times 3\frac{1}{2} \times \frac{3}{8}$-in. angles, and two $13 \times \frac{3}{8}$-in. cover plates. Assume the web equivalent as $\frac{3}{8}$, the permissible flexural stress as 16,000 lb. per sq. in., and rivets $\frac{3}{8}$ in. diameter. Net section of tension flange will be computed by the exact method of Art. 65, the rivet spacing being as shown.

From Fig. 39, it is evident that the dangerous section is S-S, cutting the holes through the cover plates and the horizontal legs of the flange angles, and located 3 in. from the centers of the holes through the vertical legs of the flange angles. Since the distance between gage lines of the developed flange angle is 5 in. the deduction for one angle is found from Fig. 55 to be $1 + 0.6 = 1.6$ holes, and, for two angles, 3.2 holes. For the cover plate it is 2 holes.

Net area of two angles is $(2)(4.50) - (3.2)(1.0)(0.5) = 7.4$ sq. in.; net area of one cover plate is $(13)(0.375) - (2)(1.0)(0.375) = 4.13$ sq. in.

Total net area of one flange proper $= 7.4 + 4.13 = 11.53$ sq. in.

Web equivalent $= (14)(60)(0.375) = 2.81$ sq. in.

Assuming $d = $ depth of web plate $= 60$ in.,

$$M = (16,000)(60)(11.53 + 2.81) = 13,760,000 \text{ in.-lb.}$$

Let it be required to find the moment of resistance using the exact value for the effective depth—that is, the computed distance between the centers of gravity of the flanges. Making the computation on the basis of gross flange area, the section that predominates, it is found that the center of gravity of the flange is 0.47 in. inside the back of the flange angles and that the true effective depth is 59.55 in. The moment of resistance using this value $M = (16,000)(59.55)(11.53 + 2.81) = 13,680,000 \text{ in.-lb.}$, or 0.75 per cent less than by using the approximate value of $d$. In girders with a pair of flange plates on each flange, there is very little error involved in assuming the effective depth as equal to the depth of the web plate.

**Illustrative Problem.**—Find the moment of resistance of the girder in the last problem, assuming that the cover plates are omitted.

As there are holes in only one leg of the flange angles, the deduction from each angle will be one hole.

Net area of one flange $= (2)(4.50) - (2)(1.0)(0.5) = 8.00$ sq. in. The web equivalent is, as before, 2.81 sq. in.

Since there are no cover plates, it is best not to assume the effective depth as the depth of the web plate. The distance of the center of gravity of each flange being 0.83 in. from the backs of the flange angles, the effective depth is $60.5 - (2)(0.83) = 58.84$ in.

The moment of resistance,

$$M = (16,000)(58.84)(8.00 + 2.81) = 10,170,000 \text{ in.-lb.}$$

If the effective depth had been assumed as the depth of the web, or 60 in., the error would have been 1.97 per cent on the unsafe side. It is thus evident that if the outstanding legs of the flange angles are considerably greater in length than the vertical legs, there is no serious error involved in assuming the effective depth as the depth of the web plate, even though there are no cover plates.

**Illustrative Problem.**—If in the last problem the flanges consist of two $6 \times 6 \times 3\frac{1}{2}$-in. angles without cover plates, the flange rivets having a staggered pitch of 6 in., as shown in Fig. 40, find the moment of resistance.

From Fig. 55 it is seen that the rivet hole deduction from one angle is $1 + 0 = 1$ hole, and 2 holes for the two angles. The net flange area is therefore $(2)(5.75) - (2)(1.0)(0.5) = 10.50$ sq. in.

The distance of the center of gravity of a flange being 1.68 in. from the backs of the angles, the effective depth is therefore $60.5 - (2)(1.68) = 57.14$ in.

The moment of resistance is, therefore,

$$M = (16,000)(57.14)(10.50 + 2.81) = 12,160,000 \text{ in.-lb.}$$
Had the effective depth been assumed as 60 in., the result would have been 5 per cent in error. With equal-legged flange angles and no cover plates, it is, therefore, necessary to compute the effective depth.

Illustrative Problem.—If the section of a plate girder be as shown in Fig. 41—that is, with a $60 \times \frac{3}{4}$-in. web plate, bottom flange of two $6 \times 6 \times 1\frac{1}{4}$-in. angles and two $14 \times \frac{3}{4}$-in. cover plates, and the top flange of four $6 \times 4 \times \frac{3}{8}$-in. angles, and four $7 \times \frac{3}{8}$-in. plates, with flange riveting as shown—find the amount of resistance, assuming the web equivalent as $\frac{1}{8}$ in. Permissible flange stress on tension flange $= 16,000$ lb. per sq. in. net area, and on compression flange $15,000$ lb. per sq. in. gross area. Rivets $\frac{7}{8}$ in.

Applying the method of calculating exact deductions from tension flanges, explained in Art. 65 to the riveting arrangement shown in Fig. 41, it is evident that the deduction should be two holes from each angle and two from each cover plate.

**Fig. 40.**—Effect of omission of cover plates on effective depth.

**Fig. 41.**—Moment of resistance of unsymmetrical girder section.

Net area of two $6 \times 6 \times 1\frac{1}{4}$-in. angles $= (2)(7.78) - (4)(1)(0.688) = 12.81$ sq. in.

Net area of two $14 \times \frac{3}{4}$-in. cover plates $= (2)(14)(0.5) - (4)(1)(0.5) = 12.00$ sq. in.

Web equivalent $= (0.9)(60)(0.4375) = 3.28$ sq. in.

Total net flange area $= 12.81 + 12.00 + 3.28 = 28.09$ sq. in.

The center of gravity of the compression flange is seen by inspection to be $4\frac{3}{4}$ in. down from the backs of the outer angles, and if the center of gravity of the tension flange be assumed as at the edge of the web, the effective depth is then $60.5 - (4.375 + 0.25) = 55.88$ in.

Moment of resistance of girder with respect to tension flange is

$$M = fd \left( A_f + \frac{A_r}{8} \right) = (16,000)(55.88)(28.09) = 25,100,000$ in.-lb.$$

Gross area of compression flange is made up as follows:

- 4 angles, $6 \times 4 \times \frac{3}{8} = (4)(3.61) = 14.44$ sq. in.
- 4 plates, $7 \times \frac{3}{8} = (4)(7)(0.5) = 14.00$ sq. in.
- $\frac{3}{8}$ web $= (\frac{1}{8})(60)(0.4375) = 3.28$ sq. in.

Total gross area $= 31.72$ sq. in.

 Moment of resistance with respect to compression flange is

$$M = (15,000)(55.88)(31.72) = 26,600,000$ in.-lb.$$

The girder is therefore, stronger in the compression flange than in the tension flange, but the make-up of the flanges is in accordance with the principle that the gross area of the compression flange should be as nearly as possible equal to the gross area of the tension flange (see Art. 46).
45. Comparison of Exact and Approximate Methods.—Before using the approximate or truss-chord method of calculating the moment of resistance, designers should be aware of the degree of accuracy attainable by it. Although for the ordinary cases of plate girders with T-flanges it is sufficiently exact, for very shallow girders it cannot safely be used, particularly if these girders have heavy or 4-angle flanges. The assumptions of uniform stress over the flanges and that the effective depth equals the depth of the web plate are too much in error. In such cases the method of the moment of inertia should be employed instead.

Two characteristic examples will make this point clear.

Illustrative Problem.—Find the moment of resistance of the plate girder shown in Fig. 38B by the approximate method and compare it with the moment of resistance for this girder already found by the moment of inertia method. Assume permissible bending stress on both flanges = 16,000 lb. per sq. in. Gross area of one flange is:

Two angles, $6 \times 6 \times \frac{3}{8}$ in. = (2)(4.36) = 8.72 sq. in.
Two plates, (2)(14)(0.375) = 10.50 sq. in.

$\frac{1}{6}$ of web = (16)(36)(0.375) = 1.69 sq. in.

Total gross area = 20.91 sq. in.

Net area of one flange with the rivet pitch assumed = gross area less 2 holes out of each angle and 2 holes out of each cover plate = 20.91 - (4)(1)(0.375) - (4)(1)(0.375) = 17.91 sq. in.

Assuming the effective depth as equal to the depth of the web, as is commonly done the moment of resistance is

$$M = (16,000)(36)(17.91) = 10,320,000 \text{ in.-lb.}$$

Referring to Table 12, it is seen that the probable net section modulus of this girder, considering the effect of the holes on the position of the neutral axis is $(0.95)(569) = 626$ The moment of resistance would, therefore, be

$$M = S''f = (626)(16,000) = 10,025,000 \text{ in.-lb.}$$

The approximate method in this case gives a result in error on the safe side by 3 per cent.

Illustrative Problem.—Calculate the moment of resistance of the unsymmetrical girder shown in Fig. 41 by the exact method and compare it with the moment of resistance already found by the approximate method.

Taking statistical moments of gross areas about the center line of the web, the center of gravity of the gross section is found to be 1.64 in. below the center line of the web. Momen t of inertia of gross section with respect to its gravity axis is found to be 52,827.

Moment of inertia of holes about gravity axis of gross section = 4,572.

Net moment of inertia = 48,255, and section modulus = 48,255/31.89 = 1,515.

Multiplying this by the recommended correction factor (Table 12), adjusted net section modulus = 1,440.

Moment of resistance of net section by the exact method = (1,440)(16,000) = 23,000,000 in.-lb.

Least moment of resistance found by the approximate method (last problem under Art. 44), = 25,100,000 in.-lb.

Hence the approximate method gives in this case a result 9 per cent too great.

From the first problem it is evident that the approximate method gives reasonably accurate results for girders of moderate depth with T-flanges. Had the girder been much shallower with the same flange material, or if it had had much heavier flanges with the same depth, the error would have been more serious, justifying the use of the exact method of the moment of inertia.

From the second problem, it is seen that with a top flange of 4 angles, even where the girder is of moderate depth, the approximate method cannot be relied
upon. Had the flanges been heavier, or the girder shallower, the error involved in the use of the approximate method would have been greater.

45. Composition of Flanges.—In making up a flange section, such as any of those shown in Fig. 35, it is necessary to decide upon the proportion of the added flange material that must be directly connected to the web. Many specifications require that not over one-half of the total flange section must be in the form of plates not directly connected to the web, and this rule is very commonly observed. However, the 1920 A.R.F.A. specifications merely require that flange angles shall form as large a part of the area of the flange as practicable. If a very large amount of material in the form of cover plates were attached to relatively light flange angles, the latter might not be able to transmit safely to the plates the stress they should bear. For flanges of such large section that approximately half the area cannot be provided by the number and size of angles adopted, flange plates are added between the angles and the web, as shown in Fig. 35c and f. They are then directly connected material.

For a T-flange in which unequal-legged angles are used and there is no danger of concentrated loads such as ties, it is best to have the long legs outstanding, since this throws the center of gravity of the flange farther out than if the short legs were outstanding, and hence increases both the vertical and lateral flexural efficiency. Unequal-legged angles so placed are more efficient than equal-legged ones, but the latter are often required to accommodate the necessary flange rivets.

Four-angle flanges, such as shown in Fig. 35d, e and f, should be as shallow as possible, in order to throw the center of gravity well out, and if the angles are of unequal areas the inner pair of the two should be the smaller, since they are less effective than the other pair. Unequal-legged angles with the long legs outstanding are more efficient than equal-legged angles.

It has long been the custom to specify that if the flange plates on a flange are of unequal thickness, the thinnest plate should be on the outside. There does not appear to be any good reason for this rule. On the contrary, with a given rivet spacing there is less likelihood of a thick plate separating from the remainder of the flange than there is for a thin plate to do so. Besides, if one plate of the top flange must be carried the full length of the girder, it is more economical to employ the thinnest one for this purpose. The width of flange plates should be so fixed that they do not extend more than 6 in. outside the outer line of rivets nor more than 8 times the thickness of the thinnest plate on the flange.

Since no deduction need be made for rivet holes in the compression flange, the gross area of this flange might theoretically be made less than the gross area of the tension flange, even though the working stress on the former may be less than on the latter, due to allowance for buckling. However, as it is desirable to keep the neutral axis as nearly as possible in the center of the web, and its position depends largely on gross areas, it is frequently specified that the gross area of the compression flange shall be the same as the gross area of the tension flange.

For the purpose of determining the flange area required to resist a given moment by the approximate method, Formula (2) of Art. 44 may be written

\[ A' + A_w = \frac{M}{8fd} \]  
(1)

or

\[ A' = \frac{M}{fd} - A_w \]  
(2)
From Formula (1) the total flange area required (including web equivalent) is found, while from Formula (2) the area of the angles and flange plates only is determined.

If no part of the web is counted as flange material, Formula (1) or (2) becomes

$$A_f' = \frac{M}{fd}$$

(3)

While this latter method of proportioning is too severe; it is useful in making rough preliminary approximations of the size of the flanges required.

In proportioning by the exact or moment of inertia method it is useful to make a preliminary calculation of section by the approximate method and then test it by the exact method. If the stresses are too large or too small, the section must then be revised. Proportioning by the exact method is much more laborious than by the approximate one.

Illustrative Problem.—Determine by the approximate method the required composition of the two flanges of a girder for which the maximum bending moment is 3,000,000 ft.-lb., if the web is $72 \times \frac{3}{4}$ in. and the upper flange is to be of the I-angle type, with vertical flange plates if necessary. Permissible flexural stress = 16,000 lb. per sq. in. on tension flange and 15,000 lb. per sq. in. on compression flange. Web equivalent, $\frac{3}{8}$ in. dia.

Assume the top flange angles as $6 \times 6$ in. the upper pair being set $\frac{3}{4}$ in. out from the edge of the web and the pairs $12 \frac{1}{4}$ in. back to back. Assume also that the two angles of the bottom flange are set out $\frac{3}{4}$ in. from the edge of the web plate. The effective depth may therefore be taken with sufficient accuracy as $72.5 - (6.25 + 0.25) = 66$ in.

Required net area in angles and flange plates of bottom flange, from Formula (2)

$$A_f' = \frac{M}{\frac{fd}{8}} = \frac{(3,000,000)(12)}{(16,000)(66)} - \frac{(72)(0.5)}{8} = 29.6 \text{ sq. in.}$$

Required gross area in angles and flange plates of top flange,

$$A_f' = \frac{(3,000,000)(12)}{(15,000)(66)} - \frac{(72)(0.5)}{8} = 31.9 \text{ sq. in.}$$

For the bottom flange, the riveting arrangement will be assumed as such that the two holes should be taken out of each angle and two out of each flange plate. The flange may then be made up as follows:

- Two angles, $6 \times 6 \times 1\frac{1}{2}$ in., less four 1-in. holes = 14.93 sq. in. net
- Two plates, $14 \times 5\frac{8}{16}$ in., less four 1-in. holes = 15.00 sq. in. net

The total gross area of the bottom flange is 35.68 sq. in.

The top flange will be made up as below, the small excess of area over stress requirements being provided to make the gross areas of the two flanges nearly equal.

- Four angles, $6 \times 6 \times 3\frac{8}{16}$ in. = 17.44 sq. in.
- Four plates, $10 \times 3\frac{8}{16}$ in. = 15.00 sq. in.

32.44 sq. in.

47. Moment of Resistance of Girders with Sloping Flanges.—Variation of the depth of a girder to suit the shear and moment requirements from point to point is sometimes carried out in girders for travelling cranes, turntables, bridge floor beams and viaduct girders. This is done by sloping one or both flanges, as shown in Fig. 42, or sometimes in the case of the lower flange by curving it
upward from the center towards the ends where it becomes horizontal. Such a girder is termed a "fish-bellied" girder.

To obtain the total stress in either flange of a girder with sloping flanges, it is only necessary to divide the moment at the section by the perpendicular distance to the center of gravity of the flange concerned from the intersection of the section plane with the neutral line of the other flange, reducing the result to allow for such portion of the moment as may be taken by the web.

Thus, in the case shown in Fig. 42a, the compressive stress in the top flange

\[ C = K \cdot \frac{M}{d}, \]

where

\[ M = \text{moment at the section.} \]
\[ d = \text{effective depth at the section, measured vertically.} \]
\[ K = \frac{\text{Net area of angles and plates for one flange}}{\text{Total net area of one flange, including web equivalent.}} \]

The tension in the bottom flange is

\[ T = K \cdot \frac{M}{d \cos a}, \]

where

\[ a = \text{angle of slope of bottom flange with horizontal.} \]

In the case of the girder shown in either Fig. 42b or c, with the top as well as the bottom flange inclined, the total stresses in the top and bottom flanges respectively are:

\[ C = K \cdot \frac{M}{d \cos b} \]
\[ T = K \cdot \frac{M}{d \cos a} \]

where \( b = \text{angle of slope of top flange with horizontal.} \)

Where the flanges of a girder are inclined, they absorb some of the shear at the cross section, in the same manner as do the chords in curved chord trusses. The web shear is then less than it would be if the flanges were horizontal.

For the girder of Fig. 42a, if \( V \) be the total shear at the section, the shear that must be absorbed by the web

\[ V_w = V - T \sin a \]
\[ = V - K \cdot \frac{M \tan a}{d} \]
In the case of Fig. 42b
\[ V_w = V - C \sin b - T \sin a \]
\[ = V - K \frac{M}{d} (\tan a + \tan b) \]

For the girder of Fig. 42c, it is
\[ V_w = V + C \sin b - T \sin a \]
\[ = V - K \frac{M}{d} (\tan a - \tan b) \]

In dealing with fish-bellied girders, the flange at the section being investigated may be assumed to have the slope of the tangent to the curve at the section.

48. Flange Buckling.—To compensate for the columnar or buckling action of compression flanges of plate girders not supported continuously in a lateral direction, a reduction in the normal working stress should be made for them. In the discussion of flange buckling for beams, Art. 15, reference was made, for purpose of comparison, to the method of providing for flange buckling in plate girders. The last six formulas of Table 5, p. 196, are applicable to such a situation. Reduction is required for all values of \( \frac{l}{b} \) in the case of the A.R.E.A., C.E.S.A. and the writer’s formulas, although such is not required for values of \( \frac{l}{b} \) under 10 for any of the others.

Reduction of the working stress on the compression flange tends to bring the gross area of this flange somewhat nearer the gross area of the tension flange. It serves to offset in part the neglect to rivet holes in compression material.

Application of reduction formulas is carried out as in the problem under Art. 15.

49. Length of Flange Plates.—As in the case of the box girder, the flange area of a plate girder may be readily varied to suit the moment requirement by terminating the flange plates where they are not needed to supplement the angles. The length of flange plates, whether they be placed in vertical planes or horizontally on the backs of the flange angles, may be determined by either analytical or graphical means. If the moment be computed at sections not over 5 ft. apart, the sections being taken at all points of concentrated loading, the flange area requirements at these points may be compared with the area of the flange, counting various numbers of plates, and the point of cut-off of the plates may then be found.

49a. Graphical Method.—Such comparison, except in the simple case where the loading is, or may be considered as uniform, is best made graphically, particularly if the loading is unsymmetrical. The simplest procedure is to plot a diagram of required flange areas at the different points where moments are determined, rather than to plot a moment diagram as is sometimes done. It is unnecessary for typical plate girders to plot a diagram of required section modulus as was done for reinforced beams (Art. 13). When the approximate method of calculating moment of resistance is used, it is sufficiently accurate to work with a diagram of required areas. If the diagram be constructed by plotting required flange areas vertically and lengths horizontally, as in Fig. 43, and the assigned constituent areas be plotted on the same diagram, the length for which each is required may be readily scaled off. The points where each of the cover plates
should end theoretically are found by noting the points where the inner horizontal bounding line representing the area cuts the curve. This method may be applied whether the flange plates are in vertical planes over the vertical legs of the flange angles or are in horizontal planes on the outstanding legs of the flange angles.

It is customary to extend each plate far enough past the point where it might theoretically end to accommodate two transverse rows of rivets so that the plates may be capable of bearing stress at the points where they are first needed.

For deck plate girders in bridge work, the inner cover plate of the top flange, if it be a T-flange, is extended to the full length of the girder so as to protect the flange from corrosion and separation of the angles.

![Diagram of plate girder flange plates by graphical method.]

Fig. 43.—Length of plate girder flange plates by graphical method.

To keep the neutral axis at the same level throughout the girder and preserve the assumed distribution of stress, corresponding plates on the top and bottom flanges should, as far as possible, be cut off at the same points.

**Illustrative Problem.**—Consider a 50-ft. girder loaded with a uniform load of 1,000 lb. per lin. ft., which includes the weight of the girder, and a system of concentrated loads, as shown in Fig. 43. Find the points of theoretical and practical cut-off, if the section of each flange at the point of maximum moment is made up as follows:

\[ \frac{1}{2} \] of web = \( \frac{1}{2} \times 60 \times 0.375 \) = 2.82 sq. in.

2 angles, 6 \( \times \) 6 \( \times \) \( \frac{1}{2} \) in., net = 9.50 sq. in.

2 plates, 14 \( \times \frac{3}{8} \) in., net = 9.00 sq. in.

---

Assume the permissible flange stress = 16,000 lb. per sq. in., and effective depth = 60 in.

The moments and required flange areas at points of concentrated loading and certain intermediate points are as shown in the accompanying table.

On Fig. 43, the required flange areas and the part areas assigned are shown plotted vertically. The points of theoretical cut-off are shown dotted, but the plates are extended somewhat more than a foot at each end past those points. The total lengths required and the position of the left hand end of each plate with respect to the 70,000-lb. load are shown on the diagram.
### Table

<table>
<thead>
<tr>
<th>Distance of point from left support (ft.)</th>
<th>Uniform load moment (ft.-lb.)</th>
<th>Moment from concentrated loading (ft.-lb.)</th>
<th>Combined moment (ft.-lb.)</th>
<th>Required flange area (in.)</th>
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### 49b. Analytical Methods

Where the loading is uniformly distributed, or is such that it produces practically the same moment curve and flange area curve (a parabola), the theoretical lengths of the flange plates may be readily determined by analytical means. In Fig. 44, let \( l/2 \) be the half span of the girder,

![Fig. 44.—Analytical determination of length of flange plates.](image)

and \( A \), the maximum ordinate, be the total flange area required at the center A parabolic curve \( BD \), with vertical axis and vertex at \( D \), will then correctly represent the flange area requirement at all points of the half span. Let \( x_1/2 \), \( x_2 \), etc., be the half lengths of the flange plates, the areas of which are \( a_1, a_2 \), etc., numbered from the outside. The area \( a_1 \) is to be taken as the area required to be contributed by the outside plate.

For similar reasons to those set forth in Art. 13, the lengths of the first or outer flange may be written

\[
x_1 = l \sqrt{\frac{a_1}{A}}
\]
For the second and the $n$th plate the lengths are

$$x_2 = \frac{l}{A} \sqrt{\frac{a_1}{a_2}}$$

$$x_n = \frac{l}{A} \sqrt{\frac{a_1}{a_2} + \frac{a_n}{A}}$$

To the theoretical lengths thus found there should be added about a foot at each end to ensure action of the plates where first needed.

The above simple method is based on the assumption that the next areas of component parts of the flanges are the same at points of cut-off as at the point of maximum moment. An more precise solution may be devised in the same form as that for the length of reinforcing plates for beams (Art. 13).

**Illustrative Problem.**—If the total net area of one flange of a 50-ft. plate girder subjected to uniform loading be 18.67 sq. in., and of this 4.22 sq. in. is provided by each of two flange plates, find the theoretical and practical length of the two plates.

Theoretical length of first cover $= x_1 = 50 \sqrt[4.22]{18.67} = 23.75$ ft.

Adding a total of 2.25 ft., the practical length becomes 26 ft.

Theoretical length of second cover $= x_2 = 50 \sqrt[8.44]{18.67} = 33.6$ ft.

Practical length $= 36$ ft.

**50. Economic Depth.**—It may be shown by means of the calculus that for a girder to withstand a given bending moment, there is a depth which will give the minimum weight of material. This depth may be conveniently called the least-weight depth. In determining it, however, certain assumptions with respect particularly to the size and spacing of stiffeners have to be made which have an important influence on the result. If they are not realized in a given case—and such is possible, as there are very great differences in practice respecting stiffeners—the resulting depth may be materially in error.

As a result of such studies it may be shown that the least-weight depth varies from as much as $\frac{1}{3}$ to $\frac{1}{12}$ of the span, the former figure being for short, very heavily-loaded spans and the latter for long, slightly-loaded spans. For girders of from 40 to 50-ft. span with modern railway loading, the least-weight depth is not far from $\frac{1}{6}$ of the span.

Practical considerations often make it desirable to adopt a shallower depth than that giving the least weight. For example, for girders used in grade separation work where the upper grade must be kept as low as possible and the lower one as high as possible, it is economically wise to use the shallowest practicable girders. In long span girders the depth of web indicated by least weight calculations may entail extra cost per pound for wide plates and especial risk of damage in fabrication, shipment, and erection. In such cases, it is best to reduce the least weight to give what may be called the true economic depth.

Fortunately, little addition to the weight of a girder is caused by considerable changes in the depth. Thus, it may be shown that for a change of as much as 25 per cent, the increase in weight is only about 3 per cent. The advantages to be gained in other respects, and larger economies which may be effected, result in the true economic depth of plate girders being usually 15 or 20 per cent less than the least-weight depth.
Such depth ratios as have been given may be regarded without material error as referring to the depth of the web plate.

51. Flange Riveting.—In order that the flanges of a plate girder may act as an integral part of the whole girder, they must be attached to the web by sufficient rivets to transfer to them from the web the stress that they should properly bear. In this connection it is helpful to conceive of the web as the primary part of the girder and the flanges as edge reinforcement of the web. Up to the limit of the capacity of the flange rivets, therefore, the shortening and lengthening of the edges of the web will bring about a stress transference to the angles and plates of the flanges.

The rate of transference of stress to the flange, or the rate of increase of stress therein, on which flange rivet spacing depends, is directly proportional to the rate of increase of moment, as will be seen from the formula \( F' = M/d \), in which \( F' = f(A_f + \frac{1}{2}A_w) \) = the total flange stress, \( M \) = the moment, and \( d \) = the effective depth (which may be assumed constant). But the rate of increase of moment, horizontal distances being denoted by \( x \), is \( dM/dx \), which may be shown as equal to \( V \), the total vertical shear. Rivet spacing formulas may therefore be based upon the vertical shear at the section, a quantity that is more readily determined than the moment.

51a. Unloaded Flange, Web Takes No Moment.—Consider the section of the plate girder shown in Fig. 45. Let the distance between the rivet lines in the two flanges be \( h' \) and the total vertical shear at the section be \( V \). The increment in moment per lin. in. being \( dM/dx = V \), the growth in total flange stress per lin. in. will be \( V = V/h' \), assuming the transference of stress to be at the rivet lines and assuming that no part of the flange stress is taken by the web. If the safe resistance of one flange rivet be \( r \) lb., then the number of \( r \) inches that may be served by one rivet = \( \frac{r}{h'} \), or the pitch \( p = \frac{rh'}{V} \)  

Illustrative Problem.—Find the rivet spacing for the bottom (unloaded) flange of the girder shown in Fig. 39, assuming that the web takes no share of the moment. Vertical shear at the section = 100,000 lb.; rivets, \( \frac{3}{8} \) in. dia.; safe shearing and bearing stresses on rivets = 10,000 and 20,000 lb. per sq. in. respectively. Distance between gage lines of flanges = 56.5 in.

Least safe resistance of \( \frac{3}{8} \)-in. rivet is in bearing on the \( \frac{3}{8} \)-in. web = \((0.88)(0.38)(20,000)\) = 6,560 lb.

Theoretical pitch, \( p = (6,560)(56.5)/100,000 = 3.72 \) in.

51b. Unloaded Flange, Web Takes Moment.—If the web is assumed to take its proper share of the bending moment, the increment in flange stress per lin. in. will be less than if the web takes no moment, in the proportion of the respective effective flange areas. If, therefore, \( K = (\text{net area of flange angles and covers of one flange}) ÷ (\text{net area of flange angles and covers of one flange } + \text{web equivalent}) \), the actual increment of flange stress per lin. in.
going to the angles and covers will be \( i = K \cdot \frac{V}{h'} \). The rivet pitch then becomes

\[
\frac{r}{h'} = \frac{KV}{h'}
\]

or

\[
p = \frac{rh'}{KV}
\]  

(2)

The value of \( K \) defined above applies to rivets in the tension flange. For the compression flange, gross areas may be used in finding \( K \), but no material error results from using the same value for both flanges. The value obtained by using net areas is somewhat smaller than that found by using gross areas.

**Illustrative Problem.**—Find the rivet spacing for the unloaded flange of the girder of the last problem if one-eighth of the gross area of the web is considered as effective flange area. Assume the rivet spacing for purposes of computing net section as 4 in., the rivets through the outstanding legs of the flange angles staggering exactly with those in the vertical legs, similar to those in Fig. 39.

With a stagger of 2 in. and a distance between gage lines of 5 in. for a developed flange angle, the deduction for one angle is \( 1 + 0.85 = 1.85 \) holes, and for two angles it is 3.70 holes. The net area of two angles is \((2)(4.5) - (3.7)(1)(0.5) = 7.15 \) sq. in. For one cover plate it is, as previously found, 4.13 sq. in. The web equivalent is 2.81 sq. in.

\[
K = \frac{7.15 + 4.13}{7.15 + 4.13 + 2.81} = 0.8
\]

\[
p = \frac{0.8(5.660)(56.3)}{0.8(100,000)} = 4.64 \text{ in.}
\]

51c. Loaded Flange, Web Takes No Moment.—Where the imposed load on a girder is applied directly to one or other of the flanges, as in deck plate girder bridges and also in occasional through plate girder spans with ties bearing on the outstanding legs of the flange angles, the flange rivets have a dual function to perform. Not only must they transmit to the flange angles and covers the same increments of flange stress as in the unloaded flange, but they must be able to carry the directly applied load into the web for proper distribution. The stress per lin. in. on rivets, therefore, is a resultant stress, compounded of the horizontal increment of flange stress \( i' \) and the vertical load per lin. in. \( w \), or \( R = \sqrt{i'^2 + w^2} \). The rivet pitch then becomes \( r/R = r/\sqrt{i'^2 + w^2} \), or since \( i' = V/h' \)

\[
p = \frac{r}{\sqrt{(V/h')^2}}
\]  

(3)

**Illustrative Example.**—Consider the top flange of the girder of Fig. 39 as supporting a directly applied load of 7,200 lb. per lin. ft. and the web as taking no share of the moment. Other data will be as for the last two problems. Find the rivet spacing.

Increment of flange stress per lin. in. = \( V/h' = 100,000/56.5 = 1,770 \) lb.

Directly applied vertical load on flange rivets per lin. in. = \( 7,200/12 = 600 \) lb.

Resultant stress on rivets per lin. in. of girder = \( R = \sqrt{(1,770)^2 + (600)^2} = 1,870 \) lb.

Pitch, \( p = 6,560/1,870 = 3.51 \) in.
51d. Loaded Flange, Web Takes Moment.—If it be assumed that the web resists its appropriate share of the bending moment, the increment of flange stress per lin. in. will be \( i = K\frac{V}{h'} \), and Formula (3) becomes

\[
p = \frac{r}{\sqrt{\left(\frac{K V}{h'}\right)^2 + w^2}}
\]  

(4)

This is the general form of the rivet spacing formula, reducing to Formula (1) when \( K = 1 \) and \( w = 0 \), to Formula (2) when \( w = 0 \), and to Formula (3) when \( K = 1 \).

Illustrative Problem.—Adding to the data of the last problem the stipulation that one-eighth of the gross area of the web is to be assumed as effective flange area, find the spacing. For deduction purposes assume the pitch as 4 in., the stagger being similar to that shown in Fig. 39. Assume \( K = 0.80 \), the same values as used for the tension flange.

\[
R = \sqrt{\left(\frac{(0.8)(100,000)}{56.5}\right)^2 + (600)^2} = 1,540 \text{ lb.}
\]

The pitch is, therefore,

\[
p = \frac{6,500}{1,540} = 4.26 \text{ in.}
\]

51e. Multiple Rows of Flange Rivets.—Formulas (1), (2), (3) and (4) apply directly to flanges containing one row of rivets, or to those containing two rows if they be staggered, as in Fig. 46a. In the latter case the distance \( h' \) is to be measured to the center of gravity of the rivet lines, as also when the flange contains two rows of rivets opposite each other, as in Fig. 46b. When the pitch is to be computed for rivets arranged as in Fig. 46b, it should be remembered that the rivets are in pairs and hence the general Formula (4) becomes

\[
p = \frac{2r}{\sqrt{\left(\frac{K V}{h'}\right)^2 + w^2}}
\]  

(5)

In this case the directly applied load \( w \) is sometimes considered as taken up entirely by the upper pair of angles, since the load bears directly on them and the short stiffeners between the angles transfer load only at distances of several feet apart.

Illustrative Problem.—Find the pitch of the rivets in the top (loaded) flange of the girder shown in Fig. 41 if the total shear at the section is 80,000 lb. and a uniformly distributed load of 6,000 lb. per lin. ft. is applied to the top flange. Rivets, \( \frac{3}{16} \)-in. dia.; safe shearing and bearing stresses on rivets = 10,000 and 20,000 lb. per sq. in. respectively. Web equivalent = \( \frac{3}{16} \). Consider vertical load as transmitted to both pairs of angles.

Least resistance of one rivet in bearing on \( \frac{3}{16} \)-in. web = \((0.875)(0.4375)(20,000) = 7,660 \text{ lb.} \)

Ratio \( K \) for the compression flange from the problem under Art. 44 relating to this girder = \( 28.44/31.72 = 0.89 \).
Distance \( h' \) from Fig. 41 = 52.5 in.

Hence required pitch

\[
p = \frac{(2)(7,660)}{\sqrt{\frac{(0.89)(80,000)}{52.5}} + (500)^2} = 10.6 \text{ in.}
\]

As the minimum permissible pitch in the line of stress is 6 in. for this case, the theoretical pacing would need to be reduced to the latter figure.

51f. Rivet Pitch in Flange Plates.—Since the function of the rivets connecting the flange plates to the other material of the flange is to transmit to them the part of the increment of flange stress per lin. in. which they should bear, a formula similar to (2) will apply. The part of the total increment of flange stress going to the flange plates or to any one plate will be some fraction \( K' \) of the whole increment, or

\[
i'' = \frac{K'V}{h'}
\]

The rivet value employed will in general be the single shearing value \( r' \) and not the bearing value \( r \). If the rivets in any plate or group of plates are in pairs, as is usual, the pitch will be

\[
p = \frac{2r'h'}{K'V}
\]

where \( K' = \frac{\text{Net area of plate (or plates) considered on one flange}}{\text{Total net area of one flange including web equivalent}} \)

Illustrative Problem.—Find the theoretical required pitch of rivets in the cover plates of the girder shown in Fig. 39 at a section where the shear is 100,000 lb., if \( \frac{1}{8} \)-in. rivets are used and the safe shearing and bearing stresses on rivets are 10,000 and 20,000 lb. per sq. in., respectively. Web equivalent = \( \frac{1}{8} \).

Least value of rivet is single shearing value = \( \frac{0.661(10,000)}{h'} = 6,010 \) lb.

\( \frac{h'}{\text{from Fig. 39} = 56.5 \text{ in.}}\)

\( K = \frac{\text{Net area of one cover plate}}{\text{Total net flange area including web equivalent}} = \frac{4.13}{13.94} = 0.30, \)

counting four holes out of the flange angles and two out of the cover plate.

Hence,

\[
p = \frac{(2)(6,010)(56.5)}{(0.30)(100,000)} = 22.7 \text{ in.}
\]

This would need to be reduced to not over 6 in. for practical reasons.

51g. Rivet Spacing in Sloping Flanges.—Where a flange is inclined to the horizontal, as shown in Fig. 42, Formulas (1) and (2), Arts. 51a and 51b must be modified to take into account the fact that the increment of flange stress per lin. in. is not the total shear at the section divided by the vertical distance between rivet lines in the flanges, for part of the shear is resisted by the inclined flanges. The amount of this increment for typical cases will be found.

Consider a vertical strip of web of width \( dx \) and height \( h' \) between flange rivet lines at the section considered, as shown in Fig. 47. Let the shear actually absorbed by the web be \( V \), and let the total compressive and tensile flange stresses
to the left of the strip be \( C \) and \( T \) respectively, and to the right of the strip \( C' \) and \( T' \). Then if \( i \) be the actual increment in the top flange stress per lin. in.,

\[
C' - C = i \; dx \; \text{sec} \; b,
\]

Taking moments about \( A \),

\[
 idx \; \text{sec} \; b \; h' \cos b = V_w \; dx
\]

or

\[
i = \frac{V_w}{h'}
\]

Hence the pitch measured along the top flange is

\[
p = \frac{rh'}{V_w}
\]

or substituting the value of \( V_w \) from Art. 47,

\[
p = \frac{r}{h'} V - K_d M (\tan a + \tan b)
\]

(7)

Proceeding in the same manner for the bottom flange the same formula may be shown to apply, indicating that for the case considered the pitch measured along the flanges is the same for top and bottom flanges.

51h. Practical Considerations in Rivet Spacing.—To simplify both office and shop work, it is desirable to use one spacing of rivets for a section of flange several feet in length. Accordingly, the theoretical pitch is computed for several points in the half span, usually at the center of the panels marked off by stiffeners, and a spacing to the nearest lower practicable fraction of an inch is adopted for the entire panel, putting any odd spaces near the stiffeners. If one flange is loaded, it may be desirable to make the spacing in the unloaded flange correspond with it so that the flange angles may be kept alike.

In no case must the adopted spacing depart from the usual minimum of 3 diameters of the rivet or the maximum of 16 times the thickness of the outside material, or 6 in. The restriction on the minimum side often necessitates increasing the thickness of the web to keep the rivets at the end of the girder from being too close.

Where the girder is shallow and has heavy flanges, the approximate method of determining spacing outlined above cannot safely be applied. The horizontal shear method explained and illustrated for box girders in Art. 37 must then be employed.

52. Intermediate Web Stiffeners.—One way of strengthening the web of a plate girder or of making, what otherwise might be too thin, a web available for a given girder is as has been pointed out in Art. 17, to restrain it from buckling, in some measure at least, by means of stiffeners. These consist usually of pairs of vertical angles, one on each side of the web, riveted firmly thereto at intervals of approximately the depth of the web. Stiffeners in place in a typical girder are shown in Fig. 34.

In addition to their service of stiffening the web, intermediate stiffeners may help to transmit concentrated loads to the web and distribute them over it. For example, the stiffeners in a deck plate girder, or any girder loaded along the top flange, help to carry the applied load down into the web and relieve the top flange rivets of much of the vertical component of their stress (see Art. 51). They also to some extent lessen the deflection of the inner outstanding legs of the top
flange angles in deck railway girders carrying ties. If definite concentrated loads are applied to the top flange over stiffeners, or are applied to them anywhere within their length, they must be treated as concentrated load stiffeners, as described in Art. 53.

Formerly, stiffeners were sometimes placed in a diagonal direction pointing downward towards the near support at an angle of 45 deg. with the neutral axis. The idea was virtually to construct columns in the line of maximum stress at a number of points. However, between the columns so constructed little support would be given the web, and so, for reasons of greater efficiency as well as simplicity, vertical stiffeners were employed. These as shown in Fig. 48 restrict the length of a laterally unsupported strip of web to the length of the diagonal of the rectangle bounded by consecutive stiffeners and by the flanges. Moreover, there is but one narrow strip of this length in each rectangle. All other strips are intercepted by stiffeners and prevented from buckling over their full length.

Whether stiffeners are to be used or not is usually determined by the application of the rule, stated in Art. 42, that the clear or unsupported distance between the flange angles must not exceed 50 or 60 times the thickness of the web. Assuming the latter ratio, the slenderness ratio \( \frac{L}{t} \) of a 45-deg. strip of web might thus be as high as 295. Such is permissible under the circumstances, however, since the strip receives important support from the adjacent material.

Even though a strict application of the above rule might not indicate the necessity of stiffeners, it is often considered desirable by designers to insert them to stiffen the web in fabrication, transportation, and erection.

The spacing of stiffeners has an important influence on the buckling stability of the web. Since at the sections where web crippling is an important factor—that is, where the shear is large—the direction of the maximum diagonal compressive stresses makes an angle of about 45 deg. with the neutral axis throughout the unsupported depth of the web (Art. 42), the critical diagonal strips are those inclined at 45 deg. Along steeper ones, as \( AD \), or along flatter ones as \( AC \), Fig. 48, the intensity of the diagonal compressive stress is reduced. To determine, therefore, the spacing of stiffeners that will permit the employment of a certain thickness of web in a girder, it is necessary to find how long a 45-deg. strip of web, Fig. 48, may be without support in order that the safe buckling strength of the strip may not be exceeded.

Let \( AB \) be a strip of web 1 in. wide and \( t \) in. thick inclined at 45 deg. to the flanges or to the stiffeners, and let \( h' \) be the clear distance between stiffeners.
This strip may be regarded as a modified column of length \( l = \sqrt{2} \cdot h' \) and radius of gyration at right angles to the web of \( \frac{l}{\sqrt{12}} \). It is more favorably situated than a column, in that tensile stresses in the web at right angles to its length prevent it from buckling in the plane of the web, and the presence of the adjacent material hinders its buckling at right angles to the plane of the web. Reduction of the normal safe stress should, therefore, be at a much less rate than for ordinary columns. If the A.R.E.A. column formula

\[
p = 15,000 - 50 \frac{l}{r}
\]

be applied to this situation, it is probable, considering available experimental evidence, that the reducing term should be somewhere between \( 20 \frac{l}{r} \) and \( 30 \frac{l}{r} \).

Assuming the average value, and making it \( l = \sqrt{2} \cdot h' \) and \( r = \frac{l}{\sqrt{12}} \), the formula becomes very nearly

\[
p = 15,000 - 120 \frac{h'}{l}
\]

(1)

If now the diagonal compressive stress at the section in question, which at critical sections very nearly equals the average shearing stress be represented by \( v \), then for complete realization of the buckling strength of the web

\[
v = p = 15,000 - 120 \frac{h'}{l}
\]

If the existing stress, \( v \), is greater than \( p \), the section is unsafe, and the stiffeners require to be closer together, or the web should be thickened.

The necessary spacing of stiffeners to permit the use of a given web may be found by re-arranging the last equation, or

\[
h' = \frac{t}{120} \left( 15,000 - v \right)
\]

(2)

If a more conservative reduction of the A.R.E.A. column formula be adopted—that is, \( 30 \frac{l}{r} \)—we have very nearly

\[
p = 15,000 - 150 \frac{h'}{l}
\]

(3)

and for the safe clear spacing of stiffeners

\[
h' = \frac{t}{150} \left( 15,000 - v \right)
\]

(4)

These latter formulas are seen to be in accord with the recommended conservative formula for the buckling strength of beam webs derived in Art. 17 and plotted in Fig. 11, p. 206. If \( \frac{h'}{l} \) is over 60, they would give results that are too severe. Adopting Formula (7) of Art. 17 for values of \( \frac{h'}{l} \) over 60, they would become respectively

\[
p = 10,200 - 70 \frac{h'}{l}
\]

(5)

and

\[
h' = \frac{t}{70} \left( 10,200 - v \right)
\]

(6)
The provisions of the A.R.E.A. (1920) Specification covering the spacing of intermediate stiffeners contain the following:

The webs of plate girders shall be stiffened by angles at intervals not greater than the distance given by the formula

\[
d = \frac{t}{40} (12,000 - v)
\]

where

- \(d\) = distance between rivet lines of stiffeners in inches.
- \(t\) = thickness of web in inches.
- \(v\) = web shear at point considered, in lb. per sq. in.

This formula gives spacings in excess of those given by either Formulas (2), (4) or (6). The spacing is, of course, between rivet lines, but this accounts for only a small part of the difference. Actual tests of built-up girders with stiffeners show surprisingly large diagonal compressive strength, as will be seen from the three plotted points for such girders in Fig. 11, p. 206. The spacing formula of the A.R.E.A. specification appears to conform very well with these tests, although it does not conform closely to the tests of I-beams. More conservative formulas would, therefore, appear to be required in the investigation of I-beam webs.

In the determining of safe stiffener spacing by any of the formulas above, if the quantity \(k\)' should come out equal to or greater than \(k\), the clear depth between flanges, the inference is that stiffeners are not required. If they were spaced in the clear at distances apart greater than the clear depth of the web, they would uncover diagonal strips of considerable width along which buckling could take place.

In recent years the practice has developed of spacing stiffeners practically uniformly from end to end of girders, thus disregarding the effect of intensity of diagonal compressive stresses on the situation. Where such practice is followed, the spacing is generally about equal to the depth of the web. If, however, such a spacing is adequate at points of maximum shear and diagonal compression it is more than adequate in regions of low diagonal compression. Of course, if the shearing stress is low at the section, or if the ratio of unsupported depth of the web to its thickness is low, the practice cannot be said to be objectionable. For example, the specifications of the Department of Railways and Canals, Canada, provide that for girders under 4 ft. in depth, stiffeners may be spaced 4 ft. apart. Such a rule should not be applied if the shearing stresses are over about 50 per cent of the maximum permissible shearing stress.

The effort to utilize a comparatively thin web for high diagonal compressive stresses by very close stiffener spacing is likely to prove inadvisable. Where the necessary stiffener spacing works out to be less than half the depth of the web, it is more economical to use a thicker web.

Most specifications prescribe an arbitrary maximum permissible spacing for stiffeners. For the A.R.E.A. specifications, the limit, center to center, is 6 ft. or the depth of the web. The specifications of the Canadian Engineering Standards Association fix it at the depth of the web, or 7 ft. These rules should not be interpreted to mean that such spacings may be adopted without question for the whole girder. The spacing should be regarded as dependent upon the actual web stresses and in the typical girder these require a gradually diminishing spacing near the ends.
Although calculation of the web stresses may indicate a low shearing stress, stiffeners must nevertheless be used if the ratio of unsupported depth to thickness is over the limit set down in the specification, say 50 or 60. An arbitrary rule such as this is open to question, but it is of the same kind as the one fixing the upper limit of the slenderness ratio for columns.

Proportioning of Stiffeners.—Experimental investigation has shown that under working conditions the deformation of an unstiffened web or of ordinary intermediate stiffeners along vertical lines is small. The function of stiffeners not carrying concentrated loads is, therefore, merely to stay the web laterally. They are to be regarded as vertical beams rather than columns. For this reason, the outstanding legs should be relatively wide, but the legs next to the web may be as narrow as the riveting will permit.

No recognized method for the scientific proportioning of intermediate stiffeners exists. A useful empirical rule given in the A.R.E.A. specifications is that the outstanding leg of each angle shall not be less than 2 in, plus \( \frac{1}{50} \) of the depth of the girder, nor more than 16 times its thickness. The thickness for railway girders is usually \( \frac{3}{8} \) in., while for highway bridges and building girders it is usually \( \frac{5}{16} \) in.

The following sizes for intermediate stiffeners of railway girders are fixed in the specifications of the Department of Railways and Canals, Canada:

<table>
<thead>
<tr>
<th>Depth of Web (FEET)</th>
<th>Size of Stiffener Angles (INCHES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ft. and under</td>
<td>( 3 \times 2\frac{1}{2} \times \frac{3}{8} )</td>
</tr>
<tr>
<td>4</td>
<td>( 3 \times 3 \times \frac{3}{8} )</td>
</tr>
<tr>
<td>5</td>
<td>( 3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8} )</td>
</tr>
<tr>
<td>6</td>
<td>( 4 \times 3\frac{1}{2} \times \frac{3}{8} )</td>
</tr>
<tr>
<td>7</td>
<td>( 5 \times 3\frac{1}{2} \times \frac{3}{8} )</td>
</tr>
</tbody>
</table>

While so far as duty is concerned, intermediate stiffeners not carrying concentrated loads may be crimped over the flange angles, opinion differs considerably as to the economic advantages of crimping. For stiffeners shorter than 3 ft., it is doubtful if the saving in filler material will offset the cost of crimping, unless the fillers be very thick. For depths greater than 3 ft., most shops crimp the stiffeners.

In case forces are applied to the flange of a girder acting towards the neutral axis, the stiffeners should be carefully fitted to the flange to which the loads are applied. It is not theoretically necessary that the other end should be fitted to the flange, but it is found easier to make a tight fit at two ends than at one. If a tight fit is specified at one end, therefore, the fabricating shop would probably fit stiffeners tight at both ends.

Unless known concentrated loads are to be provided for, the rivet spacing in stiffeners is a matter of judgment. The maximum spacing permitted is usually 6 in.

**Illustrative Problem.**—Recommend a size for the intermediate stiffeners of a building girder 54 in. deep.

Based on the A.R.E.A. rule, the outstanding leg should be not less than

\[
2 + \frac{5\frac{1}{2}}{50} = 3.8 \text{ in.}
\]

For building work the stiffeners would probably be two \( 4 \times 3 \times \frac{3}{8} \)-in. angles, with the 4-in. legs outstanding.
53. Concentrated Load Stiffeners.—If at any point a concentrated load is applied to a girder flange or web, reinforcement of the web is usually required. Should the load be applied on the tension side of the neutral axis, the stresses due to the concentration will augment the existing diagonal tensile stresses in the web and it may be necessary to add distributing plates or angles to carry the load up into the web.

It is more usual, however, for concentrated loads to be applied to the compression side of the girder and very commonly to the top flange. If these would raise the existing diagonal compressive stresses in the web above the limit of safety, distributing angles or stiffeners, such as described for beams in Art. 18, must be introduced. The principles of design of these stiffeners, as given in Art. 18, apply when they are used for plate girders.

A very common occurrence of the concentrated load stiffener is at the end or reaction stiffeners for a girder. These consist usually of two or three pairs of vertical angles tightly fitted in between the outstanding legs of the flange angles and arranged in one or other of the ways shown in Fig. 49.

Arrangement (a) is very commonly adopted when the girder reaction is transmitted to the masonry through a shoe plate and a bed plate only. Type (b) is preferred by some designers for the same situation on the ground that the load is not applied to the edges of the shoe plate, as in (a), but is fairly well placed on the four quarters of the plate. If the girder rests on a bolster or rocker, arrangements (c) or (d) should be used, the former for light reactions and the latter for heavy ones.

End stiffeners have a dual function to perform: (1) They serve as vertical beams to stay the web against buckling under diagonal compression; and (2) they receive the concentrated load of the end reaction and distribute it to the web. In the latter role they act in some degree as columns, but since their loading varies gradually from a maximum at one end to zero at the other, they are commonly assumed to have a length as columns of only one-half the depth of the girder. In order that they may be well adapted to carry loads as columns, they should not be crimped, but should be kept straight by the use of fillers between them and the web.

Since the lower edge of the web cannot be counted upon as bearing on the shoe plate, and since the rivets passing through the flange angles and the web are already stressed to their safe limit for flange purposes, and also since the bearing of the ends of the fillers on the inner edges of the flange angles is uncertain, it is customary to assume that the entire end reaction is carried into the web by the stiffener angles. This load they receive in end bearing on the outstanding legs of the flange angles. No bearing value can be credited to the legs of the stiffeners in contact with the vertical legs of the flange angles by reason of the grinding or
bevelling necessary to clear the flange angle fillets, as shown in Fig. 50. If the outstanding legs of the stiffener angles should extend beyond the rounded corners of the flange angles, the projection cannot be counted, so the bearing length is restricted to 0, shown in Fig. 50. Close fitting of the lower ends of the stiffener angles to the bottom flange angles is imperative for all girders and in the case of deck girders to the top flange angles as well. While theoretically it is unnecessary to have a close fit to the upper flange angles in through girders, it is found that the fitting at the bottom is facilitated if they are fitted tightly at both ends.

To give large bearing area as well as to increase the lateral stiffness of stiffeners, the outstanding legs are usually specified to be as wide as the flange angles will permit. The legs in contact with the fillers may be as narrow as riveting will allow, for although these legs have value in the column it is small as compared with the outstanding legs and as has been seen, their bearing value is zero.

By reason of the deflection of a girder resting on ordinary shoe and bed plate bearings, the stiffeners nearest the face of the support probably receive much more load than those farther back. Practice varies with respect to the assumed distribution of load amongst the pairs of angles, but if there are two pairs at the extreme inner and outer edges of the shoe plate, it is probably fair to assume that the inner pair receives about twice as much load as the outer pair. If the pairs are placed closer together, or in somewhat from the edges of the bearing, the inequality of loading would be somewhat lessened.

Rivets connecting the stiffeners to the web plate must be sufficient in number to transmit safely to the web the maximum stress calculated to be borne by each pair. As has been pointed out, the flange rivets should not be counted. If loose fillers are used, the number of rivets required, figured in bearing on the web, would, under most specifications, need to be increased by 50 per cent.

Illustrative Problem.—The total end reaction of a plate girder is 120,000 lb., the end section consisting of a 48 × 3/8-in. web and four 6 × 4 × 5/8-in. angles, as shown in Fig. 51. Determine the size and arrangement of the end stiffeners. Permissible stresses in compression and bearing = 15,000 lb. and 16,000 lb. per sq. in., respectively. Rivets 3/4 in. diameter. Safe shearing and bearing stresses on rivets 12,000 and 24,000 lb. per sq. in., respectively. Rivets through loose fillers to be increased 25 per cent.

Load on inner pair of stiffener angles is approximately (3/4)(120,000) = 80,000 lb.
Outstanding legs must extend at least to rounded corners of flange angles, which will require 5-in. legs. Length of bearing, \( b \), for each angle is approximately 4.5 in.

Required total bearing area for two angles = \( 80,000 / 10,000 = 8.0 \) sq. in.

Required thickness of each stiffener angle \( t = \frac{3.0}{(2)(4.5)} = 0.55 \) in., say \( 3/16 \) in.

Assuming 3-in. legs next the web, the angles would be \( 5 \times 3 \times 3/16 \) in.

Radius of gyration of two such angles about axis in plane of web

\[
\sqrt{\frac{2[(10.4) + (4.18)(2.58^2)]}{8.36}} = 3.02 \text{ in.}
\]

Assuming effective length of stiffener column of \( 48/2 = 24 \) in.,

\[
p = 15,000 - (50) \left( \frac{24}{3.02} \right) = 14,600 \text{ lb.}
\]

Required area for column action = \( 80,000 / 14,600 = 5.48 \) sq. in.

Area provided = \( (2)(4.18) = 8.36 \) sq. in. and hence section is adequate.

Least value of one rivet in bearing on \( 3/16 \)-in. web = \( (0.75)(0.375)(24,000) = 6,750 \) lb.

Number of connecting rivets required = \( 80,000 / 6,750 = 12 \).

If loose fillers be employed, this must be \( 12 + 25 \) per cent = 15, exclusive of those in the flange angles.

This number cannot be driven without adopting practically the minimum permissible spacing and so it is best to use tight fillers. These will be 13 in. wide and extend under both angles on each side of the web. The rivets driven in the fillers may be counted as effective for the stiffeners, since they increase the bearing area of the rivets through the stiffener angles. The same riveting arrangement will be adopted for the outer pair of angles as for the inner pair, as shown in Fig. 51. For the stress borne by the inner angles there are 8 rivets in the angles themselves plus one-half of those in the line along the center of the fillers, or 12 in all.

54. Girders Under Lateral Flexure.—Apart from the accidental lateral flexure to which girders are subjected, due to vibration, swaying of trains, wind, or flange buckling, there are sometimes definite lateral loads to be provided for in design. In the case of crane runway girders, for example, the end thrust of the crane due to sudden stopping of the loaded trolley on the crane bridge or due to lifting heavy loads by inclined pull is often definitely fixed in specifications. In Schneider’s “General Specifications for the Structural Work of Buildings,” it is required that the top flanges of crane girders shall withstand, in addition to the vertical load, a lateral loading based upon one-fifth the lifting capacity of the crane equally divided amongst the four wheels.

In proportioning the top flanges of such girders a section is first assumed which appears suitable for the combination of vertical and lateral moment. The stress due to vertical moment may then be found at the extreme fibers by the approximate Formula (2) of Art. 44 (using the subscript \( v \) to indicate quantities having to do with vertical moment),

\[
f_v = \frac{M_v}{d(A_v + A_w)}
\]

or preferably by the exact flexure formula

\[
f_v = \frac{M_v}{I_v}
\]

The stress due to lateral moment is then found by the flexure formula

\[
f_t = \frac{M_t}{I_{t_v}}
\]
the subscript \( l \) indicating quantities pertaining to lateral moment and \( c' \) being the half width of the flange.

If \( f_e + f_t \) exceeds the permissible working stress, the section must be increased.

55. Torsion on Plate Girders.—If girders are curved horizontally or are of such slope in plan that vertical loads do not come on the line joining the supports, a condition of torsion on the cross-section is produced. The comments on the design of beams for torsion in Art. 23 will apply to built-up girders.

**BEARING PLATES AND BASES FOR BEAMS AND GIRDERS**

**By C. R. Young**

56. Types and Uses.—In order to transmit its loads to the masonry, a beam or girder must in general have bearing plates or bases placed under its ends. The bearing strength of the masonry is so much less than that of a steel beam flange that the flange must in effect be enlarged to prevent the masonry from being crushed.

The simplest bearing is the flat, rectangular plate, Fig. 52a or b, extended out on either side of the beam or girder flange, and of a length parallel to the beam equal to the bearing on the support. When the plate is of cast iron, it is possible to taper it out towards the ends and so vary the section in accordance with the bending requirements.

When the reaction becomes heavy, a much deeper bearing must be employed in order to transmit loading to a large area without exceeding the safe bending stresses or the allowable upward deflection. Ribbed bases of cast iron or cast steel, such as shown in Fig. 52c, can be made to distribute heavy loads over large areas. They are also incidentally useful to raise the end of the girder to a greater height above the masonry, thus permitting the more ready attachment of bracing and serving to keep the masonry at the fixed end of a long span girder at the same elevation as at the roller end. Cast steel is preferable to cast iron for such pedestals on account of its greater reliability, but it costs more.

If the span is longer than about 60 to 80 ft., it is desirable to use hinged bolsters so as to prevent the high intensification of stress near the inner edge of the bearing due to the deflection of the girder. Some engineers make this limit as low as 50 ft. These bolsters may be of cast steel, as shown in Fig. 53b, or may be built up of plates and angles, as in Fig. 53c and d. They consist of an upper and a lower part connected by a pin, pivot, or disc.

For all spans over 25 or 30 ft. it is necessary to provide for changes of length due to expansion and contraction. One end of a beam or girder must, therefore,
be allowed to slide freely. At the fixed end of a girder the base or bolsters described above are placed directly on the masonry, with perhaps a sheet of lead between, and are bolted or anchored down with usually two anchors, one on each side of the girder, so as to prevent movement.

At the sliding end it is necessary to place a bed plate or bed casting on the masonry to provide a smooth surface on which the adjustment may take place. For girders up to about 60- or 80-ft. span, adjustment is allowed to take place merely by the upper or sole or shoe plate sliding on the lower plate, as in Fig. 53a. If bolsters are used, rollers must be interposed between the sole plate of the bolster and the bed plate at the sliding end. These consist of a group, or “nest” of either round or segmental rollers held against shifting endwise by planed grooves in the shoe and bed plates or a ridge on them engaging a turned recess in the rollers, and kept at the proper distance apart by spacing bars attached to the ends, as shown in Fig. 53d. Segmental rollers are flattened on their vertical sides so as to take advantage of the higher strength of large rollers without having to provide an unduly large shoe or bed plate. Sometimes they are concaved on the sides, thus making a greater rotation possible for the same clear spacing. To prevent segmental rollers from tipping over, it is frequently required that at least one of them be geared to the upper and lower plates. With close spacing and substantial side bars this danger is minimized.

57. Design of Simple Bearing Plates.—In proportioning a bearing plate, Fig. 52a or b, the required area is first found by dividing the maximum reaction by the allowable pressure on masonry of the class employed. If there is no specification for this, an average value may be found in any of the civil engineering handbooks. The dimensions are then fixed so as to utilize readily-obtained
plates, one of them being equal to the bearing on the support. Recommended bearings and sizes of plates for building work are found in all of the steel handbooks.

To determine the thickness required, consider a strip of plate at right angles to the beam or girder, Fig. 52a or b. It is in effect a cantilever of span $l$ subjected to an upward loading. This is commonly assumed as uniformly distributed, although the pressure at the outer edge is much less than near the beam flange. Let $p$ be the uniformly distributed upward pressure in pounds per square inch, $f$ the allowable flexural stress on steel plates, and $t$ the thickness required. Then

$$t = 1.73(l \sqrt{\frac{p}{f}})$$  

The strength of the combined flange and bearing plate in bending should also be investigated. They form a cantilever beam of span $l'$ and thickness $t'$, Fig. 52a or b. It is not always safe to assume this cantilever as a single beam of depth $t'$, since there may be insufficient connection between the flange and the bearing plate to resist the horizontal shear accompanying beam action for the whole depth. The moment of resistance of each should in such a case be found and the two added.

Tables may be prepared of the allowable projections of bearing plates based on Formula (1). They will vary in accordance with the allowable pressure and flexural stress assumed, but are usually from 3 to 5 times the thickness.

58. Design of Ribbed Bases.—The load applied to a ribbed base, such as shown in Fig. 52c, must be transmitted to the masonry by means of the ribs acting as short columns or prisms. These ribs should be so placed as to receive the load from the beam or girder in the most direct manner possible. One rib should be directly under the web and, when cast pedestals are used for plate girders, transverse ribs should be directly under the outstanding legs of the end stiffener angles. The rib under the pair of stiffeners nearest the edge of the support is much more highly stressed than the others by reason of the deflection of the girder. Some designers assume that the whole reaction is applied over this rib. If it be proportioned for the whole load and the other ribs made of the same thickness, the base will be amply strong. While it is desirable to have three ribs running longitudinally, as shown in Fig. 52c, only two need be run transversely if there be but two pairs of stiffener angles. With three pairs of angles there should be three ribs. The top plate of the pedestal must be fixed largely by judgment and experience, but should not be less than $1\frac{3}{4}$ in. The bottom plate is designed by assuming it as a cantilever projecting past the most heavily-loaded rib. The moment of the uniformly varying pressure on the projection taken about the edge of the rib is then found and the projection, including both base plate and extension of ribs, is figured in the same manner as a cast iron lintel section (Art. 30). The ribs should not be thinner than 1 in. for important work and the bottom plate not thinner than $1\frac{1}{2}$ in.

To equalize the pressure on the masonry, it is desirable to make the top plate of the casting as narrow as possible in a direction parallel to the length of the girder.

59. Design of Bolsters.—The design of the upper and lower parts of bolsters involves the same principles, although, by reason of its bearing on masonry, the lower part is usually the larger. Considering Fig. 53b or c, it is seen that the
ribs which transfer the load from the girder to the pin, and from the pin to the lower part of the bolster, must be of sufficient thickness to carry their loads without exceeding the safe bearing pressure on the ribs or on the pin. To determine their thickness, the size of pin must, therefore, be assumed first. If the rib thicknesses are satisfactorily fixed and the moment or shear on the pin is excessive, the pin size will need to be increased, which may make it possible to use thinner ribs. The bottom plate of the lower part of the bolster must be investigated as a beam continuous under the ribs and cantilevered past them. The upward pressure may be assumed as uniformly distributed over the lower parts of the bolster.

Care must be taken not to make the ribs so thin as to render them likely to buckle laterally. To safeguard against this it may be necessary to introduce transverse diaphragms between them.

As both the upper and lower parts of the bolster are in effect beams resting on a single central support, they should be figured in bending at the vertical section through the pin hole. Due to the lateral deformation of the pin under pressure, causing it to bear against the right and left faces of the hole, no deduction need be made for the hole.

60. Design of Expansion Bearings.—If sliding is provided for merely by allowing a sole or shoe plate to slide on a bed plate, as in Fig. 53a, the shoe plate is designed in the same manner as an ordinary simple bearing plate (Art. 57) and the bed plate is made about the same thickness. Each of these plates should either be planed or straightened to ensure true contact. They are rarely less than \( \frac{7}{8} \) in. thick.

Round holes for anchors are provided in both sole and bed plates at the fixed end, but slotted holes must be provided in the sole plate at the sliding end. These must be elongated about \( \frac{1}{5} \) in. for each 10 ft. of span.

Where the span is in excess of from 60 to 80 ft., rollers must be provided. Since round rollers take up a great deal of room for their pressure value, segmental ones are now very commonly used. Large rollers also tend to overcome the frictional difficulties incident to the use of small rollers. This is evident from the formula for the permissible pressure on cast steel rollers which in pounds per linear inch is usually placed at 600 times the diameter in inches. Railway bridge specifications now frequently require that rollers be not less than 6 in. in diameter. The bearing length provided must be clear of all recesses engaging ribs on the shoe or bed plates.

To make the masonry of the same height at the two ends and at the same time keep the bolsters alike, a cast or built-up bed equal to the height of the rollers and the bed plate at the expansion end should be placed under the bolster at the fixed end, as shown in Fig. 53c and d. In the case illustrated, this bed is made up of a cellular casting resting on a steel plate.

To give true bearing on the masonry, beds are sometimes set \( \frac{1}{2} \) to 1 in. high and grouted underneath. Another method is to place a sheet of lead \( \frac{1}{8} \) in. thick under the bed, as shown in Fig. 53c and d.

61. Anchors.—Beams and girders are prevented from shifting horizontally on their supports by means of anchors. For ordinary rolled beams resting on loose bearing plates, a round rod is frequently used, preferably bent so as to engage the masonry beyond the end of the beam. Sometimes two angles are
bolted to the end of the beam, the outstanding legs securing the beam to the masonry. Rod anchors are usually \( \frac{3}{4} \) in. in diameter, projecting about 9 in. on either side of the web. Angle anchors should have a 6-in. outstanding leg and be \( \frac{3}{6} \) or \( \frac{3}{6} \) in. thick. Such anchors are illustrated in the Carnegie Pocket Companion.

If beams or girders rest on sole plates riveted thereto, anchor bolts are inserted vertically in the masonry through holes left in it for this purpose or specially drilled after the erection of the steel, care being taken to so locate the holes that this can be done. Usually two anchors, one on each side of the beam or girder, are employed. For building work, they should not be smaller than \( \frac{3}{4} \) in. diameter, while for railway bridge work the minimum should be \( \frac{3}{4} \) in. They should extend at least 12 in. into the masonry and be well grouted or otherwise secured thereto. To this end, split bolts with wedges in the ends, or hacked bolts, are sometimes used.

Holes for anchor bolts should be \( \frac{3}{6} \) or \( \frac{3}{6} \) in. larger than the bolts, to provide for errors in placing the bolts or for easier drilling of the holes.

**STEEL TENSION MEMBERS**

BY C. R. Young

62. Forms and Uses.—Steel tension members vary in form with the magnitude and character of the stress carried by them, with the character and situation of the structure of which they form a part, and with the methods of construction adopted.

Round or square rods, single or multiple, made adjustable by end nuts, turnbuckles or sleeve nuts, Fig. 54a, b and c, are used in many cases where loads are light and cheapness is desired. In pin-connected bridges they are employed as counters, and sometimes, though inadvisedly, as laterals in both riveted and pin-connected bridges; in buildings they serve for lateral and sway bracing, for hangers and for tie rods in arch floors; in towers they are frequently used for bracing. The end connections are made by forging the bar into a simple or a forked loop, Fig. 54b and c, or, perhaps, by attaching a clevis thereto, as shown in Fig. 54d.

Eye bars, which consist essentially of rectangular bars of metal with a head, Fig. 54e, forged at each end, through which a pin hole is bored, have an extensive use as tension members, usually in multiples of two bars. While formerly used for bridge and-roof trusses of almost all spans, their use is now confined very largely to the longer spans, where there is little likelihood of objectionable rattling or vibration. For such situations they afford a reliable, easily transported and readily erected tension member. In places where reversal of stress is possible, or where attachment of riveted work is necessary, eye bars are replaced by built-up tension members, as for the end panels of the bottom chord and for the hip hangers of railway bridge trusses.

Adjustable eye-bars are frequently used as counters in bridge trusses. The adjustment is made possible by inserting in the body of the bar at a conveniently accessible point, a turnbuckle or a sleeve nut, Fig. 54f.

Flats, or narrow plates, Fig. 54g, are little used, as they are flexible, easily become bent in transportation and erection and do not hold their length or
straightness well against drifting or reaming. They are sometimes used as hangers, but apart from such use are seldom employed in America.

Single angles, Fig. 54h, are extensively used as light tension members in trusses and as lateral and sway bracing.

Double angles, arranged as in Fig. 54i or j, are used as truss tension members of medium capacity.

Forms consisting of four angles arranged in H-shape and connected together by a single plane of latticing or tie plates, Fig. 54k, or by a web plate, are very economical and are advantageously used as bottom chords, tension diagonals, and hip hangers of riveted trusses.

Two channels with flanges turned in, as in Fig. 54l, and connected together by latticing or tie plates, are well adapted for bottom chords of through spans, and for tension diagonals and hip hangers of both deck and through trusses. If the two channels are arranged with their flanges turned out, as in Fig. 54m, the section is well adapted for the bottom chords of deck spans.

Built channels, with flanges turned in, as in Fig. 54n, or with flanges turned out, as in Fig. 54o, and with or without added side plates, are the commonest forms for riveted tension members in both riveted and pin-connected trusses of both moderate and long span.

63. Theory of Design of Tension Members.—Assuming a uniform distribution of stress over that portion of the cross-section that may be considered
as effective, it follows that the effective area required in a member for the resistance of a given force is given by the formula

\[ A = \frac{P}{p} \]

where \( A \) = effective area of section required in square inches.

\( P \) = force, or total stress, to be resisted, in pounds.

\( p \) = permissible stress in pounds per square inch.

If it should happen that the force to be resisted by the tension member is applied at some other point than the center of gravity of the net section, the member must be designed for a combination of direct and bending stress. The principles governing the design of such members have been thoroughly explained in Sec. 1. If the load is applied on a principal axis of the cross-section, then, neglecting the effect of the deflection arising from the eccentric application of the load, the maximum resultant stress at the most highly stressed fiber, distant \( c \) from the neutral axis, is given by the formula

\[ f_1 = \frac{P}{A} + \frac{Pec}{I} \]

where \( e \) = the eccentricity, and \( I \) = moment of inertia of section about an axis normal to the plane of bending.

Let \( p \) be the permissible stress on the most highly stressed fiber, and \( r \) the radius of gyration of the section in the plane of bending. Then the total required area

\[ A = \frac{P}{p} + \frac{Mc}{r^2p} \]

Should the resultant axial load be applied eccentrically and not on a principal axis of the cross-section, the maximum resultant stress\(^1\) on the most highly stressed fiber, as established by Professor C. Batho, is

\[ f_1 = \frac{P}{A} + \frac{P x_p (y_1 - x_1 \tan a)}{J - I_y \tan a} \]

where \( x_1 \) and \( y_1 \) are co-ordinates of the most highly stressed fiber, the origin being at the center of gravity.

\( x_p \) = \( x \)-co-ordinate of point of application of load \( P \).

\( a \) = angle which neutral axis makes with \( x \)-axis, found by equating \( f_1 \) to zero.

\( J \) = product of inertia of section.

\( I_y \) = moment of inertia of section about \( x \)-axis.

The necessary effective area can be found by a trial and error application of Formula (3).

**64. Choice of Section.**—Members should be composed, in so far as possible, of sections symmetrically placed and should be of forms such as to obviate large eccentricity in the end connections. In order to secure the maximum efficiency of the material employed, the form chosen should permit the direct connection at the joints of as large a proportion of the area as possible. Thus, in the case of single—or double—angle tension members, connected by one leg only, unequal-legged angles should be employed and the longer legs connected to the gusset plate. Even though both legs are connected to the gusset, it is preferable to use unequal-legged angles with the longer leg in contact with the gusset, for the

\(^1\) See Transactions Canadian Society of Civil Engineers, Vol. 20, 1912, p. 249.
reason that the avoidance of dependence upon rivets where possible requires that the amount of stress delivered through attached details—such as a lug and its two sets of rivets—should be reduced to a minimum. There is, too, the further advantage that the use of a relatively narrow outstanding leg effects a saving in the material of the lugs.

For members of structures subject to vibration, rigid or stiff forms should be used. Open sections are in all cases preferable to closed ones. Limiting sections and thicknesses prescribed by the governing specifications must be observed.

65. Net Section.—The net section of a tension member at any right section is the gross area of the member less all rivet holes, pin holes or cuts, or fractions thereof, that diminish the resistance of the member at that section. It is determined in accordance with the form of the member. As explained in Art. 68, when the stress is assumed as uniformly distributed, sometimes only a fraction of the net section is effective. In such cases it is this effective portion that must be considered in designing.

For rods, the net section is the section at the root of the thread, unless the rod is upset, when it is the section in the body of the rod.

In the case of eye bars the net section is the net area of the body, since considerable excess area is always provided in the eye. For adjustable eye bars it is also the area in the body.

For riveted or built tension members the net section depends on the types of body and end details adopted, and the arrangement and size of the rivets. Details involving the minimum practicable number of rivet holes on, or within certain critical distances of, any right section through the body of the members are desirable since they give the maximum net section. Other arrangements should be avoided if possible. Obviously the larger the rivet, the greater the loss of section. It is almost universal practice in calculating net areas to consider the diameter of the rivet hole as \( \frac{1}{8} \) in. greater than the diameter of the rivet before driving.

The number of rivet holes that must be deducted from the gross area of a right section, to give the net section, depends upon the number of gage lines and the stagger of the rivets. Obviously, the net area adopted should be such that nowhere on any diagonal or zig-zag section should the maximum stress due to the combination of normal and shearing stresses exceed that on a right section. This result will be attained if the number of rivet holes \( N \) deducted from the gross right section be taken as the maximum number given on any zig-zag section by the formula

\[
N = 1 + x_1 + x_2 + x_3 + \ldots
\]

where \( x \) = a fraction of a rivet hole

\[
g = 2 \left( g^2 + s^2 - h \sqrt{g^2 + 4s^2} \right) \\
h = \frac{h(g + \sqrt{g^2 + 4s^2})}{g}
\]

\( g \) = the gage, i.e. the distance between any two holes measured at right angles to the axis of the member.

\( s \) = stagger of these holes.

\( h \) = diameter of rivet hole as considered for deduction purposes = diameter of rivet + \( \frac{1}{8} \) in.
This formula is to be applied to alternative sections, the successive terms representing the deductions for successive holes considered in a chain across the member. The particular group of rivets to be considered is that which will give the greatest total deduction, whether the rivets lie on adjacent gage lines or not.

To obviate the large amount of work required in solving Formula (1), the diagrams of Fig. 55 have been prepared. These give the theoretically correct deductions for any assured ratio of stagger to gage. The curves have been drawn for $\frac{3}{4}-$, $\frac{3}{8}-$ and $\frac{7}{8}$-in. rivets, for staggerers up to 9 in., and for gages up to 15 in.

66. Proportioning of Rod Members.—If a tension member consisting of one or more rods or bars of uniform section is threaded for end connections or for insertion of a turnbuckle or sleeve nut, its strength will depend on the net area at the root of the thread. If, however, the rod is upset before threading, the rods may be weaker through the body than at the root of the thread. Good practice requires that this be the case and hence the standard upsets for both round and square rods tabulated in the steel handbooks provide for an area at the root of thread usually between 20 and 30 per cent greater than through the body of the bar. If, then, standard upsets are to be employed, the designer need concern himself only as to the area to be provided in the body of the member.

It is not always economical to use upset ends. If the rods are short and of small area, the actual saving of material effected by upsetting may be more than offset by the labor cost of making the upsets. For example, tie rods for floors and sag rods for roofs are not upset.

Loops at ends are so proportioned as not to constitute a source of weakness to the member. If standard loops are to be provided, no attention need be given to the looped ends in design.

For adjustable rod members, the stress may, in most cases, be assumed as uniformly distributed. The different rods, if there are more than one, may be given equal tension by adjustment, and care should be taken to see that in service they are equally stressed. Only regular inspection can ensure this. If the rod is not at right angles to the pin to which it connects, the load will be applied at one edge and allowance would then need to be made for flexural stress in addition to the tensile stress.

Illustrative Problem.—A round rod tension member of soft steel with ends threaded but not upset, is to carry safely a load of 22,000 lb. If the permissible stress in tension is 18,000 lb. per sq. in., determine the necessary size of rod.

Required net area = $\frac{22,000}{15,000} = 1.47$ sq. in.

From Carnegie or Cambria, it is found that one $1\frac{3}{8}$-in. round rod with area of 1.52 sq. in. at root of thread, or two $1\frac{3}{4}$-in. round rods with combined area of 1.78 sq. in. at root of thread would do. One rod would be cheaper, if practicable for the situation.

Illustrative Problem.—A tension member is to consist of one or two round or square soft steel rods upset and threaded at the ends. Determine the required size. Load and permissible stress as in last problem.

Required net area = 1.47 sq. in.

Area of one $1\frac{3}{8}$-in. round rod upset = 1.49 sq. in. in body of bar and 1.74 sq. in. at root of thread.

Area of one $1\frac{3}{4}$-in. square rod upset = 1.56 sq. in. in body of bar, or 2.05 sq. in. at root of thread.

The round rod is the more economical.

Two 1-in. round rods, or two $\frac{7}{8}$-in. square rods, would also be sufficient.
Fig. 55.—Theoretically-correct deductions of rivet holes for tension members.
67. Proportioning of Eye Bar Members.—For pin connected structures, the eye bar, Fig. 54e affords a very convenient unit from which to build up a tension member. The heads are now so well standardized that the designer need have no fear of the bar failing therein. Examination of the tables of proportions of heads in Carnegie or Cambria shows that the area through the eye is ordinarily from 35 to 40 per cent in excess of that through the body of the bar. The A.R.E.A. specifications require the excess to be 37 1/2 per cent.

Experience has shown that there should be a certain minimum thickness for each width of bar, for reasons of manufacture and since very thin bars tend to fail by buckling in the head. This varies from 1/2 in. for bars, 2 in. wide, to 1 3/4 in. for bars 16 in. wide. On the other hand, the thinner bars simplify packing at joints and reduce pin moments. It is commonly specified that the thickness of the bars shall not be less than 1/4 the width.

The size of head to be employed will, of course, depend on the necessary size of pin. Ordinarily, it is about 2 1/4 times the width of the bar. In selecting a bar, care should be taken to ensure that the head is not too large to fit into any built-up member, such as a top chord, to which it may be required to connect. Bars should be selected with the size of pin in mind. It is now often required that the pin be not over 7/8 the width of the widest bar attached.

Ample basis for fixing safe working stresses on eye bars exists in the many full-sized tests that have been made. Their ultimate strength is on the average less than for small specimens of the same material, usually about 85 to 90 per cent as great, due to the less perfect working received by the thick metal of the bar and to the annealing of the heads after forging. However, since the eye bar member is subjected to low secondary stress in the structure, and also since the probable percentage loss by corrosion is small and the resilience is large, the permissible working stress may be quite as great as for riveted members.

In building up eye bar tension members, the constituent bars should be packed symmetrically about the plane of the truss with the inclination of any bar thereto as small as possible and in no case greater than 1 1/16 in. per foot. By keeping the inclination or "cradling" down, the flexural stresses are thereby minimized. Bars should be secured against lateral shifting and so arranged that adjacent bars in the same panel will not be in contact with each other lest there be corrosion between them.

Illustrative Problem.—A tension member of a truss carrying a load of 285,000 lb. is to consist of two or four medium steel, non-adjustable eye bars connecting at each end to a pin estimated to be 4 1/2 in. diameter. The thickness of the bars is not to be less than one-eighth the width, nor less than 1 in., and the width of the bars not over eight-sevenths of the diameter of the pins. Permissible tensile stress, \( p = 16,000 \) lb. per sq. in. Heads of American Bridge Co. standard.

Required area in body of bars = 285,000/16,000 = 17.82 sq. in.

Maximum permissible width of bars = \((4.5)(\frac{3}{4})\) = 5.1, say 5 in. Minimum thickness for 5-in. bars = 1 in.

Two bars, \( 5 \times 1 \frac{3}{4} \) in. with a total area of 18.13 sq. in. would be sufficient. If four bars be used, they must each be 5 x 1 in. because of the rule respecting minimum thickness. This would give a considerable excess of area, although the use of the thinner bars is desirable for other reasons.

68. Proportioning of Riveted Tension Members.—Design of the simplest form of riveted tension member, the narrow plate, or flat, is vitally related to the
arrangement of rivets in the end connections. To develop the highest possible efficiency the rivets should be arranged in a triangular group, as shown in Fig. 54g, with the apex of the triangle formed by a single rivet, pointing towards the center of the member. With such a connection it is necessary to reduce the gross section by only one hole in proportioning for the full calculated stress, since at the second line of rivet holes the stress in the member is less than the full stress in the body by the amount of the stress taken out by the first rivet.

Practical objections to the arrangement of connections as shown in Fig. 54g usually result in a less efficient arrangement. The proper deduction to be made, however, can easily be determined by following the methods of Art. 65.

Although a flat as a tension member is deficient in lateral rigidity and may for that reason contribute to the vibration of a structure, the rigidity of the end connections as compared with those of the pin type is an advantageous feature.

![Diagram](image)

**Fig. 56.**—Effect of end connections on efficiency of flat plate tension members.

One inherent defect of a single flat as a tension member is that by reason of the end connections being made to a gusset in the form of a lap joint, as shown in Fig. 56a, there is an eccentricity of one-half the thickness of the plate, assuming, as appears justifiable in the light of experiment, that the load is applied at the plane of contact of the member with the gusset. If there be two flats side by side forming a single-member, and possibly stitch riveted together, with the end connections in the form of a butt joint, Fig. 56b, the effect of eccentricity is not wholly overcome, for each component part of such a tension member tends to bend in its own way, due to the eccentric application of the part of the applied load that goes to it. The bending is somewhat restrained, however, by the body details.

With the single angle, or more complicated forms of tension members built up of angles or other shapes and plates, it is desirable to have as large a portion as possible of the cross-section directly connected to the end gussets. By so doing, the length of the end connections and size of gusset plates is thereby reduced and often a more equable distribution of stress over the cross-section is brought about, thus improving the efficiency of the member. If possible, single angles should have unequal legs and, if only one leg is connected, it should be the longer one.

In most riveted tension members there is an unavoidable eccentric application of the load, by reason of the fact that the component parts lack perfect symmetry in themselves or are connected to the gussets in the manner of a lap joint. Typical cases of this kind are shown in Fig. 57.

The load is applied to a single angle tension member, Fig. 57a, along the line of connecting rivets, and slightly inside the gusset. If $G$ is the center of
gravity of the angle, there is consequently an eccentricity $KG = \epsilon$. The true maximum stress resulting from this combination of direct and bending stress can only be calculated by employing the theory of unsymmetrical bending stress, as explained in Art. 22.

The main component parts of the members shown in Fig. 57b and $c$ will receive their loads eccentrically. Each one should, therefore, properly be designed for a combination of direct and bending stress, unsymmetrical bending being considered for the case of Fig. 57b. The presence of connecting stitch rivets, tie plates, or battens does not entirely prevent this bending, but, according to Professor C. Batho\(^1\) each component part tends to bend in its own way.

\[\text{Fig. 57.—Eccentric application of load on tension members.}\]

The relation of the maximum to the mean stress is dependent on the amount of initial eccentricity and the restraining effect of the end connections. By utilizing heavy, wide gusset plates, the deflection of the member due to eccentricity is lessened, which in turn tends to equalization of stress over the cross-section. While the effect of the restraint in a direction normal to the plate is small, it is important in the plane of the plate. The average decrease of the ratio of maximum to mean stress for single angles due to the stiffness of the end plate in its own plane, was found by Prof. Batho to be about 35 per cent at the highest load applied. It, therefore, appears most desirable, in order to improve the efficiency of members connected unsymetrically, to fix the direction of the ends as far as possible.

The effort to equalize the stress in an angle member by connecting it by both legs is shown by the tests of Prof. Batho to bring comparatively little advantage. In the most favorable cases it decreased the ratio of maximum to mean stress at working loads by about 4 per cent. The earlier tests by Prof. F. P. McKibben\(^2\) showed an improvement in efficiency of under 15 per cent in the most favorable case and in most cases under 10 per cent. For single angles connected by one leg, the efficiency ranged from 75 to 83 per cent, while for single angles connected by both legs, it ranged from 86 to 96 per cent. The practice of permitting only the connected leg to be counted for angles connected by only one leg, while allowing both legs to be counted if lug angles are used, is thus seen to be unduly favorable to the use of lug angles.

Experimental investigation shows that a considerable change may be made in the position of the line of pull of the gussets with respect to the gravity line of the angles connected, without greatly affecting the stress distribution over the

\(^1\) *Transactions Canadian Society of Civil Engineers*, vol. 26, Part 1, 1912.
angles for the efficiency of the member. From Prof. McKibben's tests, it appears that changing the line of pull from the gage line of a single angle connected by only one leg to the projection of the gravity line, improved the efficiency only 5.5 per cent in the most favorable series of tests. In another series it was improved by only 2 per cent. No great sacrifice in efficiency is thus brought about by placing the gage line of an angle member on the skeleton line of a truss.

Double angle members, such as shown in Fig. 57b, have the merit of somewhat hindering each other from bending perpendicularly to the gusset plate, with the result that the ratio of maximum to minimum stress over the cross-section is thereby reduced. In Prof. McKibben's tests, no particular advantage appeared to attach to this form of member, but Prof. Batho found them to give better results than single angle members.

In the practical design of tension members composed of either single or double angles, it is desirable to proportion on the assumption that the stress is uniformly distributed over the cross-section. Allowance can be made for the loss of efficiency arising from eccentricity by reducing the net area by a percentage to give the true effective area. This percentage will vary with the amount of eccentricity and the ratio of the legs of the angle, and will depend on whether lug angles are used or not.

Although the tests cited indicate that only from 75 to 83 per cent of the net area of representative angles connected by one leg is effective, the adopted efficiency in design should be higher than this. Built tension members with an average efficiency of 87 per cent are regarded as 100 per cent effective, and consequently for consistency, single and double angles should be considered as having an efficiency about 15 per cent higher than that shown by actual test. To be on the safe side, however, it would seem desirable to limit this excess to 10 per cent. The percentage efficiency of single and double angles connected by one leg only has been found to be expressed fairly well by the formula

\[ p = 100 - 30 \cdot \frac{sg}{c^2} \]  

where \( p \) = percentage of net area effective.

\( s \) = length of outstanding leg of angle in inches.

\( g \) = gage in inches of angle if one gage line only is used, or two-thirds of the sum of the gages if two gage lines are used.

\( c \) = length of connected leg in inches.

Efficiencies of angles connected by both legs may with safety be taken as 5 per cent higher than those given for angles connected by only one leg.

These effective percentages are to be applied to the net sectional area as determined by the principles of Art. 65. The reduced net area then becomes the true effective area.

**Illustrative Problem.**—Let it be required to find the effective percentage of the net section of a 3\( \frac{1}{2} \) \times 2\( \frac{1}{2} \)-in. angle connected by the 3\( \frac{1}{2} \)-in. leg, with rivets on a 2-in. gage.

\[ p = 100 - 30sg/c^2 = 100 - (30)(2.5)(2)/(3.5)^2 = 87.8 \text{ per cent}. \]

Find the effective percentage of the net sectional area of a 6 \times 4-in. angle, connected by the 6-in. leg, with two lines of rivets driven on gages of 2\( \frac{1}{2} \) and 2\( \frac{3}{4} \) in.

\[ p = 100 - (30)(4)(0.67)(2.25 + 2.25)/(6)^2 = 90 \text{ per cent}. \]
Although no records of tests of single channels in tension are available, it is probable that single channels connected by their webs only, would, because of the relatively high ratio of eccentricity to corresponding section modulus as compared with angles of the same area, show an efficiency somewhat less than that of single or double angles. The relative areas of the flanges and the web, or the weight of a channel for a given depth, should materially affect the efficiency. It is probable that for single channels the effective area is about equal to the net area of the web plus 70 per cent of the net area of the flanges.

Due to eccentricities and imperfections of material and workmanship, built up tension members will not develop under test the same strength in pounds per square inch of net area as would be given by a small specimen. Tests reported by J. E. Greiner\(^1\) on (1) members of H-shape made up of four angles connected by latticing, battens, and solid web plates, and (2) on members composed of two built up channels connected by latticing and battens, showed that the efficiency ranged from 80.4 to 90 per cent. Some of the lower figures were for specimens with a highly eccentric pin-plate connection to the outstanding legs of the angles of the member. The lowest were, strangely enough, for box-shaped members with both legs of the four main angles connected to the end pin plates. This lack of strength was probably due to the fact that the pin plates did not extend to the near ends of the end batten plates.

In fixing working stresses, the efficiency likely to be attained should be borne in mind. In view of the fact that for tests on eye bars, the average efficiency is over 90 per cent, the working stresses on built up tension members may safely be as great as on eye bars.

**Body Details.**—When a tension member is built up of an assemblage of rolled sections, it is desirable to connect them together at certain intervals depending on the character of the member.

If angles or channels are used without web plates, the connection may be in the form of stitch rivets, Fig. 58a, or latticing, Fig. 58b, or battens, Fig. 58c, or d. Stitch rivets are used where the two connected parts are sufficiently close together to make it practicable to insert washers between them through which the rivets may be driven. Latticing, formerly much used for the larger tension members, is now chiefly employed for compression members, or for members subject to reversal of stress. Battens are found to be cheaper and practically as satisfactory as latticing for tension members. If web plates are used, the connection of angles or channels thereto is made by lines of rivets spaced as described below.

Body details such as shown in Fig. 58 serve several purposes. They lessen the transverse vibration by so connecting the parts that they will act practically as one unit with a width or depth equal to the overall lateral dimension. This

\(^1\) *Transactions American Society of Civil Engineers*, vol. 35, 1897.
reduces general vibration in the structure and minimizes the likelihood of loosening of the rivets in the end connections. Web or batten plates near the end of the member, Fig. 58d, serve to lessen the effect of the eccentric application of load by restraining the end of the member against bending. Solid webs or closely spaced battens or latticing help to equalize the stress over the cross-section by transferring stress from a heavily loaded to a lightly loaded part.

Where stitch rivets are used, they are spaced from 2 to 4 ft. apart in angle members and, where they connect shapes to web plates or various plates together, they are spaced in the line of stress a distance not over 10 times the thickness of the thinnest outside metal, nor over 6 in.

Latticing, if used, is designed and arranged in the same manner as for compression members.

Battens are spaced center to center about 3 or 4 ft. apart.

The thickness of battens for tension members may be determined by the rule applied to single lattice bars—that is, not less than \(\frac{b}{4}\) of the distance between the rivet lines. In no case should it be less than the minimum prescribed by the specification for secondary material. For light members the length of the batten plates parallel to the axis of the member need not be greater than is required to accommodate two rivets in each line. At the ends of members, in order to contribute to the restraint of the individual parts of the member and lessen bending due to eccentricity, the battens should be as long as the member is wide and be placed inside the end gusset plates.

**Illustrative Problem.**—In the following problems, let the permissible stress in tension be 16,000 lb. per sq. in. of effective area, and the rivets \(\frac{7}{8}\) in. dia. with holes 1 in. dia.

(a) Find the size of a double flat plate tension member to carry 47,000 lb., if a line of stitch rivets runs along its axis and if provision must be made for two rivets opposite each other at the inner edge of the end connections.

Required net area = 47,000/16,000 = 2.94 sq. in.
Net section will be at the end connection, and if \(w\) = width of plates and \(t\) the thickness of each plate, it = \(tw = 2.94\).

Hence \((w - 2)t = 2.91\) sq. in., from which assuming \(w = 6\) in., \(t\) required = 0.368 in.
Two 6 \(\times \frac{3}{8}\)-in. plates will be used.

(b) A single angle with one leg only connected is to carry a load of 28,000 lb. Find the required size.

Effective area required = 28,000/16,000 = 1.75 sq. in.
Assume a 3\(\frac{1}{2}\) \(\times\) 3 \(\times\) \(\frac{3}{8}\)-in. angle connected by the 3\(\frac{1}{2}\)-in. leg on a 2-in. gage.
Net section = 2.30 - (1)(0.38) = 1.92 sq. in.
Effective percentage of net section from Formula (1) is

\[
p = \frac{100 - (30)}{(3.5)^2} = 85.4 \text{ per cent}
\]

and effective area of angle = \((0.854)(1.92) = 1.64\) sq. in.

The angle is not large enough. A 3\(\frac{1}{2}\) \(\times\) 3 \(\times\) 1\(\frac{1}{8}\)-in. angle gives an effective area of 1.89 sq. in., and hence would be satisfactory.

(c) Find the required size of angle for problem (b) if both legs were connected and the stagger of the inner rivets were 2 in.

Assuming a 3\(\frac{1}{2}\) \(\times\) 3-in. angle with gages of 2 and 1\(\frac{1}{2}\)-in., respectively, in the 3\(\frac{1}{2}\)- and 3-in. legs, the distances of the rivet lines apart, or \(y\), assuming a thickness of \(\frac{3}{8}\) in., is 3.31 in. The deduction, from Fig. 55, is \(1 + 0.7 = 1.7\) holes, and the net area = 2.65 - (1.7)(1)(0.44) = 1.90 sq. in.

Assuming the efficiency as 5 per cent greater than for an angle connected by one leg, or say 90 per cent, the effective area = (0.90)(1.90) = 1.71 sq. in. This is slightly below the requirement but would in most cases be accepted.
(d) A truss member carrying 235,000 lb. is to be of H-shape, consisting of two pairs of angles $12\frac{1}{2}$ in. back to back with connecting battens, as shown in Fig. 59. Determine the size, assuming the rivet arrangement as shown. Assume the full net area as effective, because of the restraining effect of the end battens.

Required effective area of member = $235,000/16,000 = 14.7$ sq. in.

Assume four $6 \times 4 \times \frac{1}{2}$-in. angles. Considering one angle developed as in Fig. 59a, it is found by consulting Fig. 55 that the least net section is $S-S$, the deduction from each angle being 2 holes.

Net section = $(4)(4.75) - (8)(1)\left(\frac{1}{2}\right) = 15.0$ sq. in. which is adequate.

(e) A truss member carrying 370,000 lb. is to be made up of two channels, 12 in. deep, with flanges turned out, reinforced by two 11-in. plates on the backs of the channels, as shown in Fig. 60. Battens connect the flanges of the channels, the end batten being outside the gussets. Determine the necessary section if only 70 per cent of the net area of the channel flanges is considered effective.

Required effective area = $370,000/16,000 = 23.2$ sq. in.

![Fig. 59.—Design of four-angle tension member.](image1)

![Fig. 60.—Design of a double channel tension member.](image2)

Assume two 12-in., 30-lb. channels and two $11 \times \frac{1}{2}$-in. plates with the riveting at the inner edge of the connection as shown.

Critical section in $S-$ $S$, cutting four web holes and four flange holes.

Gross area of section = $(2)(8.79) + (2)(11)(0.5) = 28.58$ sq. in.

Area of 4 holes through webs of channels and plates (section $S-$ $S$) = $(4)(1)(0.51 + 0.50) = 4.04$ sq. in.

Gross area of flanges of channels = total area - area of webs = $(2)(8.79) - (2)(12)(0.51) = 5.34$ sq. in.

Net area of flanges, the grip being $\frac{1}{2}$ in., = $5.34 - (4)(1)(0.5) = 3.34$ sq. in.

Reduction of flange area = $(0.30)(3.34) = 1.00$ sq. in.

Effective area of member = $28.58 - (4.04 + 1.00) = 23.54$ sq. in. which is adequate.

69. Tension Members Subject to Cross Bending.—It frequently happens that tension members are subjected to transverse as well as to axial loading. This produces a combination of direct and bending stress for which the member must be designed. The problem is essentially the same as for a tension member subjected to eccentric axial loading and consequently a moment of eccentricity.

Common cases of cross bending in bending members are a tension member subjected to its own weight or to a directly applied load. The bottom chords of roof trusses in mill buildings are frequently loaded with trolleys, piping, wires, etc. Another case is a tension chord subjected to the thrust of a cross strut of a lateral system.

If the effect of the deflection in augmenting the moment is neglected, the maximum fiber stress is

$$f_1 + f_2 = \frac{P}{A} + \frac{Me}{I}$$

(1)
If the deflection be considered, assuming the ends free to turn, the maximum stress becomes

\[ f_1 + f_2 = \frac{P}{A} + \frac{Mc}{I - \frac{PL^2}{10E}} \]  

(2)

where

- \( f_1 \) = uniformly distributed stress due to axial load \( P \).
- \( f_2 \) = flexural stress on extreme fiber.
- \( P \) = axial load.
- \( A \) = area of member.
- \( M \) = bending moment.
- \( c \) = distance from neutral axis to most highly stressed fiber.
- \( l \) = length of member.
- \( I \) = moment of inertia in direction of bending.
- \( E \) = modulus of elasticity.

**Illustrative Problem.**—The bottom chord of a truss is 15 ft. long between panel points 1 consists of two 4 × 4 × 5/16-in. angles with vertical legs back to back and separated by a space of \( 3 \frac{3}{16} \) in. Washers and stitch rivets are inserted 2 ft. apart. The axial load is 40,000 lb. If a wind strut carries a load of 3,000 lb. into the chord at right angles to it and 1 \( \frac{1}{2} \) ft. from one end, as shown in Fig. 61, find the maximum resulting stress, neglecting the effect of the deflection. Rivets, \( 5/16 \)-in. dia.

As the angles are fairly closely stitch riveted, it will be assumed that they act as a single section against transverse loading.

Net area of member = \( (2)(2.40) - (2)(0.875) \) \( (0.3125) = 4.25 \) sq. in.

Uniformly distributed stress

\[ f_1 = \frac{40,000}{4.25} = 9,400 \text{ lb. per sq. in.} \]

Wind reaction at end of chord nearest the strut connection = \( (3,000)(13.5)/15 = 2,700 \) lb.

Wind moment at strut connection = \( (2,700)(1.5)(12) = 48,000 \) in.-lb.

Gross moment of inertia of chord about a vertical axis,

\[ I_g = 2 \left[ 3.7 + (2.40)(1.31)^2 \right] = 15.6 \]

Moment of inertia of 2 holes in outstanding legs assuming a 2\( \frac{1}{2} \)-in. gage, approximately,

\[ = (2)(0.875)(0.3125)(2.69)^2 = 3.9 \]

Net moment of inertia = 15.6 - 3.9 = 11.7.

Extreme fiber stress due to wind moment

\[ f_2 = \frac{(48,000)(4.19)}{11.7} = 17,400 \text{ lb. per sq. in.} \]

Total extreme fiber stress, \( f_1 + f_2 = 9,400 + 17,400 = 26,800 \) lb. per sq. in., an excessive stress even considering the usual increase permitted for a combination of dead load, live load and wind stresses.

**CAST-IRON COLUMNS**

**By H. S. Rogers**

**70. Use of Cast-iron Columns.**—Cast-iron columns are suitable only for small buildings of non-fireproof construction. They offer somewhat greater resistance to fire than unprotected steel columns and occupy a minimum of space
in the building, but cast iron is by no means as reliable as steel and the bolted connections of cast-iron columns allow more or less lateral movement which is serious in high buildings.

Columns of this material should not be used with fabricated steel in skeleton construction or under conditions which produce flexural stresses of any magnitude, other than those due to concentrically-loaded column action. The unreliability of cast-iron columns is due to the variation in quality of the material, defects likely to occur in casting, and the difficulty of thorough inspection.

71. Properties of Cast-iron.—Cast-iron has a very high unit compressive strength—usually considered to be about 80,000 lb. per sq. in. This material, however, is not strong in shear or tension, the average ultimate shearing stress being 18,000 lb. per sq. in., and the average ultimate tensile stress 15,000 lb. per sq. in. The ultimate intensity of stress which can be developed in a piece of cast-iron varies with its fineness of grain, and depends largely upon its thickness and the rate of cooling, as well as its composition. The high compressive stresses make it a very desirable material to use in compression, but because of the somewhat treacherous nature of cast-iron, the high compressive stresses found are often misleading. Also, the low shearing and tensile values preclude its use under any condition other than that of direct compression. It does not rust as quickly as steel and resists fire somewhat better, but may, however, be subjected to serious strains because of sudden cooling with water from a fire stream. It is very hard and brittle, and fractures suddenly without warning. No riveted connections should be made to cast-iron. All connections of girders to columns, or column to column, must therefore be made by bolts which impair the rigidity of a structure by the allowance for clearance.

72. Manufacture of Cast-iron Columns.—Cast-iron columns may be cast in sand molds either upon the side or on end. In either case a baked core molded to the dimensions of the inside of the column must be made of sand, flour, and water, and supported within the sand mold. There are practical conditions surrounding every part of the work which will determine the quality of the column produced. Many pronounced defects found in columns are due to the method of pouring used in their manufacture.

If the column is cast on its side, the core will be buoyed up within the mold because of the great difference in density between it and the molten metal. Provision must, therefore, be made to prevent the core from rising toward the top side of the mold, or from being sprung from line so that the mid-portion of the top side of the casting will be thinner than the desired thickness. This defect produced by “floating cores” is one which is frequently found in cast-iron columns. The molten metal rising in the mold carries dirt and air above, in which will form “honeycomb” and “blowholes” along the top side of the column, unless provision is made by vents for the escape of the air. This provision can be made by forcing a wire rod through the mold at intervals. When these difficulties have been overcome, there are still others which may arise due to unequal cooling produced by the manner or speed of pouring, by the condition of part of the mold, or by the unequal radiation in the molds. The last may be due to an unequal uncovering of the mold. Unequal cooling may produce stresses which will crack the column before any load is placed upon it.
The end method of casting avoids some of these difficulties if the molten metal is introduced at the bottom of the mold. The dirt, sand, and air that collect will thus be borne to the top of the mold so that they can be removed, but the pressure produced by the head of molten metal will often be greater than the mold can withstand, if the column is of any considerable length. The defects found in columns cast on end will not, however, be so numerous as those found in columns cast on the side. These defects can be eliminated to some extent by careful foundry work. If not eliminated, they should be caught at the time of inspection.

73. Inspection of Cast-iron Columns.—Cast-iron columns may have defects either in the surface, or within the metal, or may have insufficient strength due to variation in the section of the metal due to displacement of the core. Defects in the surface can be found by a careful examination of the column. Defects within the metal can be discovered by a careful tapping of the column with a hammer, as the honeycomb or sand spots will sound dead. In hollow square or round columns, variation in thickness of the metal can be determined by drilling two or three \( \frac{1}{4}\)-in. holes through the column. If this variation is more than \( \frac{1}{4}\)-in., the column should be rejected. The H-section affords easy access to the surface for inspection and painting, and opportunity to measure the section. Columns with brackets should be carefully inspected at these details, especially if the column has been poured on its side through the bracket.

74. Tests of Cast-iron Columns.—The Department of Buildings of New York City made a series of tests upon cast-iron columns some years ago at the works of the Phoenix Bridge Co. Nine columns were tested to destruction and a tenth to the capacity of the testing machine. Six of the ten columns had a diameter of 15 in., a length of 15 ft. 10 in., and a thickness of shell of 1 in.; two had a diameter of 8 in., a ratio of \( L/d \) equal to 20, and a shell thickness of 1 in.; two had a diameter of 6 in., a ratio of \( L/d \) equal to 20, and a shell thickness of 1 in.

The columns broke at loads varying from 22,700 lb. per sq. in. to over 40,400 lb. per sq. in., the latter being the intensity of stress in one of the 15-in. columns which withstood the total capacity of the machine. The other five 15-in. columns all exhibited foundry dirt, honeycomb, cinderpockets, or blowholes.

75. Cast-iron Column Formulas.—The following are some of the formulas for cast-iron columns used by different authorities:

New York Building Law .......................... \( P/A = 9,000 - 40I/r \)

Cambria Steel Handbook (Round Columns) ...... \( P/A = \frac{10,000}{1 + \frac{800d^2}{L^2}} \)

Cambria Steel Handbook (Rectangular Columns) \( P/A = \frac{10,000}{1 + \frac{1,067w^2}{L^2}} \)

Chicago Building Law .......................... \( P/A = 10,000 - 60I/r \)

Boston Building Law ............................ \( P/A = 11,300 - 30I/r \)

Watertown Arsenal Tests ........................ \( P/A = 34,000 - 88I/r \)

In these formulas \( r \) is the radius of gyration, \( d \) is the outside diameter, and \( w \) is the least lateral dimension. The formulas are all for flat ends, and all but one are for working loads. The Watertown Arsenal Tests formula, which is for ultimate loads, is quoted by J. B. Johnson, who says it fits very well the results obtained on certain tests made on full sized cast-iron columns.
In order to compare the above column formulas, the allowable unit load has been calculated for a column 15 ft. long, outside diameter 10 in., and inside diameter 8 in. These dimensions give a radius of gyration of 3.2 and a slenderness ratio of 56.2. The results are as follows:

<table>
<thead>
<tr>
<th>Formula</th>
<th>P/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York Building Law</td>
<td>6,750</td>
</tr>
<tr>
<td>Cambria Steel Handbook</td>
<td>9,020</td>
</tr>
<tr>
<td>Chicago Building Law</td>
<td>6,630</td>
</tr>
<tr>
<td>Boston Building Law</td>
<td>9,615</td>
</tr>
<tr>
<td>Watertown Arsenal Tests</td>
<td>7,250</td>
</tr>
</tbody>
</table>

A factor of safety of 4 was used with the Watertown Arsenal Tests formula.

The results indicate that the Cambria Steel Handbook and the Boston Building Law formulas give results which are probably somewhat too high. Any one of the other three formulas would be a safer one to use in design.

76. Design of Cast-iron Columns. — The sections of cast-iron columns in general use are shown in Fig. 62. The hollow cylindrical section gives the best distribution of metal in a column, but the connection details do not work as nicely as those for the hollow square section, which is almost as efficient in distribution of material. The hollow square section, on the other hand, has disadvantages which are not found in the hollow cylindrical section. The corners of the square section are very liable to crack, due to the cooling of the column; but this can be obviated by an outside curved corner and an inside fillet. The H-section, though not affording a distribution of material as efficient as the hollow cylindrical or hollow square column, has the advantages of being open to inspection, of being cast without a core, and of being easily built into a brick wall. It meets the greatest favor as a wall column.

In all the formulas given in the preceding article it will be noted that the area, \( A \), and the radius of gyration, \( r \), both appear in the formula. Therefore, for the ordinary column, in a design problem, the designer is confronted with two unknowns, neither of which can usually be expressed in terms of the other. For this reason it is necessary to design columns by trial and error methods. The procedure involves choosing a size of column which is assumed to be satisfactory, and then calculating the load which it can carry. If the column is too small or too large, then the dimensions must be increased or decreased, respectively, and a second trial calculation must be made.

The following specifications should be observed in the design of the shafts of cast-iron columns:

The minimum thickness of the shell should not be less than \( \frac{3}{4} \) in.; the maximum thickness should not be greater than \( 1\frac{3}{4} \) to \( 2\frac{1}{2} \) in.

The maximum diameter should not be greater than 16 in.; the minimum diameter should not be less than 5 or 6 in.

The slenderness ratio, \( L/r \), should not exceed 70; the unsupported length of the column should not exceed 20 times the least diameter.

All corners should be filleted\(^1\) with a radius of \( \frac{1}{4} \) to \( \frac{3}{8} \) in.

\(^1\) By J. B. Kommens.
No inside offset nor any sudden change in the thickness of shaft should be made.

**Illustrative Problem.**—A hollow, round cast-iron column is 16 ft. long, flat ended, and is to carry safely a load of 200,000 lb. The New York Building Law formula is to be used.

One rough method of making a first guess as to the size required is to assume that the column is a short compression member with \( I/r \) equal to zero. According to the formula this would mean that the unit load could be 9,000 lb. per sq. in. The area required on this basis is \( \frac{200,000}{9,000} = 22 \text{ sq. in.} \) This is known to be too small, so that an area of 30 sq. in. will be chosen for a first trial. Assuming an outside diameter of 10 in., \( \pi (10^2 - d^2) = 30 \), \( d = 7.85 \text{ in.} \), and an inside diameter of 8 in. will be used. For the diameters of 10 and 8, the radius of gyration is 3.2 in. and the area is 28.3 sq. in. From the formula the load

\[
P = 28.3 \left[ 9,000 - \frac{(40)(16)(12)}{3.2} \right] = 187,000 \text{ lb.}
\]

which is a little too small. If the thickness of the column is increased to 1\( \frac{1}{4} \) in., the inside diameter will be 7.75 in., the radius of gyration will be 3.16 in. and the area will be 31.4 sq. in. Substituting in the formula,

\[
P = 31.4 \left[ 9,000 - \frac{(40)(16)(12)}{3.16} \right] = 206,000 \text{ lb.}
\]

which is satisfactory.

The handbooks published by the steel companies contain tables of safe loads for various sized columns, so that the work of computation may be considerably reduced by using these tables.

**77. Column Caps and Bases.**—Hollow cylindrical and square cast-iron columns are generally fastened together by a simple flanged base and cap as shown in Fig. 63a and 63b. The flanges should not be thinner than the shaft of the column and should be at least 3 in. wide; which width will be sufficient for hexagonal nuts on \( \frac{3}{4} \)-in. bolts. These flanges should be faced at right angles to the axis of the column. The bolt holes in the flanges should be drilled to a template so that the columns can be fitted together in proper alignment and the flanges should be spot-faced at bolt holes so that they will give a square firm bearing to bolts and nuts. If the ends of cast-iron columns must be left rough, sheets of lead or copper should be placed between flanges of columns bolted together, so that an even bearing will be obtained by the soft metal taking up the inequalities of the surface. In no case should shims be used to wedge up one side of a column.

If it is desired to give any architectural pretensions to the caps or bases of cast-iron columns, the design of such should be made so as not to weaken the shaft section of the column by change of dimensions or offsets that will throw transverse stresses into the column. Ornamental caps or bases of large size should be cast separate from the column.

**78. Bracket Connections.**—The usual forms for the connections of beams and girders of cast-iron columns are shown in Fig. 63c, \( d \), and \( e \) and in the table
of "Manufacturers' Standard Cast-iron Column Connections." The beam rests upon the bracket shelf and is bolted to the lug on the column through the web. The holes in the web of the beam for bolting to the lugs should be drilled in the field in order to match the cored holes of the lug.

Connections should be designed with a bracket directly below the web of a single girder or below each web of a box girder so that no transverse bending strains will be thrown into the bracket shelf. The bracket shelf should be given a slope of \(\frac{1}{8}\) in. to the foot away from the column so that the load cannot be applied at the end of the shelf. A bracket will bear only about one-half as great a load applied eccentrically at the edge of the shelf as one distributed over the shelf. A bracket shelf may fail in one of three ways, (1) by shearing through shelf and bracket next to the column, (2) by transverse bending, or (3) by tearing out a section of the column as shown in Fig. 63f.

### Manufacturers' Standard Cast-iron Column Connections
Dimensions in Inches

<table>
<thead>
<tr>
<th>Depth of beam</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>K</th>
<th>Thickness of lugs</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>10(\frac{1}{2})</td>
<td>1(\frac{1}{4})</td>
<td>1(\frac{1}{2})</td>
<td>2</td>
<td>(\frac{1}{2})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>10(\frac{1}{2})</td>
<td>1(\frac{1}{4})</td>
<td>1(\frac{1}{2})</td>
<td>2</td>
<td>(\frac{1}{2})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>3(\frac{1}{2})</td>
<td>5(\frac{1}{2})</td>
<td>9(\frac{1}{2})</td>
<td>1(\frac{1}{2})</td>
<td>2</td>
<td>(\frac{1}{2})</td>
<td>1(\frac{1}{4})</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>3</td>
<td>4(\frac{1}{4})</td>
<td>7(\frac{1}{4})</td>
<td>1(\frac{1}{2})</td>
<td>2</td>
<td>(\frac{1}{2})</td>
<td>1(\frac{1}{2})</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

### Holes cored for \(\frac{3}{4}\)-in. bolts

<table>
<thead>
<tr>
<th>Depth of beam</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>K</th>
<th>Thickness of lugs</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>3(\frac{3}{4})</td>
<td>3(\frac{3}{4})</td>
<td>4</td>
<td>7</td>
<td>1(\frac{1}{4})</td>
<td>1</td>
<td>2</td>
<td>1(\frac{1}{2})</td>
<td>1(\frac{1}{4})</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1(\frac{1}{2})</td>
<td>1(\frac{1}{2})</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>2(\frac{3}{4})</td>
<td>3</td>
<td>4</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1(\frac{1}{2})</td>
<td>1(\frac{1}{2})</td>
<td>1(\frac{1}{4})</td>
</tr>
<tr>
<td>7</td>
<td>2(\frac{3}{4})</td>
<td>2(\frac{3}{4})</td>
<td>4</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1(\frac{1}{2})</td>
<td>1(\frac{1}{2})</td>
<td>1(\frac{1}{4})</td>
</tr>
</tbody>
</table>
Tests by the Building Department of New York City have shown that brackets will not fail by shear or transverse bending on columns of more than 6-in. diameter if designed according to standard practice. Of 22 brackets tested, those on 8- or 15-in. columns failed by tearing holes in the body of the column, and 4 on 6-in. columns failed by shearing or transverse stress.

The design of bracket shelves by any rigorous analytical method is impossible. Some of the factors which complicate it are the rate of cooling, variations in the thickness of metal, and imperfections. The design should, however, be checked against failure due to shear or transverse bending.

STEEL COLUMNS

By J. B. Kommers

79. Steel Column Formulas.—A diagram of the allowed unit stresses for structural-steel columns as given by the principal column formulas which have received sanction among engineers is shown in Fig. 64, given by C. E. Fowler, Eng. News-Rec., Feb. 13, 1919. The formulas graphically represented are as follows:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Value (ps/lin. ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Am. B.</td>
<td>19,000 – 100L/(r)</td>
</tr>
<tr>
<td>A.R.E.A.</td>
<td>16,000 – 70L/(r)</td>
</tr>
<tr>
<td>A.R.E.A., 1919</td>
<td>13,000 – 0.25(L/(r))^2</td>
</tr>
<tr>
<td>E.I.C.</td>
<td>12,000 – 0.3(L/(r))^2</td>
</tr>
<tr>
<td>F., 1893</td>
<td>12,500 – 41(\frac{1}{2})(L/(r))</td>
</tr>
<tr>
<td>F., 1919 (Cl. A.)</td>
<td>15,000 – 60L/(r)</td>
</tr>
<tr>
<td>F., 1919 (Cl. B.)</td>
<td>20,000 – 80L/(r)</td>
</tr>
<tr>
<td>McK-F.</td>
<td>12,500 – 50L/(r)</td>
</tr>
<tr>
<td>N. Y. (Old)</td>
<td>15,200 – 58L/(r)</td>
</tr>
<tr>
<td>B.</td>
<td>16,000/1 + (L^2/20,000)</td>
</tr>
<tr>
<td>G.</td>
<td>12,500/1 + (L^2/36,000)</td>
</tr>
<tr>
<td>P.</td>
<td>16,250/1 + (L^2/11,000)</td>
</tr>
</tbody>
</table>

The limitations of the formulas as to maximum unit stresses and maximum values of \(L/\(r\)\) are shown by the diagram. All of the formulas lie in a diagonal zone, the upper limit of which is 18,000 – 60L/\(r\) and the lower limit of which is 12,000 – 60L/\(r\) with the exception of Fowler’s 1919 (Cl.B.). The average of the zone would be 15,000 – 60L/\(r\) which is the formula that has been adopted in a 1919 edition of “General Specifications for Steel Roofs and Buildings” by C. E. Fowler. The A.R.E.A. formula, 16,000 – 70L/\(r\), with a maximum stress of 14,000 lb. per sq. in. and maximum limit of \(L/\(r\)\) at 120 has received very wide sanction in building codes, being found in the codes of New York, Detroit, Chicago, St. Louis, and Seattle.

80. Forms of Cross-section.—For economy, the radius of gyration of the section should be as large as possible. This makes it desirable to place as much of the material as possible as far from the axis of the column as is consistent with good design. The hollow cylinder is theoretically the most economical form of column cross-section, for in this form all of the material is at a maximum distance from the axis.

1 By Clyde T. Morris.
Steel pipe columns are frequently used for light loads where the loads are quiescent and there is no probability of a lateral component to the forces acting on the column. The caps and bases of these are usually cast-iron and the use of this form of column has the same limitations as that of cast-iron columns.

Figure 65 shows the more common forms of cross-section for steel columns and struts.

Struts of 2 angles (Fig. 65a) are commonly used for light lateral bracing. The section is unsymmetrical and for this reason is undesirable for main compression members. Columns composed of 2 channels laced (Fig. 65g, h, and k) or 2 pairs of angles laced (Fig. 65b) are not as rigid in the plane of the lacing as those in which the parts are connected by plates. Care should be used in proportioning the lacing in such columns. Types i and l are forms which are commonly used for top chords and end posts of bridges. The lattice on the lower side permits access for cleaning and painting. The Bethlehem H-section (Fig. 65e and f) is a form much used in building work. Type e without cover plates is very economical on account of the small amount of fabrication necessary. Type f is much more expensive as it is necessary to drill the holes in the heavy flanges of the H-section for riveting on the cover plates. These flanges are too thick to punch. Z-bar columns (Fig. 65g and r) are seldom used in modern structures. The Grey column (Fig. 65s) and the 4-angle column (Fig. 65t) are frequently used in combined steel and concrete columns.

81. Design of Cross-section.—The method of designing steel columns is quite similar to that used for cast-iron columns. Here also the radius of gyration and the area both appear in the formulas, and it is usually not possible to express one in terms of the other. This means that a column size must be chosen and tried out to determine whether it is satisfactory.

The nature and size of the work will determine whether a single structural shape may be used as a column, or whether several parts must be riveted together to obtain sufficient area. The method used in design will be illustrated by working a problem using an I-beam, and a second problem using a section built up of two channels and two plates.

Illustrative Problem.—A steel column is 8 ft. long, flat ended, and is to carry safely a load of 100,000 lb. The American Railway Engineering formula, \( P/A = 16,000 - 70l/r \) is to be used.
If the column were a short compression member with slenderness ratio zero, it could carry a unit load of 16,000 lb. per sq. in. On this basis an area of \( \frac{100,000}{16,000} = 6.25 \text{ sq. in.} \) would be required. This is known to be too small. Therefore, a size will be chosen from the tables of standard I-beams giving a larger area. A 9-in. 30-lb. I-beam has an area of 8.82 sq. in. and a least radius of gyration of 0.85. Substituting in the formula

\[ P = 8.82 \left[ 16,000 - \frac{70)(8)(12)}{0.85} \right] = 71,400 \text{ lb.} \]

which means that the I-beam is too small. To carry 100,000 lb. the area of the above column would have to be increased about 40 per cent, therefore the next choice will be a 12-in. 40-lb. I-beam with an area of 11.76 sq. in. and a least radius of gyration of 0.90. Substituting in the formula

\[ P = 11.76 \left[ 16,000 - \frac{70)(8)(12)}{0.90} \right] = 100,000 \text{ lb.} \]

which is satisfactory.

In designing columns it is good practice to calculate the slenderness ratio after choosing a size for a tentative design, in order to determine whether the column is a long column or not. In the above problem, if the column had been 15 ft. long the slenderness ratio for the first tentative design would have been 212. This means that the column is a long column and that therefore the straight line formula cannot be used. In such a case Rankine’s or Euler’s formula may be employed.

Illustrative Problem.— A steel plate and channel column is to be designed to carry a load of 300,000 lb. The column has flat ends and is 20 ft. long. The Milwaukee Building Law formula \( P/A = 17,100 - 57/r \) is to be used.

The standard plate and channel columns, taken from the steel handbooks, will be used, because the work of computation is greatly simplified when the area and least radius of gyration can be found in the handbook tables. For \( l/r = 0 \), a unit load of 17,100 lb. per sq. in. could be carried, so that on this basis an area of \( \frac{300,000}{17,100} = 17.6 \text{ sq. in.} \) will be required. Since this is known to be small, the first choice will be a column of 19.93 sq. in. area, made up of two plates 5\( \frac{1}{8} \)-× 9-in. and two 7-in. channels, each weighing 14.75 lb. per ft. For this column the least radius of gyration is 2.53 in. Substituting in the formula,

\[ P = 19.93 \left[ 17,100 - \frac{57)(20)(12)}{2.53} \right] = 233,000 \text{ lb.} \]

which means that the column is too small. To carry 300,000 lb. the above column area should be increased about 30 per cent. Therefore, the next choice will be a column made up of two plates 5\( \frac{1}{8} \)- × 10-in. and two 8-in. channels, each weighing 21.25 lb. per ft. The area in this case is 25 and the least radius of gyration is 2.80. Substituting in the formula

\[ 25 \left[ 17,100 - \frac{57)(20)(12)}{2.8} \right] = 305,000 \text{ lb.} \]

which is satisfactory.

82. Eccentrically Loaded Columns.—When a column carrying direct loading is also subjected to bending moment due to the column load, or any part of it, being applied away from the axis of the column, the resulting fiber stresses may be determined by the formulas given in the chapter on Bending and Direct Stress in Sec. 1. The fiber stress may also be determined from the equation in Sec. 1, Art. 80, p. 132, which may be written in the form

\[ f = \frac{P}{A} + \frac{Mc}{KI} \]
Values of $K$ for pin-ended columns are given in Fig. 66. For columns with fixed ends use one-half the column length in determining values of $l/r$ for use in Fig. 66.

**Values of $l/r$**

**Fig. 66.**—Use for eccentrically loaded columns with pin ends. For columns with fixed ends use $l/2r$ in determining $l/r$.

In all cases the radius of gyration should be taken about an axis normal to the plane of bending. Note that this value of $r$ may not give the greatest value of $l/r$ which should be used in the column formula.

**Illustrative Problem.**—Figure 67 shows a building column to which floor beams are connected unsymmetrically, causing an eccentric load on the column. Determine the fiber stress in the column section. Solve by means of eq. (1), and also by means of the formulas given in the chapter on Bending and Direct Stress in Sec. 1.

If the beams are riveted to the column in addition to resting on shelf angles, it is safe to assume that the load is applied at the face of the column. The deflection of the shelf angles would probably be sufficient to bring the center of pressure very near to the face of the column in any case.

The total load, $P = 90,000 + 32,000 + 32,000 + 40,000 = 194,000$ lb.

The bending moment, $M = (40,000)(3)^2(4) = 235,000$ in.-lb.

**Solution by eq. (1):**
For the given conditions \( l = 192 \), \( r = 5.13 \) = 38. From Fig. 66 with \( l_r = 38 \) and \( P_A = \frac{194,000}{19} = 10,210 \), we find \( K = 0.935 \). Then from eq. (1),
\[
\dot{f} = \frac{194,000}{19} + \frac{235,000(5.875)}{499(0.935)(499.0)}
\]
\[
= 10,210 + 2,960 = 13,170 \text{ lb. per sq. in.}
\]

**Solution by eq. (5), p. 139:**
\[
\dot{f} = \frac{194,000}{19} + \frac{235,000(5.875)}{499}
\]
\[
= 10,210 + 2,760 = 12,970 \text{ lb. per sq. in.}
\]

**Solution by eq. (14), p. 143:**
Since the ends of the column are probably partially fixed, use \( C = \frac{1}{\sqrt{10}} \).
\[
M = \frac{235,000}{1 - \frac{(194,000)(16)(12)}{(10)(30,000,000)(499)}} = 246,000 \text{ in.-lb.}
\]

Then
\[
\dot{f} = \frac{194,000}{19} + \frac{246,000(5.875)}{499}
\]
\[
= 10,210 + 2,890 = 13,100 \text{ lb. per sq. in.}
\]

Since the results given by these three solutions are practically identical, we conclude that the second solution is preferable because it is more simple than the others.

### 83. Column Details

No element of a column should be left in a condition which will make it possible for this element to fail locally. A column made up of several parts must be so designed that no element can fail as a column between the rivets attaching it to the adjacent column parts. It is evident from Fig. 64 that if the slenderness ratio for any element is made less than about 40 to 50, this possibility of failure will be provided against. Specifications cover this matter by prescribing rules for the pitch of rivets for lacing, the pitch of rivets for attaching plates, the thickness of side plates and cover plates, and the maximum pitch of rivets which attach plates to other shapes.

### 84. Shear in Column

Because a column fails partly by direct stress and partly by bending, it is necessary to make provision for the shear which is produced in a column because of the bending. In a column made up of two channels latticed together it is necessary to design the details so that the column may act as a unit. These details must be designed so that the column will have the necessary stiffness as well as the necessary strength.

It may be well to recall that when two wooden beams, each 4 × 4 in. in cross-section, are placed one on top of the other to form a beam, the strength is proportional to the section modulus of two 4- × 4-in. beams, or \( \frac{(2)(4)^3}{6} = 21.3 \). If, however, instead of separate beams a solid beam 4 × 8 in. is used, then the strength is proportional to \( \frac{(4)(8)^2}{6} = 42.7 \). In the second case the beam is twice as strong as the two separate beams because of the horizontal shearing stresses which it can resist. The same effect could have been produced by fastening the two 4- × 4-in. beams together in some other way so that they would act as a unit.

In a latticed column the lacing bars must provide the material for taking care of the shearing stresses which are developed because of the bending which occurs. If the column is made up of two 15-in. 40-lb. channels, it will have a strength equal only to twice the strength of a single channel unless they are properly fastened together. Two such single channels 12 ft. long could carry a load of 114,600 lb.
according to the Carnegie Steel Company handbook. If, however, the channels are laced together they will develop a strength of 377,000 lb.

Shear in columns may occur in the case of a rather long column bent into a single loop (Fig. 68), in the case of a short column bent in double curvature due to the fact that the load is applied with opposite eccentricity at the two ends (Fig. 69), or in the case of a short column subjected to secondary bending moments.

One method of estimating the shear in a column is that specified by the specifications of the American Railway Engineering Association. These require that the shear be calculated as equal to that produced by a uniformly distributed load, assuming that the column is loaded as a beam, and that the bending stress produced is equal to that assumed in the column formula. The American Railway formula is \( P/A = 16,000 - 70 \, l/r \). Using the form \( P/A + 70 \, l/r = 16,000 \), it is evident that the direct stress, \( P/A \), plus the bending stress, \( 70 \, l/r \), is limited to 16,000 lb. per sq. in. The bending moment produced by a uniformly distributed load, \( W \), is \( \frac{WL}{8} \). Therefore,

\[
\frac{WL}{8} = \frac{70 \, l/r \, I}{c}
\]

From which

\[
W = \frac{560 \, I}{rc} = \frac{560 \, Ar}{c}
\]

For such a beam the maximum shear at the ends is \( V = \frac{W}{2} \). Hence

\[
V = \frac{280 \, Ar}{c}
\]

In which

- \( A = \) area of cross-section, in square inches.
- \( r = \) radius of gyration of cross-section, in inches.
- \( c = \) distance from bending axis to extreme fiber, in inches.

Since this formula was developed from a safe load formula, it may be used for working conditions.

85. Design of Latticing.—When the shear which must be provided for in a column is known the lattice bars may be so designed as to take care of this shear.

In a column with single lacing on each side, Fig. 70, the total stress in each lattice bar is

\[
F = \frac{V}{2} \cdot \frac{a}{b}
\]

in which

- \( a = \) length of the lattice bar.
- \( b = \) width between rows of rivets.

This follows from the fact that if \( F \) is the total stress in the lattice bar it will have a horizontal component of \( F \frac{b}{a} \), and the equilibrium equation is

\[
2F \frac{b}{a} = V
\]
An alternative method makes use of the formula developed for the horizontal shearing unit stress in a beam, which was

\[ v = \frac{VQ}{It} \]

in which

\( v \) = shearing unit stress, horizontal or vertical.
\( V \) = total shear at the section.
\( Q \) = statical moment of one-half the area of the cross-section with respect to the neutral axis.
\( I \) = moment of inertia of the cross-section.
\( t \) = thickness of beam at neutral axis.

If the unit shear is uniform, then the total shear for 1 in. along the beam, either horizontally or vertically, would be \( (v) (1) (t) \). From this it is evident that the shear per lineal inch is \( \frac{VQ}{I} \). If the lacing is the same on both sides, then the total shear carried by a section covered by one lacing bar is \( \frac{VQb}{2I} \). If the force in the bar is \( F \), then again, for single lacing

\[ F = \frac{VQb}{2I} \]

and

\[ F = \frac{VQa}{2I} \] \hspace{1cm} (4)

If the lacing is double, as in Fig. 71, then

or

\[ F = \frac{VQa}{4I} \] \hspace{1cm} (5)

\[ F = \frac{VQa}{4I} \] \hspace{1cm} (6)

If a column has three webs, as in Fig. 72, then again the shear per lineal inch is \( \frac{VQ}{I} \); and, if the lacing is the same on both sides, then the total shear carried by a section covered by one lacing bar is \( \frac{VQb}{2I} \). Here \( Q \) = statical moment of the outer rib section with respect to the column center.

If the force in the bar is \( F \), then for horizontal equilibrium,

\[ Fh = \frac{VQb}{2I} \]

and

\[ F = \frac{VQa}{2I} \] \hspace{1cm} (7)

**Illustrative Problem.**—A column 15 ft. long is composed of two 15-in. 45-lb. channels placed as shown in Fig. 73. Determine the size of the lacing bars required.

Equation (2) may be used to estimate the shear carried by the lacing bars. For the given column \( A = 26.48 \) sq. in., \( r = 5.48 \) in., and \( c = 8 \). Hence

\[ V = \frac{(280)(26.48)(5.48)}{8} = 5,100 \text{ lb.} \]
Since the column has single lacing, eq. (4) is to be used. For the given column \( b = 14\frac{3}{4} \text{ in.} \), and \( a = 17 \text{ in.} \). The stress in a lacing bar at the end of the column is then

\[
F = \frac{(5.100)(17)}{(2)(14.75)} = 2,940 \text{ lb.}
\]

This stress may be either tension or compression.

For the given channels the usual specifications require a \( \frac{3}{8} \)-in. rivet. The minimum size of lacing bar is generally taken as \( 2\frac{1}{2} \times \frac{3}{8} \text{ in.} \). Assuming a working stress in tension of 16,000 lb. per sq. in., the net area required for the lacing bar is

\[
2,940 \quad \frac{16,000}{0.184} = 0.184 \text{ sq. in.}
\]

The net area provided, allowing for a 1-in. rivet hole, is

\[
(2\frac{1}{2} - 1)(\frac{3}{8}) = 0.562 \text{ sq. in.}
\]

Since the ends of the bars are rigidly fastened, the length used in the column formula may be taken as half the length of the bar. The allowable working stress for the \( 2\frac{1}{2} \times \frac{3}{8} \)-in. bar is then

\[
16,000 - 70 l = 16,000 - \frac{(70)(8.5)}{0.1082} = 10,500 \text{ lb. per sq. in.}
\]

and the area required is

\[
2,940 \quad \frac{10,500}{0.28} = 0.28 \text{ sq. in.}
\]

The area furnished is \( (2\frac{1}{2})(\frac{3}{8}) = 0.937 \text{ sq. in.} \). Hence the assumed bar is larger than required, but since it is the minimum allowed by good practice it will be adopted.

86. Design of Tie-plates and Forked Ends.—At the ends of compression members the lacing is generally replaced by tie-plates, as shown in Fig. 74. These plates act as lacing and in addition they hold the segments of the member rigidly in line, and assist in transmitting the stress uniformly over the cross-section of the column.

The end connections for compression members, whether riveted or pin connected, are generally so constructed that moments are set up due to eccentricity of application of the applied load. Thus in Fig. 74a, the usual type of connection is so arranged that the stress at the end of the member is transmitted to the web of the channel at lines \( b-b \). In the body of the member the load may be considered as applied at the center of gravity of the segments, as shown by the lines \( a-a \). The moment due to eccentricity is then the load on that segment times the distance between lines \( a-a \) and \( b-b \). This moment must be resisted by the tie-plate and the rivets connecting the tie-plate to the segments of the member.

Tie-plates should be placed as near the end of the member as possible. The loads should be transferred from the member to the joint or bearing plates as outlined in the chapter on Column Bases which follows. In pin-connected structures
it often happens that the tie-plates cannot be placed at the ends of the members due to interference with other members at the joint. This makes necessary the use of a forked end, as shown in Fig. 74b. These forks must be designed to carry the shear to the tie-plate and lacing. The total moment carried by both forks is \( Vc \), where \( V \) = shear determined as in Art. 84. The distribution of the shear \( V \) between the two forks is indeterminate. It will probably be best to design each fork for a moment equal to \( \frac{1}{3} Vc \). If the forks are overstressed, they may be strengthened by side plates placed on the webs of the channels and extending beyond the edge of the tie-plate.

**COLUMN BASES**

By C. R. Young

87. Types and Uses.—To transmit the load of a column to the masonry without exceeding the safe bearing pressure on the latter, the lower end of the column must be enlarged by constructing a base for it. This may be either an independent construction or an enlargement of the column itself.

![Fig. 75.—Cast bases for columns.](image)

If the former, it may be a separate steel plate or slab; it may be an iron or steel casting, as shown in Fig. 75a, b or c; or it may be a built up steel bolster or grillage, such as shown under the casting in Fig. 15. The solid tapered cast plate, shown in Fig. 75a, can be used for only light loads, and, if its thickness would exceed about 4 in., it should be replaced by a ribbed pedestal. This may be either rectangular or circular, usually the former. The advantage of the separate base is that is can be placed and levelled much more easily than can a column with a base riveted to it. Steel grillages are preferred by some engineers as being more reliable than cast-iron bases, and cheaper than cast steel ones. They lend themselves well to situations where long narrow bases must be provided and where the bending moment on them is very large.

Steel columns resting on separate plates, slabs, or cast bases require at most only side connection angles to the base merely to hold them in position. For
light columns not subjected to lateral forces or uplift, no connection between the column and the separate base is required.

If the base is riveted to the bottom of the column, forming an enlargement of it, the spread must be large, and projecting side angles must be used, supplemented perhaps by side plates and by stiffener angles. The type shown in Fig. 76a is the simplest of these, consisting of a base plate and two pairs of side angles. Type b shows the addition of distributing gussets or side plates; type c shows the further addition of stiffener angles to assist in the distribution of load; while the type illustrated in Fig. 78 shows a base with stiffener angles arranged to transfer the pull of anchor bolts to the column shaft. This latter type is used only where an uplift on the column is likely to occur.

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88. Design of Plain Bases.—To find the size of any base, the load to be transferred by it must be divided by the allowable pressure on the masonry as fixed by the specification. If the base consists of a plain steel plate or slab of rectangular shape, the thickness may be determined by figuring the maximum moment on the plate, or slab, and applying the common flexure formula to the dangerous section. If it is assumed that the moment is a maximum at the center of the base—an assumption on the side of severity—the moment in the direction of the length is

\[ M = \frac{1}{6} W (L - l) \]  

and in the direction of the width

\[ M = \frac{1}{6} W (B - b) \]

where \( W \) = total upward reaction on base.

\( L \) = length of slab in inches.

\( B \) = breadth of slab in inches.

\( l \) = outside dimension of column parallel to \( L \).

\( b \) = outside dimension of column parallel to \( b \).
If the moment be taken at the edge of the column shaft, corresponding formulas may be readily written. The whole width of the plate or slab is assumed to be effective in calculating the resistance.

Knowing the bending moment on the base, the thickness may then be found.

89. Design of Ribbed Cast Bases.—Having found the required size of the base in plan, and having fixed the dimensions of the top plate so as to provide the necessary area to receive the column and to accommodate the connecting bolts, the height of the base, the number and arrangement of ribs, and the thicknesses of all parts must be determined.

It has been found that the height of cast bases is best made between one-third and one-half the side of the bottom plate. This enables the upper edges of the ribs to be sloped down at approximately an angle of 45 deg. If the slope is appreciably flatter than this to the horizontal, the flexural stresses in the part of the base projecting past the upper plate become high and lessen the efficiency of the base. For the same reason it is found that as the reacting pressure in pounds per square inch increases, the economical ratio of height to side of the base also increases.

The arrangement of the ribs underneath the column shaft should correspond as closely as possible to the shape of the column, so that the pressure may be taken directly down to the bottom plate without putting much flexure in the top plate. The plan views of the bases shown in Fig. 75b and c show how two bases were arranged to suit an H-column. Frequently, a circular hub is provided at the center of the casting, as in Fig. 75c, with ribs radiating to the edges. A portion rib across the space within this hub is provided, if the column has one central web, so as to receive the load from the web. The number of ribs to be provided will depend on the size of base and the load to be carried. There may be as many as 16 radiating from the center to the edges. The sectional area of the ribs should be such that the portions under the load will take the vertical load safely as short columns or prisms. It is best to limit their ratio of clear height to thickness to about 15 and proportion for a compressive stress of not over 8,000 lb. per sq. in. if they are of cast-iron. The heaviest ribs are, of course, under the load. Those radiating to the edges are the thinnest. In no case should ribs be thinner than 1 in.

The bottom plate is proportioned as a beam continuous under the various ribs. Between ribs, it should be calculated as a restrained beam at the allowable tensile flexural stress for cast iron, or about 3,000 lb. per sq. in., if the base be of this material. The projecting portions should be calculated as cantilevers and similarly proportioned. To strengthen the edges of the bottom plate, a flange is frequently provided around the outer edge, as shown in Fig. 75c. This is commonly from 3 to 5 in. deep overall. The bottom plate itself may be from 1 to 3 in. thick.

To test the sufficiency of both the ribs and the bottom plate, the moment on the projection past the edge of the top plate on one of the four sides should be calculated, and the moment of resistance of the section cut by a vertical plane passing through this edge should be computed and compared with the bending moment. The moment of resistance is found in the same manner as that of a cast-iron lintel, Art. 30, p. 231.
To ensure that the pressure is uniformly applied to the top of the base, it should be planed. If it rest on a steel grillage, both the latter and the bottom of the base should be planed.

Holes should be left through the bottom plate to enable grout to be poured in under the base plate after it is brought to the required height and levelled.

90. Design of Built-up Bases.—The size of the base plate for a built-up base is found by dividing the total load by the permissible bearing on the masonry. Its thickness should be sufficient to withstand the upward uniform pressure without exceeding the allowable flexural stress on steel, or without undue deflection. The portions that project farthest past the column shaft, or span the greatest distances between column flanges or side plates, should be investigated as cantilever or continuous beams, as the case may be. The thickness of base may vary from $\frac{3}{4}$ in. for light angle columns to $1\frac{1}{2}$ in. for very heavy columns. It is frequently somewhat less in practice than a strict calculation of bending stresses would warrant.

To attach the base plate to the column shaft, one or two pairs of angles may be used, two pairs being used for the larger columns. These transfer pressure to the base plate up to the limit of capacity of the rivets that attach them to the column shaft. The strength of the outstanding leg in flexure may need to be investigated to discover if the angle can transfer outward at right angles to its length, the load that its connecting rivets would warrant. The thickness of angles commonly used varies from $\frac{3}{8}$ to $\frac{3}{4}$ in. The length of vertical leg is commonly 6 in., but the horizontal leg is usually $3\frac{1}{2}$ or 4 in.

Side plates are from $\frac{5}{16}$ to $\frac{1}{2}$ in. thick and should be attached by sufficient rivets to the column shaft to ensure that the load which they are supposed to transmit to the base plate may be developed safely. It should be remembered that the rivets through both the base angles and the side plates have to do double duty. The upper edges of the side plates and ends of the base angles riveted over them are usually cut to one slope, as shown in Fig. 76b and c. This is not usually less than 45 deg. with the horizontal.

Stiffeners, where used, are $\frac{3}{8}$ or $\frac{1}{2}$ in. thick with outstanding legs wide enough to cover the outstanding legs of the base angles on which they bear. Their attachment must be sufficient to develop the load they are supposed to transmit.

The proportion of the total column load to be taken by the side plates, base angles, and stiffeners, will depend on how much is assumed as transmitted to the base plate directly by the faced end of the column shaft. This is commonly taken at only 40 or 50 per cent of the total load, so that the side details must account for the other 50 or 60 per cent.

Illustrative Problem.—Design a riveted steel plate and angle base for a 10-in. 49-lb. Bethlehem H-column of the type shown in Fig. 77. The vertical centric load is 170,000 lb. Consider 40 per cent of the total axial load as carried directly to the base plate by the faced end of the column shaft. Rivets, $\frac{3}{4}$ in. Anchor bolt holes $\frac{3}{8}$ in. larger than the bolts. Permissible stresses:

| Bonding | 16,000 lb. per sq. in. |
| Bearing on end of column | 16,000 lb. per sq. in. |
| Shearing on shop rivets | 12,000 lb. per sq. in. |
| Bearing on shop rivets | 24,000 lb. per sq. in. |
| Bearing on concrete | 500 lb. per sq. in. |

Base Plate.—Required area of plate, $A = \frac{170,000}{500} = 340$ sq. in.
To facilitate details, adopt a plate 18 × 19 in., giving an area of 342 sq. in. The 18-in. dimension is made parallel to the web to accommodate two base angles with 3 1/2-in. horizontal legs and two 3/8-in. side plates.

Calculations of the thickness required, assuming the base plate as an overhanging or continuous beam, gives results in excess of the thickness found satisfactory by experience. For a base of this character, the base plate is usually about 3/8 in. This thickness will be adopted.

Side Plates and Base Angles.—Since only 40 per cent of the total axial load is assumed to be transferred to the base plate by the faced end bearing of the column shaft, the remaining 60 per cent, or (170,000)(0.60) = 102,000 lb., must be delivered to the base plates by the side plates and base angles. To make this possible, enough rivets must be placed through the column shaft to develop 102,000 lb. Assume 16 rivets through the flanges, for which the least value (single shear) is (0.44)(12,000) = 5,280 lb., and 4 rivets through the web, for which the least value (bearing on 0.36-in. web) is (0.36)(0.75)(24,000) = 6,480 lb. The total safe resistance of these two groups of rivets, therefore, = (16)(5,280) + (4)(6,480) = 110,500 lb., which is adequate for the load.

Fig. 77.—Design of a built-up base with side plates.

The side plates, which for a column of the section considered should be about 3/8 in. thick, will extend across the full width of the base plate to help transfer load out to its edges, and will be 12 in. deep so as to accommodate two rows of rivets outside the base angles, which are riveted to it.

The base angles riveted to the column flanges are run full width of the base and two rivets are driven through each angle into the side plate. There are, therefore, 6 rivets in single shear through each angle, so that the angles will deliver to the base plate (12)(5,280) = 63,400 lb., or 37 per cent of the total column load, leaving 23 per cent to be delivered by the side plates and the base angles on the web.

Four rivets through the base angles in the column web will develop (4)(6,480) = 25,900 lb., or 15 per cent of the total column load. This leaves only 8 per cent of the total column load to be delivered to the base plate by the two side plates.

All vertical rivets through base angles must be countersunk on the under side. Only sufficient rivets are employed to hold the angles and plate tightly together.

Anchor bolt holes are provided 1/6 in. larger than the anchors, which will be 1 in. diam.

91. Anchorage.—If there be no appreciable lateral force or uplift exerted on columns, the bases do not really need to be anchored down to the masonry. The frictional resistance of the base on the top of the pier, once the column has received as full dead load, is sufficient to prevent it being displaced by blows or shock.
In case there is considerable lateral force exerted on the base, anchor bolts will need to be provided. To resist sliding, their shear value should be equal to the difference between the lateral force and the frictional resistance of the base. As a safeguard against overturning of the column, the anchor bolts should be embedded far enough in the masonry, or sufficiently anchored thereto, to develop the maximum tension likely to come on them. The mass of masonry engaged should weigh at least 1½ times the tension on the bolt.

In order to develop high resistance to overturning, the bolts should be placed as far apart as possible in the direction of the moment.

**Illustrative Problem.**—A column consisting of a 24 × ¾-in. web, two 5 × 3¼ × ¾-in. angles and two 5 × 3¾ × ¾-in. angles with the 5-in. legs outstanding, as shown in Fig. 78, is subjected to an overturning wind moment of 65,000 ft-lb. The minimum axial load is 20,000 lb. applied 1 in. off center on the side towards the wind. Assuming the base plate, side angles, and side plates shown as already fixed, design the anchor bolts and an attachment for them to develop the required tension.

Net overturning moment about leeward edge of base plate, \( M = (65,000)(12) - (20,000)(18) = 420,000 \text{ in.-lb}. \)

If the bolts pass through the outstanding legs of the base angles on the column flange, their distance from the far edge of the plate would be 31½ in.

Tension in windward bolt = 420,000/31.63 = 13,300 lb.

Required area of one bolt at root of thread = 13,300/16,000 = 0.83 sq. in.

One 1¾-in. diam. bolt with a net area of 0.89 sq. in. at root of thread will be adopted.

Shelf angles, 6 × 4 × ¾ in., riveted to the sides of the column, as shown, will take the anchor bolt tension into the column shaft. Two stiffeners, 3½ × 3½ × ¾ in., will be employed under each shelf angle. The rivets through the shelf angles and the stiffeners are ample at any ordinary working stresses to carry the stress into the column.

The embedment of the anchors in the masonry must be such as to develop the tension in them. Each should engage a mass of masonry weighing at least 1½ times the amount of the uplift in it. The pressure on the masonry under the leeward side of the base plate should also be investigated to ensure that the safe bearing pressure on it at the leeward edge of the base plate is not exceeded.